CLASS XII : SAMPLE QUESTION PAPER - 2

SUBJECT: MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General instructions:

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains **38** questions. **All** questions are compulsory.

(ii) This Question paper is divided into five Sections - **A, B, C, D and E.**

(iii) In **Section A,** Questions no. **1 to 18** are **multiple choice questions (MCQs)** and Questions no. **19 and 20 are Assertion-Reason** based questions of 1 mark each.

(iv) In **Section B**, Questions no. **21 to 25** are **Very Short Answer (VSA)-type questions**, carrying 2 marks each.

(v) In **Section C**, Questions no. **26 to 31** are **Short Answer (SA)-type questions**, carrying 3 marks each.

(vi) In **Section D**, Questions no. **32 to 35** are **Long Answer (LA)-type questions**, carrying 5 marks each.

(vii) In **Section E**, Questions no. **36 to 38** are **Case study-based questions**, carrying 4 marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.

(ix) Use of calculators is **not** allowed.

SECTION — A

(Multiple Choice Questions : Each question carries 1 mark)

1. Given that matrices A and B are of order $3 \times n$ and $m \times 5$ respectively, then the order of matrix $C = 5A + 3B$ is

(a) 3×5 and $m = n$ (b) 3×5 (c) 3×3 (d) 5×5

2. Given that $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ and $A^2 = 3I$, then

(a) $1 + \alpha^2 + \beta\gamma = 0$ (b) $1 - \alpha^2 - \beta\gamma = 0$ (c) $3 - \alpha^2 - \beta\gamma = 0$ (d) $3 + \alpha^2 + \beta\gamma = 0$

3. The interval, in which function $y = x^3 + 6x^2 + 6$ is increasing, is

(a) $(-\infty,-4) \cup (0, \infty)$ (b) $(-\infty,-4)$ (c) $(-4, 0)$ (d) $(-\infty, 0) \cup (4, \infty)$

4. Given that $A = [a_{ij}]$ is a square matrix of order 3×3 and $|A| = -7$, then the value of $\sum_{i=1}^{3} a_{i2}A_{i2}$, where A_{ij} denotes the cofactor of element a_{ij} is

(a) 7 (b) -7 (c) 0 (d) 49

5. Solution of differential equation $xdy - ydx = 0$ represents

(a) a rectangular hyperbola (b) parabola whose vertex is at origin
(c) straight line passing through origin (d) a circle whose centre is at origin

6. The area of a triangle with vertices $(-3, 0)$, $(3, 0)$ and $(0, k)$ is 9 sq. units. The value of k will be

(a) 9 (b) 3 (c) -9 (d) 6

7. If $A = \begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}, X = \begin{bmatrix} n \\ 1 \end{bmatrix}$ and $B = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$ and $AX = B$, then value of n is

(a) 0 (b) 2 (c) 4 (d) 6

8. If A and B are two events such that $P(A) = 0.2$, $P(B) = 0.4$ and $P(A \cup B) = 0.5$, then value of $P(A/B)$ is

(a) 0.1 (b) 0.25 (c) 0.5 (d) 0.08

9. If the projection of $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ on $\vec{b} = 2\hat{i} + \lambda\hat{k}$ is zero, then the value of λ is

(a) 0 (b) 1 (c) $\frac{-2}{3}$ (d) $\frac{-3}{2}$

10. If $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along three mutually perpendicular directions, then

(a) $\hat{i} \cdot \hat{j} = 1$ (b) $\hat{i} \times \hat{j} = 1$ (c) $\hat{i} \cdot \hat{k} = 0$ (d) $\hat{i} \times \hat{k} = 0$

11. If the minimum value of an objective function $Z = ax + by$ occurs at two points (3, 4) and (4, 3), then

(a) $a + b = 0$ (b) $a = b$ (c) $3a = b$ (d) $a = 3b$

12. Evaluate : $\int_0^1 \left\{e^x + \sin\frac{\pi x}{4}\right\} dx$

(a) $1 - \frac{4}{\pi} + \frac{2\sqrt{2}}{\pi}$ (b) $1 + \frac{2}{\pi} - \frac{2\sqrt{2}}{\pi}$ (c) $1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}$ (d) None of these

13. $\int \cos x \, e^{\sin x} dx$ is equal to ______ .

