CLASS XII : SAMPLE QUESTION PAPER - 1

SUBJECT: MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General instructions:

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains **38** questions. **All** questions are compulsory.

(ii) This Question paper is divided into five Sections - **A, B, C, D and E.**

(iii) In **Section A,** Questions no. **1 to 18** are **multiple choice questions (MCQs)** and Questions no. **19 and 20 are Assertion-Reason** based questions of 1 mark each.

(iv) In **Section B**, Questions no. **21 to 25** are **Very Short Answer (VSA)-type questions**, carrying 2 marks each.

(v) In **Section C**, Questions no. **26 to 31** are **Short Answer (SA)-type questions**, carrying 3 marks each.

(vi) In **Section D**, Questions no. **32 to 35** are **Long Answer (LA)-type questions**, carrying 5 marks each.

(vii) In **Section E**, Questions no. **36 to 38** are **Case study-based questions**, carrying 4 marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.

(ix) Use of calculators is **not** allowed.

SECTION — A

(Multiple Choice Questions : Each question carries 1 mark)

1. The function $f: R \to R$ defined by $f(x) = 5^x + 5^{|x|}$ is

(a) One-one and onto (b) Many-one and onto (c) One-one and into (d) Many-one and into

2. If $f(x)=\begin{cases}\cos x, & \text{if } x\ge 0\\ x+k, & \text{if } x<0\end{cases}$, then find the value of k for which $f(x)$ is continuous at $x = 0$.

(a) 0 (b) −1 (c) 1 (d) 2

3. If $\begin{vmatrix}2x & 5\\ 8 & x\end{vmatrix}=\begin{vmatrix}6 & -2\\ 7 & 3\end{vmatrix}$, then find the value of x.

(a) ± 6 (b) ± 4 (c) 0 (d) ± 5

4. Evaluate : $\int_0^1 \log\left(\frac{1}{x}-1\right)dx$

(a) 1 (b) 2 (c) 0 (d) 3

5. The number of all the possible matrices of order 3×3 with each entry 2 or 0 is

(a) 9 (b) 27 (c) 81 (d) 512

6. If $f(x) = x^3$ is continuous at $x = 2$, then find $f(2)$

(a) 2 (b) 4 (c) 3 (d) 8

7. Write the integrating factor of the differential equation $\frac{dy}{dx} + y\cot x - \sec x = 0$.

(a) $\cos x$ (b) $\sec x$ (c) $\tan x$ (d) $\sin x$

8. If $\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{40}\begin{bmatrix}5 & 10 & -5\\ -5 & -2 & 13\\ 10 & -4 & 6\end{bmatrix}\begin{bmatrix}5\\ 0\\ 5\end{bmatrix}$, then the value of $x + y + z$ is

(a) 3 (b) 0 (c) 2 (d) 1

9. $\int e^{x\log a} e^x \, dx$ is equal to

(a) $(ae)^x + c$ (b) $\frac{(ae)^x}{\log(ae)} + c$ (c) $\frac{e^x}{1+\log a} + c$ (d) none of these

10. Write the sum of the order and the degree of the differential equation.

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = 5$

(a) 2 (b) 1 (c) 3 (d) 4

11. The value of k for which the function $f(x) = \begin{cases} \left(\frac{4}{5}\right)^{\frac{\tan 4x}{\tan 5x}}, & 0 < x < \frac{\pi}{2} \\ k + \frac{2}{5}, & x = \frac{\pi}{2} \end{cases}$, is continuous at $x = \frac{\pi}{2}$, is

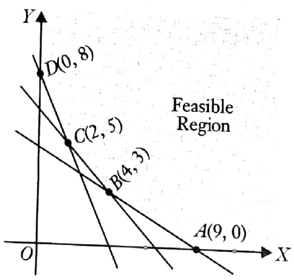
(a) $\frac{17}{20}$ (b) $\frac{3}{5}$

(c) $-\frac{2}{5}$ (d) $\frac{2}{5}$

12. A problem in mathematics is given to 3 students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. What is the probability that the problem is solved ?

