5. If $\int \frac{dx}{(x+2)(x^2+1)} = a\log|1+x^2| + b\tan^{-1}x + \frac{1}{5}\log|x+2| + C$, then
   a. $a = \frac{-1}{10}, b = \frac{-2}{5}$
   b. $a = \frac{1}{10}, b = \frac{-2}{5}$
   c. $a = \frac{-1}{10}, b = \frac{2}{5}$
   d. $a = \frac{1}{10}, b = \frac{2}{5}$

6. The area of the region bounded by the curve $y = x$, $x = 0$ and $x = 4$ is
   a. 16 sq. units
   b. 8 sq. unit
   c. 4 sq. units
   d. 2 sq. units

7. Let the relation R in the set $A = \{x \in Z: 0 \leq x \leq 12\}$, given by $R = \{(a,b): |a - b| \text{ is a multiple of } 4\}$. Then the equivalence class containing 1, is:
   a. $\{1,5,9\}$
   b. $\{0,1,2,5\}$
   c. ϕ
   d. A

8. The principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is
   a. π
   b. $\frac{4\pi}{3}$
   c. $\frac{\pi}{2}$
   d. None of these

9. If the function $f(x) = \begin{cases} e^x + ax, if\ x < 0 \\ b(x-1)^2, if\ x \geq 0 \end{cases}$ is differentiable at $x = 0$, then $(a,b) =$
   a. $(-3,-1)$
   b. $(-3,1)$
   c. $(3,1)$
   d. $(3,-1)$

10. The sides of an equilateral triangle are increasing at the rate 2 cm/sec. The rate at which the area increases, when side is 10cm is:
   a. $10\ cm^2/s$
   b. $\sqrt{3}cm^2/s$
   c. $10\sqrt{3}cm^2/s$
   d. $\frac{10}{3}cm^2/s$

11. If $f(a+b-x) = f(x)$, then $\int_a^b xf(x)dx$ is equal to
   a. $\frac{a+b}{2}\int_a^b f(b-x)dx$
   b. $\frac{a+b}{2}\int_a^b f(b+x)dx$
   c. $\frac{b-a}{2}\int_a^b f(x)dx$
   d. $\frac{a+b}{2}\int_a^b f(x)dx$

12. The value of $\int_0^{\frac{\pi}{2}} \log\left[\frac{3+5\cos x}{3+5\sin x}\right] dx$
   a. 0
   b. 1
   c. 2
   d. None of these

13. If $x = acos^3\theta, y = asin^3\theta$, then $\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ is equal to
    a. $tan^2\theta$
    b. $sec^2\theta$
    c. $\sec\theta$
    d. $|\sec\theta|$
14. Evaluate $\int \frac{(\log x)^2}{x} dx$
15. $f(x) = x^x$ has a stationary point at
    a. $x = e$
    b. $x = \frac{1}{e}$
    c. $x = 1$
    d. $x = \sqrt{e}$
16. Area of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is
    a. 20π sq units
    b. $20\pi^2$ sq units
    c. $16\pi^2$ sq units
    d. $25\pi^2$ sq units
17. The area bounded by the curve $y = \sin x$, $x-$axis, ordinates $x = 0$ and $x = 2\pi$ is
    a. 1 sq. units
    b. 2 sq. units
    c. 3 sq. units
    d. 4 sq. units
18. A cylindrical tank of radius 10m is filled with wheat at the rate of 314 cubic metre per hour. Then the depth of the wheat is increasing at the rate of
    a. 1 m/h
    b. 0.1 m/h
    c. 1.1 m/h
    d. 0.5 m/h

For questions 19 and 20, read the statements carefully and mark the correct option

a. Assertion (A) is true, Reason (R) is true; Reason (R) is a correct explanation for Assertion (A)
b. Assertion (A) is true, Reason (R) is true; Reason (R) is a not correct explanation for Assertion (A)
c. Assertion (A) is true, Reason (R) is false
d. Assertion (A) is false, Reason (R) is true

19. Assertion (A): The value of $\tan^{-1}\{\cot(\text{cosec}^{-1}2)\} = \frac{\pi}{3}$

Reasoning (R): if $y = \sin^{-1}x$, then $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

20. Consider the function $f(x) = \begin{cases} x^2, x \geq 1 \\ x+2, x < 1 \end{cases}$

Assertion (A): f is not derivable at $x = 1$ as $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$

Reasoning (R): if a function f is derivable at a point $'a'$, then its continuous at $'a'$

Section B (2 marks each)

