Name Anandita Class & Section ______ Roll No. ______

FIRST TERMINAL EXAMINATION-2014-2015

Class-XII

Subject-Mathematics

Time Allowed : 3 Hrs. **M.M. : 100**

Please Check the Total Marks

Do not write any answers on the questions paper. Check the total marks.

Attempt all the questions :

Section A consists of 6 questions carrying 1 mark each.

Section B consists of 13 questions carrying 4 marks each.

Section C consists of 7 questions carrying 6 marks each.

All the questioins are compulsory however internol choice has been given in some of the questions.

Section-A

1. Justify whether or not given operation of '*' on N as $a*b = \frac{a+b}{2}$ is a binary operation ?

2. Find the value of $\cos(2\cos^{-1}x + \sin^{-1}x)$

3. Evaluate the determinant $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \sin 15^\circ \end{vmatrix}$

4. Find the derivative of $\sin(\sin(\cos(x^2)))$ with respect to x.

5. If Rolle's theorem is justified for the function $f(x)=4x^2-12x+9$, $x \in [0, 3]$. Find the value of $c \in (1, 3)$

6. Find $\int \frac{dx}{\sin^2 x \cos^2 x}$

Section-B

7. Let $F : N \to R$ be a function defind as $f(x)=4x^2 + 12x + 15$ show that $f : N \to \text{Range}(f)$ is invertible. Find the inverse of f.

8. Let $f : X \to Y$ be a function. Define a relation R in X given by $R = \{(a,b) : f(a) = f(b)\}$. Examine, if R in an equivalence relation.

OR

Let X be a non-empty set and $P(X)$ be the power set of X. Let a binary operation '*' on $P(X)$ be defined by $A*B = (A-B) \cup (B-A)$ for all A, B $\in p(X)$. Show '*' is commutative and associative. Also find the identity and inverse elements if any.

9. Solve the following equation for x :

$\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1} 8/31$

OR

Show that : $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\sqrt{\frac{1-\cos x}{1+\cos x}}\right) = \cos^{-1}\left(\frac{a\cos x + b}{a + b\cos x}\right)$

10. If $A = \begin{bmatrix} \cos\alpha + \sin\alpha & \sqrt{2}\sin\alpha \\ -\sqrt{2}\sin\alpha & \cos\alpha - \sin\alpha \end{bmatrix}$, prove that

$$A^n = \begin{bmatrix} \cos n\alpha + \sin n\alpha & \sqrt{2}\sin n\alpha \\ -\sqrt{2}\sin n\alpha & \cos n\alpha - \sin n\alpha \end{bmatrix} \text{ for all } n \in N.$$

11. Prove that :

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$$

OR

Show that :

$$\begin{vmatrix} \sin^2 A & \sin A & \cos^2 A \\ \sin^2 B & \sin B & \cos^2 B \\ \sin^2 C & \sin C & \cos^2 C \end{vmatrix} = -(\sin A - \sin B)(\sin B - \sin C)(\sin C - \sin A)$$

12. If the function f defined by

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ c, & x = 0 \\ \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}}, & x > 0 \end{cases}$$

is continous at $x = 0$, then find the values of a, b and c.

13. Differentiate $x\sin^{-1}x$ with respect to $\sin^{-1}x$.

14. If $x = a\,(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$, find $\dfrac{d^2y}{dx^2}$ at $\theta = \pi/2$

15. The two equal sides of an isosceles triangle with fixed base b are decrearing at the rate of 3 cm/sec. How fast is the area decreasing when two equal sides are equal to the base ?

16. Find the approximate value of tan 46°, given that

$1^\circ = 0.01745$ radian.

17. Find $\displaystyle\int \frac{\sin x}{\sqrt{1+\sin x}}\,dx$

OR

Evaluate $\displaystyle\int \frac{4x+5}{\sqrt{2x^2+x-3}}\,dx$

18. Evaluate : $\displaystyle\int \frac{\sqrt{x^2+1}\,[\log(x^2+1)-2\log x)}{x^4}\,dx$

19. Find the area of the region in the first quadrant enclosed by the x–axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

Section–C

20. Using elementary operation, find the inverse of the matrix A = $\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$

21. Solve the following system of equations, using matrices :

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z} = 4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

22. Differentiate $\sin^{-1}\left(\frac{2^{x+1}\cancel{+1}}{1+4^{x}}\right)$ with respect to x.

Also find its domain.

23. Find the equation of tangents to the curve $y = \cos(x + y)$, $-2\pi \leq x \leq 2\pi$ that are parallel to the line $x + 2y = 0$

OR

Find intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$

is strictly increasing and strictly decreasing.

24. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height 'h' and semi vertical angle α is one-third that of the cone and the greatest-volume of the cylinder is

$$\frac{4}{27}\ \pi h^3 \tan^2\alpha$$

25. Evaluate $\int \sqrt{\tan x}\, dx$

OR

Evaluate $\int_1^4 \left(x^2 - x + e^x\right) dx$ using limit of a sum method.

26. Using properties of definite integral, evaluate

$$\int_0^{\pi} \log(1 + \cos x)\, dx$$