DPS, Mathura Road (Vikas...)

CLASS XII
FIRST TERM EXAMINATION 2014 – 2015
Mathematics
SET C1

Time Allowed: 3 Hrs **Max. Marks: 100**

Instructions:

1) All questions are compulsory.

2) The question paper consists of 26 questions, divided into three sections A, B and C. Section A comprises of 6 questions of 1 mark each, Section B comprises of 13 questions of 4 marks each and Section C comprises of 7 questions of 6 marks each.

3) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six mark each. You have to attempt only one of the alternatives in all such questions.

4) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION A

Q1) If A is a square matrix of order 3 such that $|Adj\ A| = 225$, *find* $|A'|$.

Q2) Write the value of the following: $\tan^{-1}\left(\frac{a}{b}\right) - \tan^{-1}\left(\frac{a-b}{a+b}\right)$

Q3) Write the smallest equivalence relation R on set A = {1, 2, 3}.

Q4) Write the value of x + y + z if $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

Q5) Give an example of a function which is continuous everywhere but not differentiable at exactly two points.

Q6) Evaluate: $\int \sec^2(7-4x)\,dx$

SECTION B

Q7) Prove that the relation R in the set A = {5, 6, 7, 8, 9} given by R = {(a, b): |a – b| is divisible by 2} is an equivalence relation. Find the set of all the elements related to the element 6.

Q8) If $y = x^x$, then prove that $\frac{d^2y}{dx^2} - \left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$

Q9) Evaluate: $\int \frac{(\sin^6 x + \cos^6 x)\, dx}{\sin^2 x . \cos^2 x}$

OR

Evaluate: $\int (x-3)\sqrt{x^2+3x-18}\, dx$

Q10) Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is

a) Strictly increasing
b) Strictly decreasing

OR

Find the equations of tangent and normal to the curve $x = a \sin^3\theta$ and $y = a \cos^3\theta$ at $\theta = \frac{\pi}{4}$.

Q11) Show that $\sin[\cot^{-1}\{\cos(\tan^{-1} x)\}] = \sqrt{\frac{(x^2+1)}{(x^2+2)}}$

OR

Prove that: $\cos\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right) = \frac{6}{5\sqrt{13}}$

Q12) By using properties of determinants, show that: $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

Q13) Evaluate: $\int \frac{5x\, dx}{(x+1)(x^2+9)}$

Q14) Using integration, find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

OR

Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$

Q15) If A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then prove that $A^2 - 4A - 5I = 0$, where I is Identity matrix of order 3.

Hence, find A^{-1}.

Q16) Using differentials, find the approximate value of $\sqrt{49.5}$

Q17) Solve the differential equation $(x^2-1)\frac{dy}{dx} + 2xy = \frac{2}{(x^2-1)}$

Q18) Solve the differential equation: $\left\{x \sin^2\frac{y}{x} - y\right\}dx + x\, dy = 0$

$\lim_{h \to 0} \frac{\cosh}{h} = 1$

Q19) Find the matrix X such that: $X\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix} = \begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$

SECTION C

Q20) If the function f defined by f(x) = $\begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x} & \text{if } x < \dfrac{\pi}{2} \\ a & \text{if } x = \dfrac{\pi}{2} \\ \dfrac{b(1-\sin x)}{(\pi - 2x)^2} & \text{if } x > \dfrac{\pi}{2} \end{cases}$ is continuous at x = $\frac{\pi}{2}$, then find the values of a and b.

Q21) Two schools P and Q want to award their selected students on the values of discipline, politeness and punctuality. The school P wants to award Rs x each, Rs y each and Rs z each for the three respective values to its 3, 2 and 1 students with a total award money of Rs. 1000. School Q wants to spend Rs 1500 to award its 4, 1 and 3 students on the respective values (by giving the same award money for the three values as before). If the total amount of awards for one prize on each value is Rs 600, using matrices, find the award money for each value.
Also from the above three values, suggest one more value for awards.

Q22) Using properties, evaluate the integral: $\int_0^{\frac{\pi}{2}} (2\log\sin x - \log(\sin 2x))dx$

OR

by parts

Evaluate: $\int \sqrt{\dfrac{1-\sqrt{x}}{1+\sqrt{x}}}\, dx$

Q23) Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$

Q24) Prove that the semi vertical angle of the right circular cone of given volume and least curved surface area is $\cot^{-1}\sqrt{2}$

OR

Of all the closed right circular cylindrical cans of volume 128π cm^3, find the dimensions of the can which has minimum surface area.

Q25) *If* $\cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = a$, *then prove that* $9x^2 - 12xy\cos a + 4y^2 = 36\sin^2 a$

Q26) Let X be a non empty set and P(X) be its power set. Let * be a binary operation defined on elements of P(X) by, A * B = A ∩ B for all A, B ε P(X).

Then,

a) Is * commutative? Justify.
b) Is * associative? Justify.
c) Find the identity element in P(X) w.r.t. *
d) Find all the invertible elements of P(X), if any

$\cos^3\theta =$

$\cos 3\theta = \cos(2\theta)\cos\theta$

$= (\cos^2\theta - \sin^2\theta)\cos\theta$

$\cos^3\theta - \cos\theta(1-\cos^2\theta)$

$\cos^3\theta - \cos\theta + \cos^3\theta$

$2\cos^3\theta - \cos\theta.$