No. of total pages : 5 SET - A GF-2-400

Roll. No. .... Name ....

NEW GREEN FIELDS SCHOOL
HALF YEARLY EXAMINATION, 2015-16
MATHEMATICS

Time : 3 hrs. **Class – XII** **M.M. : 100**

General Instructions—

(1) ***All questions are compulsory, though internal choices have been provided in 4 questions of 4 marks and two questions of 6 marks.***

(2) ***Questions have been subdivided in three sections. Section A has six questions of 1 mark each. Section B has thirteen questions (Q. 7 to Q. 19) of 4 marks each. Section C has 7 questions (Q. 20 to Q. 26) of 6 marks each.***

(3) ***Calculators are not allowed. Mathematical tables may, however, be asked for, if needed.***

(4) ***Any rough work needs to be done in a column on the right side of the answer sheet.***

(5) ***Only Blue/Blue black pen should be used to write the answers.***

(6) ***Please write the correct serial number of the question which you are attempting.***

SECTION—A

1 Evaluate :

$$\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$$

2 Find the value of x and y if :

$$2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$$

[P.T.O.]

3 Write the cofactor of the element a_{23} :

$$\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 2 & 0 & 4 \\ 6 & 2 & 3 \end{vmatrix}$$

4 Let * be a binary operation on N given by a * b = LCM (a, b) for all a, b $\in$ N. Find 5 * 7.

5 Differentiate $\sin^{-1} x$ with respect to $\tan x$. 5, 7

6 If $A = \begin{pmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{pmatrix}$, then for what value of a will A be an identity matrix.

SECTION—B

7 If the operation * on Q – {1}, is defined by a * b = a + b – ab for all a, b $\in$ Q – {1}, then find (a) the identity element and (b) the inverse of a for each a $\in$ Q – {1}.

OR

Let R be the relation on the set A of ordered pairs of positive integers defined by (x, y) R (u, v) $\Leftrightarrow$ xv = yu, show that R is an equivalence relation.

8 Prove :

$$\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x, \quad -\frac{1}{\sqrt{2}} \le x \le 1$$

9 Using properties of determinants prove the follwoing :

$$\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = -2(x^3+y^3)$$

$(x+y)(x^2+y^2-xy)$

10 If $x^y = e^{x-y}$, show that :

$$\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$$

11 If $y = (\tan^{-1} x)^2$, show that :

$$(x^2+1)^2 \frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2$$

12 Find the intervals in which the following function is (a) strictly increasing (b) strictly decreasing :

$$f(x) = -2x^3 - 3x^2 + 36x + 7$$

13 Find the values of x which satisfy the equation :

$$\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$$

14 Show that the 2 curves $x = y^2$ and $xy = k$ cut orthogonally if $8k^2 = 1$.

OR

Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is perpendicular to the line $5y - 15x = 13$.

15 From a lot of 15 bulbs which include 5 defectives, a sample of 2 bulbs is drawn at random (without replacement). Find the probability distribution of the number of defective bulbs.

16 Determine the value of k so that the following function is continuous at $x = \frac{\pi}{4}$.

$$f(x) = \begin{cases} \dfrac{k.\cos 2x}{\pi - 4x} & \text{if } x \neq \dfrac{\pi}{4} \\ 5 & \text{if } x = \dfrac{\pi}{4} \end{cases}$$

17 $x = \sec^3\theta$, $y = \tan^3\theta$ find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$

18 If $x \sin(a+y) + \sin a . \cos(a+y) = 0$, prove that :

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

OR

Using Rolle's theorem, find the point on the curve $y = x(x-4)$, $x \in [0, 4]$, where the tangent is parallel to x-axis.

19 Suppose you have two coins which appear identical in your pocket. You know that one is fair and one is 2-headed. If you take one out, toss it and get a head. What is the probability that it was a fair coin ?

OR

Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X, as well as the variance of X.

SECTION—C

20 Examine the consistency of the following system of simultaneous linear equations. If consistent, solve it :

$x - y + z = 4;\ 2x + y - 3z = 0;\ x + y + z = 2$

OR

Find the inverse of the following matrix using elementary row operations :

$$\begin{pmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{pmatrix}$$

21 A square piece of tin of side 18 cm is to be made into a box without a top, by cutting a square from each corner and folding up the flaps to form a box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible ?

22 Show that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

OR

A point on the hypotenuse of a right triangle is at distances 'a' and 'b' from the sides of the triangle. Show that the minimum length of the hypotenuse is $\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{3}{2}}$.

23 Let $f : N \rightarrow R$ be a function defined by $f(x) = 4x^2 + 12x + 15$. Show that $f : N \rightarrow S$ is invertible, where S is the range of f. Find inverse of f.

24 Using properteis of determinants prove that :

$$\begin{vmatrix} (b+c)^2 & ba & ca \\ ab & (c+a)^2 & cb \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

25 Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars. Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade. What is the probability that the student is a hosteller ?

Mr. Parth has to decide whether or not he should admit his son in the hostel in class 11th. He is very reluctant. He feels that his son will go astray. What would you do if you were in his place and why ?

26 A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is ₹ 5 each for type A and ₹ 6 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize the profit ? What is the maximum profit ?