21. $\int \frac{2x}{\sqrt{a+x}+\sqrt{a-x}}dx$

22. Show that the function $f: R \to R$, defined by $f(x) = |x| + x$ is not an onto function

23. Prove that $3\sin^{-1}x = \sin^{-1}(3x - 4x^3), x\epsilon\left[-\frac{1}{2}, \frac{1}{2}\right]$

24. Differentiate $x^{\sin x}$ w.r.t x

25. Integrate $\int \frac{sin^3x+cos^3x}{sin^2x\,cos^2x}dx$

Section C (3 marks each)

26. Differentiate w.r.t x: $(\sin x - \cos x)^{(\sin x - \cos x)}, \frac{\pi}{4} < x < \frac{3\pi}{4}$

27. Show that the relation R in the set $A = \{x \in Z: 0 \le x \le 12\}$ given by $R = \{(a, b): a, b\epsilon A, |a - b| \text{ is divisible by } 3\}$ is an equivalence relation

28. Prove that: $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}; 0 < x < \frac{\pi}{2}$

29. Determine for what values of x, the function $f(x) = x^3 + \frac{1}{x^3}(x \neq 0)$ is strictly increasing or decreasing.

30. Manufacturer can sell x items at a price of Rs $\left(5 - \frac{x}{100}\right)$ each. The cost price of x items is Rs $\left(\frac{x}{5} + 500\right)$. Find the number of items he should sell to earn maximum profit.

31. Evaluate $\int_2^4 \{|x - 2| + |x - 3| + |x - 4|\}dx$

Section D (5 marks each)

32. If $x = \sin t, y = \sin pt$, then prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$

33. Show that the height of the right circular cylinder of greatest volume which can be inscribed in a right circular cone of height h and radius r is one-third of the height of the cone, and the greatest volume of the cylinder is $\frac{4}{9}$ times the volume of the cone.

34. Using properties of definite integrals, evaluate $\int_0^{\pi} \frac{xdx}{a^2cos^2x+b^2sin^2x}$

35. Find the area of the region bounded by the circle $x^2 + y^2 = 16$

Section E (4 marks each)

36. Evaluate

a. $\int_{-1}^{3}[x]dx$

b. $\int_{-2}^{4} f(x)dx$, where $f(x) = \begin{cases} 2x^2, if\ x < 0 \\ 3x, if\ x \geq 0 \end{cases}$

37. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. water is poured into it at a constant rate of 5 cubic metre per hour. Find the rate at which the level of the water is rising at the instant when the depth of the water tank is 4m.

38. A general election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote.

Let I be the set of all citizens of India who were eligible to exercise their voting right in general election. A relation R is defined on I as follows

$R = \{(V_1, V_2): V_1, V_2 \epsilon I\ and\ both\ use\ their\ voting\ right\ in\ the\ general\ election\ 2024\}$

Answer the following questions

i. Check whether this Relation is equivalent?

j. Mr. Shyam exercised his voting right in General Election 2024, then Mr. Shyam is related to which of the following. Choose one

   a. All those eligible voters who cast their vote
   b. Family members of Mr. Shaym
   c. All citizens of India
   d. Eligible voters of India

St. Paul's School
Class – XII Half Yearly Examination (2024-25)
Mathematics (Code 041)

Time – 3 Hrs. **Max. Marks – 80**

General instructions:

(i) This question paper contains five sections A, B, C, D and E. Each section is compulsory.
(ii) Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
(iii) Section B has 5 Very Short Answer (VSA) type questions of 2 marks each.
(iv) Section C has 6 Short Answer (SA) type questions of 3 marks each.
(v) Section D has 4 Long Answer (LA) type questions of 5 marks each.
(vi) Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A (1 mark each)

1. Let $f: R \to R$ be defined by $f(x) = x^2 + 1$. Then the pre-images of 17 and -3, respectively are
   a. $\phi, \{4, -4\}$
   b. $\{3, -3\}, \phi$
   c. $\{4, -4\}, \phi$
   d. $\{4, -4\}, \{2, -2\}$
2. If $\sin^{-1} x > \cos^{-1} x$, then x should lie in the interval
   a. $\left(-1, -\frac{1}{\sqrt{2}}\right)$
   b. $\left(0, \frac{1}{\sqrt{2}}\right)$
   c. $\left(\frac{1}{\sqrt{2}}, 1\right)$
   d. $\left(-\frac{1}{\sqrt{2}}, 0\right)$
3. If $y = \log\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx} =$
   a. $\frac{4x^3}{1-x^4}$
   b. $-\frac{4x}{1-x^4}$
   c. $\frac{1}{4-x^4}$
   d. $-\frac{4x^3}{1-x^4}$
4. The function $f(x) = 2x^3 - 3x^2 - 12x + 4$ has
   a. Two points of local maxima
   b. Two points of local minima
   c. One maxima and one minima
   d. No maxima or minima