Kartik

FIRST TERM (2015-2016)

BB

CLASS XII

SUBJECT : MATHS

Time : 3 Hours **M.M. : 100**

Note:

- Q. 1 —Q. 6 carry one mark each.
- Q. 7—Q. 19 carry four marks each.
- Q. 20—Q. 26 carry six marks each.

Q. 1. If $f : R \to R$ and $g : R \to R$ are given by $f(x) = \sin x$ and $g(x) = 5x^2$, fidn $gof(x)$.

Q. 2. What is the domain of the function $\sin^{-1} x$?

Q. 3. If A is a square matrix of order 3 such tht $|\text{adj } A| = 64$, find $|A|$.

Q. 4. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, write A^{-1} in term of A?

$R_1 = R_1 - R_2$

$0 \quad 5$

$R_2 = \quad 5 \quad -2$

Q. 5. Area of a triangle with vertices (K, O), (1, 1) and (0, 3) is 5 sq units. Find the value of K.

Q. 6. Evaluate $\int \frac{1}{x + x \log x} dx$

Q. 7. Let $A = \{1, 2, 3, ... 9\}$ and R be the relation in $A \times A$ defined by $(a, b) R (c, d)$ if $a + d = b + c$, for $(a, b), (c, d) \in A \times A$.

Prove that R is an equivalence reaction. Also obtain the equivalence class $[(2, 5)]$. x & y

$2 + y = 5 + x$

Q. 8. Prove that:

$$\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$$

$(c, d) R (a, b)$

$c + b$

OR

Prove that:

$\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi$

Q. 9. Solve the following equation:

$$\cos(\tan^{-1}x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$$

Q. 10. Find the matrix X such that:

$$\begin{bmatrix} 2 & -1 \\ 0 & 1 \\ -2 & 4 \end{bmatrix} X = \begin{bmatrix} -1 & -8 & -10 \\ 3 & 4 & 0 \\ 10 & 20 & 10 \end{bmatrix}$$

Q. 11. Using proprties of determinants, prove the following:

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

Q. 12. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y such that $A^2 + xI = yA$.

Hence find A^{-1}.

Q. 13. Find the value of p and q so that:

$$f(x) = \begin{cases} x^2 + 3x + p & \text{if } x \le 1 \\ qx + 2 & \text{if } x > 1 \end{cases}$$

is differentiable at $x = 1$.

OR

Find the value of K such that function:

$$f(x) = \begin{cases} \dfrac{2^{x+2} - 16}{4^x - 16} & x \ne 2 \\ K & x = 2 \end{cases}$$

is continous at $x = 2$

Q. 14. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$

Find $\dfrac{d^2y}{dx^2}$.

Q. 15. If $x = \sin t$, $y = \sin pt$. Prove that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$$

Q. 16. The two equal sides of an isosceles triangle with fixed base. b are decreasing at a rate of 3cm/sec. How fast is the area decreasing when the two equal sides are equal to the base.

Q. 17. Find the interval in which the function f given by $f(x) = \sin x + \cos x$, $0 \le x \le 2\pi$, is strictly increasing or strictly decreasing.

Q. 18. Evaluate: $\displaystyle\int \frac{(x-4)e^x}{(x-2)^3}\,dx$

OR

Evaluate: $\displaystyle\int \frac{dx}{\cos^2 x \sin^2 x}$

Q. 19. Evaluate: $\displaystyle\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2 \sin^2 x}\,dx$

Q. 20. Let A be the set of all real numbers except –1. Let * be defined on A as a * b = a + b + ab for all a, b ∈ A. Prove that operation * is commutative and associative. Find the identity element of operation *. Also find the inverse of elements if it exists.

Q. 21. Evaluate: $\displaystyle\int_1^3 \left(e^{2-3x} + x^2 + 1\right)dx$. Using limit of sum.

OR

Evaluable: $\displaystyle\int_{-1}^{3/2} |x \sin \pi x|\,dx$.

$\frac{3x}{5} + \frac{4}{5}\sqrt{1-x^2}$ $\frac{3}{5}$ x $\frac{1}{3} + \frac{7}{3}$ $\frac{5}{23}$

Q. 22. Evaluate $\int \sqrt{\tan x}\, dx$

Q. 23. Find $\frac{dy}{dx}$ for each of the following function:

(i) $y = \cos^{-1}\left(\frac{3x + 4\sqrt{1-x^2}}{5}\right)$

(ii) $(\cos x)^y = (\cos y)^x$

Q. 24. Two institution decided to award their employes for the three values of resourcefulness, competence and determination in the form of prizes at the rate of ₹ x, ₹ y and ₹ z respectivel per person. The first institution decided to award respectively 2, 3 and 2 employees with a total prize money of ₹ 37,000 and the second institution decided to award respectively 5, 3 and 4 employees with a total prize moeny of ₹ 47,000. If all the three prizes per person together amount to ₹ 12,000, then using matrix method, find the value of x, y and z. Apart from these values, suggest one or more value which institutions must include for awards.

OR

Using elementary transformation, find the inverse of the matrixz;

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

Q. 25. Find the equation of the normal at a point on the curve $x^2 = 4y$ which passes through the point (1, 2). Also find the equation of the corresponding tangent.

Q. 26. Show that volume of greatest cylinder which can be inscribed in a cone of height h and semivertical angle α is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

$x^2 = 4y$

$4 - \frac{35}{3}$ $\frac{12-35}{3} = -\frac{23}{3}$