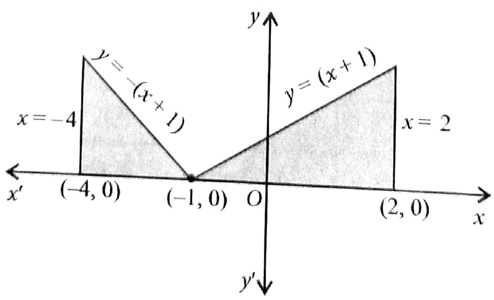
(a) 3/4 (b) 1/4 (c) 1/2 (d) 1/3

13. Feasible solution for an L.P.P. is shown shaded in the following figure. Minimum of $Z = 4x + 3y$ occurs at the point



(a) (0, 8) (b) (2, 5) (c) (4, 3) (d) (9, 0)

14. Find the shaded area shown in the given figure.



(a) 8 sq. units (b) 7 sq. units (c) 9 sq. units (d) 10 sq. units

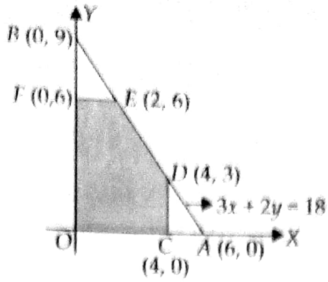
15. Solution of the differential equation $y - x\frac{dy}{dx} = 0$ is

(a) $xy = c$ (b) $y = xc$ (c) $xy = 0$ (d) $y = x$

16. The function $f(x) = \cos^2 x$ is strictly decreasing on

(a) $\left[0, \frac{\pi}{2}\right]$ (b) $\left[0, \frac{\pi}{2}\right)$ (c) $\left(0, \frac{\pi}{2}\right)$ (d) $\left(0, \frac{\pi}{2}\right]$

17. In the given graph, the feasible region for a LPP is shaded. The objective function $Z = 3x + 5y$ will be maximum at



(a) (2, 6) (b) (4, 3) (c) (0, 6) (d) (4, 0)

18. Find the value of $(\vec{a} \cdot \hat{i})^2 + (\vec{a} \cdot \hat{j})^2 + (\vec{a} \cdot \hat{k})^2$.

(a) $|\vec{a}|$ (b) $|\vec{a}|^2$ (c) 0 (d) 1

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (a), (b), (c) and (d) as given below.)

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

19. **Assertion (A) :** If the radius of circle is increasing at the rate of 4 cm/s, then the rate of change in the area of the circle when $r = 3$ cm, is 24π cm^2/s.

Reason (R) : Rate of change of area of circle is $2\pi r$.

20. **Assertion (A) :** The angle between the pair of lines with direction ratios proportional to a, b, c and $b - c, c - a, a - b$ is $\frac{\pi}{4}$.

Reason (R) : The two lines will be parallel iff $\theta = 0°$ or $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$.

SECTION B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. (a) Show that $f(x) = e^{1/x}, x \neq 0$ is a strictly decreasing function.

OR

21. (b) Show that the function $f(x) = x^2 - [x], x \in [1, 2)$ is strictly increasing.

22. If A is a square matrix of order 3 such that $A^2 = 2A$, then find the value of $|A|$.

23. Find $\int \frac{\log x}{(1 + \log x)^2} dx$.

24. Find the integrating factor of the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$.

25. (a) A bag contains 1 red and 3 white balls. Find the probability distribution of the number of red balls if 2 balls are drawn at random from the bag one-by-one without replacement.

OR

25. (b) Two cards are drawn at random from a pack of 52 cards one-by-one without replacement. What is the probability of getting first card red and second card Jack?

SECTION C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. (a) Find : $\int_0^{2\pi} e^x \sin\left(\frac{\pi}{4}+\frac{x}{2}\right)dx$

OR

26. (b) Evaluate : $\int \frac{8}{(x+2)(x^2+4)}dx$

27. Show that $y = be^x + ce^{2x}$ is a solution of the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$.

28. (a) Find a vector whose magnitude is 3 units and which is perpendicular to the vectors $\vec{a}$ and $\vec{b}$, where $\vec{a} = 3\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 6\hat{i} + 5\hat{j} - 2\hat{k}$.

OR

28. (b) Find the shortest distance between the lines l_1 and l_2 whose vector equations are $\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$.

29. (a) If $x = a\sec\theta$, $y = b\tan\theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$.

OR

29. (b) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

30. Evaluate : $\tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right)$.

31. Find the number of points at which the objective function $z = 3x + 2y$ can be maximized subject to $3x + 5y \le 15$, $5x + 2y \le 20$, $x \ge 0$, $y \ge 0$.

