

Raatik

FIRST TERM (2015-2016)

BB

CLASS XII

SUBJECT : MATHS

Time : 3 Hours

M.M. : 100

Note:

- Q. 1 — Q. 6 carry one mark each.
- Q. 7 — Q. 19 carry four marks each.
- Q. 20 — Q. 26 carry six marks each.

Q. 1. If  $f : \mathbb{R} \rightarrow \mathbb{R}$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  are given by  $f(x) = \sin x$  and  $g(x) = 5x^2$ , find  $\text{gof}(x)$ .

Q. 2. What is the domain of the function  $\sin^{-1} x$ ?

Q. 3. If  $A$  is a square matrix of order 3 such that  $|\text{adj } A| = 64$ , find  $|A|$ .

Q. 4. If  $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ , write  $A^{-1}$  in term of  $A$ ?

Q. 5. Area of a triangle with vertices  $(K, 0)$ ,  $(1, 1)$  and  $(0, 3)$  is 5 sq units. Find the value of  $K$ .

Q. 6. Evaluate  $\int \frac{1}{x+x \log x} dx$

Q. 7. Let  $A = \{1, 2, 3, \dots, 9\}$  and  $R$  be the relation in  $A \times A$  defined by  $(a, b) R (c, d)$  if  $a + d = b + c$ , for  $(a, b), (c, d) \in A \times A$ .

Prove that  $R$  is an equivalence relation. Also obtain the equivalence class  $[(2, 5)]$ .

Q. 8. Prove that:

$$\tan\left(\frac{\pi}{4} + \frac{1}{2} \cos^{-1} \frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2} \cos^{-1} \frac{a}{b}\right) = \frac{2b}{a}$$

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P.T.O.

$(c, d) R (a, b)$   
 $c + b$

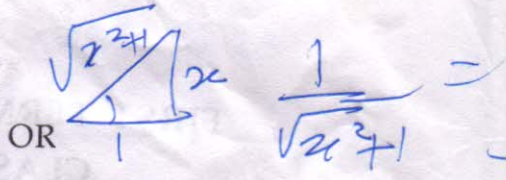
$R_1 = R_1 - R_2$

$OS$

$R_2 = S - 2$

$3 - 1 - 0 = 2$

$2 + y = 5 + x$

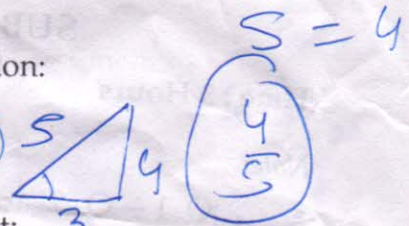


Prove that:

$$\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$$

Q. 9. Solve the following equation:

$$\cos(\tan^{-1} x) = \sin\left(\cot^{-1} \frac{3}{4}\right)$$



Q. 10. Find the matrix X such that:

$$\begin{bmatrix} 2 & -1 \\ 0 & 1 \\ -2 & 4 \end{bmatrix} X = \begin{bmatrix} -1 & -8 & -10 \\ 3 & 4 & 0 \\ 10 & 20 & 10 \end{bmatrix}$$

$\sin(\sin^{-1})$

Q. 11. Using properties of determinants, prove the following:

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

3 1 3) 2 5 7 5

Q. 12. If  $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ , find x and y such that  $A^2 + xI = yA$ .

Hence find  $A^{-1}$ .

Q. 13. Find the value of p and q so that:

$$f(x) = \begin{cases} x^2 + 3x + p & \text{if } x \leq 1 \\ qx + 2 & \text{if } x > 1 \end{cases}$$

is differentiable at  $x = 1$ .

OR

Find the value of K such that function:

$$f(x) = \begin{cases} 2^{x+2} - 16 & x \neq 2 \\ K & x = 2 \end{cases}$$

is continuous at  $x = 2$



Q. 14. If  $x = a(\cos t + t \sin t)$  and  $y = a(\sin t - t \cos t)$

Find  $\frac{d^2y}{dx^2}$ .

Q. 15. If  $x = \sin t$ ,  $y = \sin pt$ . Prove that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$$

Q. 16. The two equal sides of an isosceles triangle with fixed base  $b$  are decreasing at a rate of  $\beta$  cm/sec. How fast is the area decreasing when the two equal sides are equal to the base.

Q. 17. Find the interval in which the function  $f$  given by  $f(x) = \sin x + \cos x$ ,  $0 \leq x \leq 2\pi$ , is strictly increasing or strictly decreasing.

Q. 18. Evaluate:  $\int \frac{(x-4)e^x}{(x-2)^3} dx$

OR

Evaluate:  $\int \frac{dx}{\cos^2 x + \sin^2 2x}$

Q. 19. Evaluate:  $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$

Q. 20. Let  $A$  be the set of all real numbers except  $-1$ . Let  $*$  be defined on  $A$  as  $a * b = a + b + ab$  for all  $a, b \in A$ . Prove that operation  $*$  is commutative and associative. Find the identity element of operation  $*$ . Also find the inverse of elements if it exists.

Q. 21. Evaluate:  $\int_1^3 (e^{2-3x} + x^2 + 1) dx$ . Using limit of sum.

OR

Evaluable:  $\int_{-1}^{\frac{3}{2}} |x \sin \pi x| dx$ .



$$\frac{3x}{5} + \frac{4\sqrt{1-x^2}}{5} \quad \frac{1}{3} + \frac{7}{3} \frac{5}{23}$$

Q. 22. Evaluate  $\int \sqrt{\tan x} dx$

Q. 23. Find  $\frac{dy}{dx}$  for each of the following function:

(i)  $y = \cos^{-1} \left( \frac{3x + 4\sqrt{1-x^2}}{5} \right)$

(ii)  $(\cos x)^y = (\cos y)^x$

$a * b = b = 0 + a$

Q. 24. Two institutions decided to award their employees for the three values of resourcefulness, competence and determination in the form of prizes at the rate of ₹ x, ₹ y and ₹ z respectively per person. The first institution decided to award respectively 2, 3 and 2 employees with a total prize money of ₹ 37,000 and the second institution decided to award respectively 5, 3 and 4 employees with a total prize money of ₹ 47,000. If all the three prizes per person together amount to ₹ 12,000, then using matrix method, find the value of x, y and z. Apart from these values, suggest one or more value which institutions must include for awards.

OR

Using elementary transformation, find the inverse of the matrix:

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

Q. 25. Find the equation of the normal at a point on the curve  $x^2 = 4y$  which passes through the point (1, 2). Also find the equation of the corresponding tangent.

Q. 26. Show that volume of greatest cylinder which can be inscribed in a cone of height h and semivertical angle  $\alpha$  is  $\frac{4}{27} \pi h^3 \tan^2 \alpha$ .

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$$\frac{40 - 35}{3} = \frac{5}{3}$$

$$\frac{12 - 35}{3} = \frac{-23}{3}$$

$$0 - 3 \frac{1}{3}$$