(a) $e^{\sin x} + c$ (b) $e^{\cos x} + c$ (c) $-e^{\cos x} + c$ (d) $-e^{\sin x} + c$

14. For what value of n, the following differential equation $\frac{dy}{dx} = \frac{x^3 - y^n}{x^2 y + xy^2}$ is homogeneous?

(a) 1 (b) 2 (c) 3 (d) 4

15. $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) -1 (d) 1

16. The graph of the inequality $2x + 3y > 6$ is

(a) half plane that contains the origin
(b) half plane that neither contains the origin nor the points of the line $2x + 3y = 6$
(c) whole XOY-plane excluding the points on the line $2x + 3y = 6$
(d) entire XOY-plane

17. The function $f(x) = [x]$, where $[x]$ is the greatest integer function that is less than or equal to x, is continuous at

(a) 4 (b) -2 (c) 1.5 (d) 1

18. Area lying between the parabola $y^2 = 4x$ and its latus rectum is

(a) $\frac{1}{3}$ sq. units (b) $\frac{2}{3}$ sq. units (c) $\frac{5}{3}$ sq. units (d) $\frac{8}{3}$ sq. units

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (a), (b), (c) and (d) as given below.)

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

19. Assertion (A) : Let $f(x)$ be a function such that $f'(x) = \frac{1}{x+1}$. If $g(x) = f(2x - 1)$, then $g'(x)$ is x.

Reason (R) : Derivative of $\cos x$ with respect to $\sin x$ is $-\tan x$.

20. **Assertion (A) :** The function $f: R \rightarrow R$ defined by $f(x) = \frac{3x}{1+3x}$ is onto.

Reason (R) : If $f: R \rightarrow R$ defined by $f(x) = \frac{4x+7}{3}$ is an invertible function, then $f^{-1}(x) = \frac{3x-7}{4}$.

SECTION B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. (a) If $x, y, z \in [-1, 1]$ such that $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then find the value of $xy + yz + zx$.

OR

21. (b) If $\operatorname{cosec}^{-1} x + \operatorname{cosec}^{-1} y + \operatorname{cosec}^{-1} z = -\frac{3\pi}{2}$, find the value of $\frac{x}{y} + \frac{y}{z} + \frac{z}{x}$.

22. Determine the values of a, b and c for which the function $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ c, & x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{b\sqrt{x^3}}, & x > 0 \end{cases}$ may be continuous at $x = 0$.

23. Find whether the following function is differentiable at $x = 1$ and $x = 2$ or not.

$$f(x) = \begin{cases} x, & x < 1 \\ 2 - x, & 1 \le x \le 2 \\ -2 + 3x - x^2, & x > 2 \end{cases}$$

24. (a) Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{b} = -\hat{i} + 3\hat{k}$.

OR

24. (b) Find the angle between two vectors $\vec{a}$ and $\vec{b}$ having the same length $\sqrt{2}$ and their scalar product is -1.

25. Find the equation of a line passing through the point $(-3, 2, -4)$ and equally inclined to the axes.

SECTION C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. (a) Find : $\int_0^{2\pi} e^x \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) dx$

OR

26. (b) Evaluate : $\int \frac{8}{(x+2)(x^2+4)} dx$

27. Find the number of points at which the objective function $z = 3x + 2y$ can be maximized subject to $3x + 5y \le 15$, $5x + 2y \le 20$, $x \ge 0$, $y \ge 0$.

28. (a) Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$. Find p if A and B are (i) mutually exclusive (ii) independent.