SECTION D

(This section comprises of 4 long answer (LA) type questions of 5 marks each.)

32. (a) Assume that the chances of a patient having a heart attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chance by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga.

OR

32. (b) The bag '*A*' contains 5 white and 3 black balls while the bag '*B*' contains 4 white and 7 black balls. One of the bags is chosen at random and a ball is drawn from it. What is the probability that the ball is white?

33. Find the shortest distance between the lines

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$

and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$.

If the lines intersect, then find their point of intersection.

34. (a) If $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$, then find A^{-1}. Hence, solve the system of equations $2x + 3y + 4z = 17$; $x - y = 3$; $y + 2z = 7$.

OR

34. (b) Show that the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ satisfies the equation $A^2 - 4A - 5I_3 = O$ and hence find A^{-1}.

35. Find all points of discontinuity of f, where f is defined as follows :

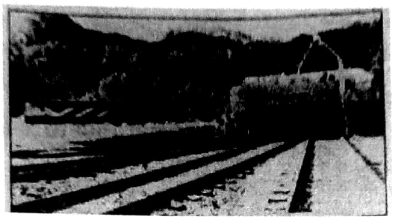
$$f(x)=\begin{cases} |x|+3, & x \le -3 \\ -2x, & -3 < x < 3 \\ 6x+2, & x \ge 3 \end{cases}$$

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each.)

Case Study-1

36. Students of a school are taken to a railway museum to learn about railways heritage and its history. An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by $R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$.



On the basis of the above information, answer the following questions.

(i) Find whether the relation R is symmetric or not.

(ii) Find whether the relation R is transitive or not.

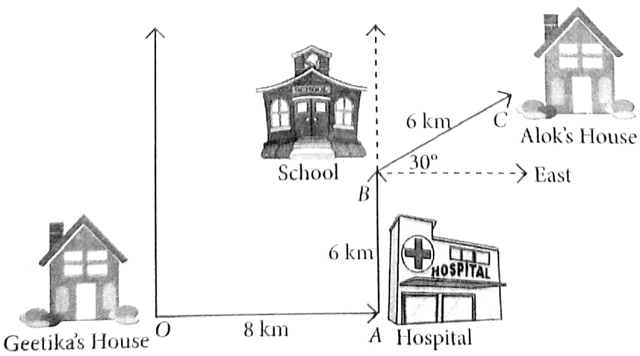
(iii) (a) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of all rail lines in R related to it.

OR

(iii) (b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$ check whether the relation S is symmetric and transitive.

Case Study-2

37. Geetika's house is situated at Shalimar Bagh at point O, for going to Alok's house she first travels 8 km by bus in the East. Here at point A, a hospital is situated. From Hospital, Geetika takes an auto and goes 6 km in the North, here at point B school is situated. From school, she travels by bus to reach Alok's house which is at 30° East, 6 km from point B.



On the basis of the above information, answer the following questions.

(i) Write the position vector of B. Also, find the distance travel by Geetika from her house to school.

(ii) What is the vector distance from Geetika's house to Alok's house?

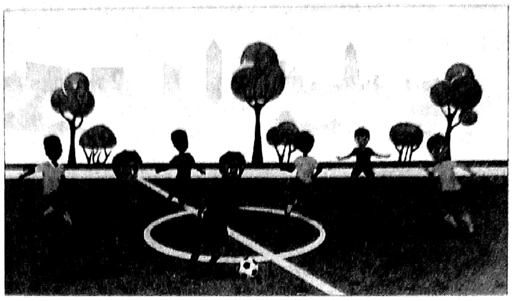
(iii) (a) What is the vector distance from school to Alok's house?

OR

(iii) (b) What is the total distance travelled by Geetika from her house to Alok's house? Also, find $|\overline{OB}|$.

Case Study-3

38. Two teams were playing football on a ground, where David is running on the ground with the football along the curve given by $y = x^2 + 7$. He wanted to pass the football to the goalkeeper of his team. The goalkeeper was standing on the point (3, 7).



On the basis of above information, answer the following questions.

(i) At what point, the distance between David and goalkeeper is minimum?

(ii) At what point the slope of David's position is parallel to x-axis?