SURAJ BHAN DAV PUBLIC SCHOOL, VASNAT VIHAR

Class: -XII

First Term Examination

Mathematics (Code-041)

Time: 3 hours | Maximum marks: 80

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

Section -A

(Multiple Choice Questions)

Each question carries 1 mark.

Q1. How many diagonal matrices of order 3×3 can be formed by using the numbers 0,1,2 and 3.

(a) 4 (b) 6 (c) 8 (d) 27

Q2. If $A=\begin{bmatrix} a & -1 & -2 \\ b & c & -3 \\ d & e & f \end{bmatrix}$ then $a+2b+3c+4d+5e+6f=$

(a) 22 (b) 23 (c) 24 (d) 25

Q3. If A is a square matrix such that $A^2 = A$ then $(I-A)^2 + A =$

(a) A (b) $I + A$ (c) $I - A$ (d) I

Q4. The symmetric part of $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ is

(a) $\begin{bmatrix} 2 & 2 \\ 2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Q5. The value of k for which the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & k & 0 \\ 1 & 0 & 1 \end{bmatrix}$ is invertible is equal to

(a) 0 (b) 1 (c) 2 (d) -1

Q6. If A is a square matrix of order 2 such that $|A| = 5$ then $A \times \text{Adj}(A) =$

(a) $\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 5 \\ 5 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q7. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ then $|\text{Adj}\, A| =$

(a) 4 (b) 16 (c) 32 (d) 64

Q8. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then $A^{-1} =$

(a) $\begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix}$

Q9. The relation R defined in the set $A = \{1,2,3\}$ by $R = \{(1,2),(2,1),(1,1)\}$ is

(a) Reflexive (b) Reflexive and Symmetric

(c) Symmetric and Transitive (d) Reflexive and Transitive

Q10.The function $f: R \to R$ by $f(x) = |x|$ is

(a) one-one and onto (b) one-one but not onto

(c) onto but not one-one (d) neither one-one nor onto

Q11.The function $f: R \to R$ by $f(x) = x + 1$ is

(a) one-one and onto (b) one-one but not onto

(c) onto but not one-one (d) neither one-one nor onto

Q12.The total number of one-one and onto functions from $A = \{1,2,3\}$ to $B = \{4,5,6\}$ are

(a) 3 (b) 6 (c) 27 (d) 81

Q13.If $\sin^{-1} x + \sin^{-1} y = \pi, -1 \le x, y \le 1$ then $x^2 + y^2 =$

(a) 1 (b) 2 (c) 3 (d) 4

Q14.The principal value of $\sin^{-1} \sin 3$ is given by

(a) 3 (b) $\pi - 3$ (c) $\pi + 3$ (d) $-\pi + 3$

Q15. The principal value of $\cos^{-1} \cos \frac{5\pi}{3}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $-\frac{\pi}{3}$ (d) $-\frac{2\pi}{3}$

Q16. The value of $\sin^{-1}(-1) + \cos^{-1}(1) + \tan^{-1}\sqrt{3} =$

(a) $-\frac{\pi}{6}$ (b) $\frac{\pi}{6}$ (c) $-\frac{\pi}{3}$ (d) $\frac{\pi}{3}$

Q17. If $f(x) = \begin{cases} \frac{\sin x}{x}, x < 0 \\ k + x, x \ge 0 \end{cases}$ is continuous at $x = 0$, then value of k is

(a) -1 (b) 0 (c) 1 (d) 2

Q18. The derivative of $\sin^{-1}\frac{2x}{1+x^2}$ with respect to $\tan^{-1}x$ is

(a) -1 (b) 0 (c) 1 (d) 2

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

Q19. Let $f(x)=x^3+1$, then

Assertion (A) f is one-one function.

Reason (R) A function f is said to be one-one function if $f(x_1)=f(x_2)\Rightarrow x_1=x_2$

Q20. Let $f(x)=\sin x+\cos x, 0\le x\le\frac{\pi}{2}$ then

Assertion (A) f has maximum value at $x=\frac{\pi}{4}$

Reason (R) $f'\left(\frac{\pi}{4}\right)=0$ and $f''\left(\frac{\pi}{4}\right)<0$

Section – B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

Q21. Find the value of $\cos^{-1}\sin\left(\frac{5\pi}{3}\right)$

OR

Prove that $\sin^{-1}2x\sqrt{1-x^2}=2\sin^{-1}x, -\frac{1}{\sqrt{2}}\le x\le\frac{1}{\sqrt{2}}$

Q22. Find the intervals in which the function $f:R\to R$ defined by $f(x)=-2x^3+15x^2-36x+15$ is increasing.

OR

Show that the function $f(x)=x+\frac{1}{x}$ is increasing on $R-[-1,1]$.

Q23. If A and B are invertible matrices of order 3, $|A|=2$ and $\left|(AB)^{-1}\right|=-\frac{1}{6}$. Find $|B|$.

Q24. If A and B are symmetric matrices, such that AB and BA are both defined, then prove that $AB-BA$ is a skew-symmetric matrix.

Q25. Show that the function $f(x)=x^2+x+1$ is neither one-one nor onto.

Section – C

[This section comprises of short answer type questions (SA) of 3 marks each]

Q26. If $\sin^{-1}x+\sin^{-1}y+\cos^{-1}z=2\pi, -1\le x,y,z\le 1$, then find the value of $x^3+y^3+z^3-xyz$.