OR

28. (b) The random variable X can take only the values 0, 1, 2, 3. Given that $P(X = 0) = P(X = 1) = p$ and $P(X = 2) = P(X = 3)$ such that $\Sigma p_i x_i^2 = 2\Sigma p_i x_i$, find the value of p.

29. (a) Prove that the lines $x = py + q$, $z = ry + s$ and $x = p'y + q'$, $z = r'y + s'$ are perpendicular, if $pp' + rr' + 1 = 0$.

OR

29. (b) Find the angle between the lines whose direction cosines are given by the equations $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$

30. Find the intervals in which the function f given by $f(x) = \tan x - 4x$, $x \in \left(0, \frac{\pi}{2}\right)$ is

(i) strictly increasing (ii) strictly decreasing

31. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when depth of the tank is half of its width.

SECTION D

(This section comprises of 4 long answer (LA) type questions of 5 marks each.)

32. Using integration, find the area of the region bounded by the curve $x^2 + y^2 = 4$, $y = \sqrt{3}x$ and x axis in the first quadrant.

33. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find $A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = O$.

34. (a) Show that the function $f(x) = |x - 1| + |x + 1|$, for all $x \in R$, is not differentiable at the points $x = -1$ and $x = 1$.

OR

34. (b) Find whether the following function is differentiable at $x = 1$ and $x = 2$ or not.

$$f(x) = \begin{cases} x, & x < 1 \\ 2 - x, & 1 \le x \le 2 \\ -2 + 3x - x^2, & x > 2 \end{cases}$$

35. (a) Find the value of p, so that the lines $l_1 : \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$ and $l_2 : \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other. Also find the equation of a line passing through a point (3, 2, –4) and parallel to line l_1.

OR

35. (b) The cartesian equations of a line are $6x - 2 = 3y + 1 = 2z - 2$. Find the direction cosines of the line. Write down the cartesian and vector equations of a line passing through (2, –1, –1), which is parallel to the given line.

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each.)

Case Study-1

36. A relation R on a set A is said to be an equivalence relation on A iff it is
Reflexive *i.e.*, $(a, a) \in R \ \forall \ a \in A$.
Symmetric *i.e.*, $(a, b) \in R \Rightarrow (b, a) \in R \ \forall \ a, b \in A$.
Transitive *i.e.*, $(a, b) \in R$ and $(b, c) \in R$
$\Rightarrow (a, c) \in R \ \forall \ a, b, c \in A$.
Based on the above information, answer the following questions.

(i) If the relation $R = \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$ defined on the set $A = \{1, 2, 3\}$, then find the relation R on set A.

(ii) If the relation $R = \{(1, 2), (2, 1), (1, 3), (3, 1)\}$ defined on the set $A = \{1, 2, 3\}$, then find the relation R on set A.

(iii) (a) If the relation R on the set N of all natural numbers defined as $R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$, then find the relation R on set N.

OR

(iii) (b) If the relation R on the set $A = \{1, 2, 3, \ldots, 13, 14\}$ defined as $R = \{(x, y) : 3x - y = 0\}$, then find the relation R on set A.

Case Study-2

37. Neelam and Ved appeared for first round of an interview for two vacancies. The probability of Neelam's selection is 1/6 and that of Ved's selection is 1/4.

On the basis of above information, answer the following questions.

(i) Find the probability that both of them are selected.

(ii) Find the probability that none of them is selected.

(iii) (a) Find the probability that only one of them is selected.

OR

(iii) (b) Find the probability that atleast one of them is selected.

Case Study-3

38. Sonam wants to prepare a sweet box for Diwali at home. For making lower part of box, she takes a square piece of cardboard of side 18 cm. Let x cm be the length of each side of the square cardboard which is to be cut off from corner of the square piece of side 18 cm.

On the basis of above information, answer the following questions.

(i) Find the expression for the volume of the open box formed by folding up the cutting corners.

(ii) Find the value(s) of x for which $\frac{dV}{dx} = 0$.