Q27. Show that the function $f:[-5,5]\to[0,5]$ defined by $f(x)=\sqrt{25-x^2}$ is onto but not one-one.

OR

Let $A=\{1,3,5\}$. Find the number of reflexive rela

Q28. If $y=\left(\sin^{-1}x\right)^2$, prove that $\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-2=0$

OR

If $x\sqrt{1+x}+y\sqrt{1+x}=0$ and $x\neq y, x, y>-1$, prove that $\frac{dy}{dx}=\frac{1}{(x+1)^2}$

Q29. Marginal revenue is defined as the rate of change of total revenue. If the total revenue (in rupees) received from the sale of x units of a product is given my $R(x)=3x^2+36x+5$, find the marginal revenue, when $x=5$

OR

Show that the function defined by $f(x)=(x-1)e^x+1$ is an increasing function for all $x>0$ and decreasing for all $x<0$.

Q30. If $A=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$, find $A^2-5A+4I$ and hence find matrix X such that

$A^2-5A+4I+X=0$

Q31. If area of triangle is 35 square units with vertices $(2,-6),(5,4)$ and $(k,4)$, then find value(s) of k.

Section –D

[This section comprises of long answer type questions (LA) of 5 marks each]

Q32. Let $f(x)=\begin{cases}ax+b & ;0<x\leq 1\\ 2x^2-x & ;1<x<2\end{cases}$ is a differentiable function in $(0,2)$ then find the values of a and b

Q33. If $y = e^{m\sin^{-1}x}$, $-1 \le x \le 1$, then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$

OR

If $x = a\sin t$ and $y = a\left(\cos t + \log\tan\frac{t}{2}\right)$, find $\frac{d^2y}{dx^2}$.

Q34. An open box with a square base is to be constructed out of a given quantity of cardboard of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

OR

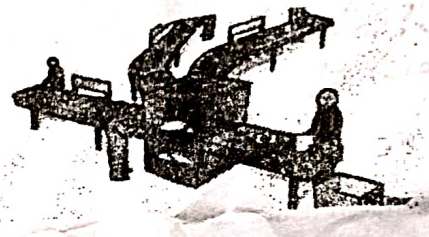
Find the intervals in which the function $f(x) = (x-1)^3(x-2)^2$ is increasing or decreasing.

Q35. A water tank has the shape of inverted right circular cone with its axis vertical and vertex lower most. Its semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic metre per minute. Find the rate at which the water is rising at the instant when the depth of the water in the tank is 10m.

Section –E

[This section comprises of 3 case- study/passage-based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.

Q36. A company produces three products every day. Their production on certain day is 45 tons. It is found that the production of the third product exceeds the production of first product by 8 tons while the total production of the first and the third product is twice the production of the second product.



Using the concepts of matrices and determinants, answer the following questions.

(i) If x, y and z respectively denote the quantity in (tones) of the first, second and third product produced, then which of the following is true?

(a) $x+y+z=45$ (b) $x+8=z$ (c) $x-2y+z=0$ (d) all of these

(ii) If $\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & -2\\ 1 & -1 & 1\end{bmatrix}^{-1}=\frac{1}{6}\begin{bmatrix}2 & 2 & 2\\ 3 & 0 & -3\\ 1 & -2 & 1\end{bmatrix}$, then the inverse of $\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & -2\\ 1 & -1 & 1\end{bmatrix}$ is

(a) $\begin{bmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{2} & 0 & -\frac{1}{2}\\ \frac{1}{6} & -\frac{1}{3} & \frac{1}{6}\end{bmatrix}$ (b) $\begin{bmatrix}\frac{1}{2} & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{6} & -\frac{1}{3} & \frac{1}{6}\end{bmatrix}$ (c) $\begin{bmatrix}\frac{1}{3} & \frac{1}{2} & \frac{1}{6}\\ \frac{1}{3} & 0 & -\frac{1}{3}\\ \frac{1}{3} & -\frac{1}{2} & \frac{1}{6}\end{bmatrix}$ (d) none of these

(iii) $x : y : z$ is equal to

(a) 12:13:20 (b) 11:15:19 (c) 15:19:11 (d) 13:12:20

Q37. Let $f(x)$ be a real valued function, then its

- Left hand derivative (LHD): $Lf'(a) = \lim_{x \to a^-} \frac{f(x) - f(a)}{x - a}$
- Right hand derivative (RHD): $Rf'(a) = \lim_{x \to a^+} \frac{f(x) - f(a)}{x - a}$
- Also, a function $f(x)$ is differentiable at $x = a$ if LHD and RHD at $x = a$ exists and are equal. In this case the common value of RHD and LHD is $f'(a)$

For the function $f(x) = \begin{cases} x^2 + x + 1 & ; x \leq 1 \\ x^3 + 2 & ; x > 1 \end{cases}$, find

(a) LHD of $f(x)$ at $x = 1$

(b) RHD of $f(x)$ at $x = 1$

(c) Is $f(x)$ differentiable at $x = 1$? If, yes, find $f'(1)$

Q38. The Government declare that the farmers can get ₹300 per quintal for their onions on 1st July and after that, the price will be dropped by ₹3 per quintal per extra day. Shyam's father has 80 quintals of onion in the field on 1st July and estimates that the crop is increasing at the rate of 1 quintal per day.

Based on the above information, answer the following questions:

(i) If x is the number of days after July 1., find the revenue function in terms of x.

(ii) Find the intervals in which the revenue is strictly increasing/ decreasing.

OR

On which day after 1st July should he harvest the onions to get maximum revenue?