Name Shreshth Class & Section XII-C Roll No. 23

BIRLA

FIRST TERMINAL EXAMINATION—2016-17

CLASS-XII

SUBJECT–MATHEMATICS

Time : 3 Hours **M.M. : 100**

Please Check the Total Marks

General Instructions :

Do not write any answer on the question paper.

Check the total marks. This question paper contains 29 questions.

Attempt all the questions. However, internal choice has been provided in a few questions.

Section A consists of 4 questions carrying 1 mark each.

Section B consists of 8 questions carrying 2 marks each.

Section C consists of 11 questions carrying 4 marks each.

Section D consists of 6 questions carrying 6 marks each.

Section-A

1. Let R be an equivalence relation in the set A = {0, 1, 2, 3, 4, 5} given by R = {(a, b) : 2 divides $a - b$}. Write the equivalence class of 0.

2. Find the value of expression
$\sin [\cot^{-1} \{\cos (\tan^{-1} 1)\}]$

3. Give an example of a function which is onto but not one-one.

4. If $y^2 + \tan (xy) = 100$, find $\frac{dy}{dx}$.

Section-B

5. Prove that the diagonal elements of a skew symmetric matrix are all zeroes.

6. If $f : [-5, 5] \to R$ is a differentiatable function and if $f'(x)$ does not vanish anywhere, then prove that $f(-5) \neq f(5)$

7. Evaluate : $\int_{-1}^{1} e^{|x|}\, dx$

8. Evaluate $\int \tan^8 x \sec^4 x\, dx$

9. Determine $f(0)$ so that the function $f(x)$ defined by

$$f(x) = \frac{\left(4^x - 1\right)^3}{\sin\frac{x}{4}\log\left(1 + \frac{x^2}{3}\right)}$$ is continuous at $x = 0$

10. If $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$, then find $|\text{adj (adj A)}|$

11. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating the volume.

12. Let '$*$' be a binary operation on N given by $a * b = \text{LCM}\,(a, b)\ \forall\ a, b \in \text{N}$. Then find

(a) the identity of '$*$' in N.

(b) the elements of N which are invertible for the operation '$*$'.

Section-C

13. Evaluate : $\tan^{-1}\left(\frac{3\sin 2\alpha}{5 + 3\cos 2\alpha}\right) + \tan^{-1}\left(\frac{1}{4}\tan\alpha\right)$ where $\frac{-\pi}{2} < \alpha < \frac{\pi}{2}$

OR

If $y = \cos^{-1}\left\{x\sqrt{1-x} + \sqrt{x}\sqrt{1-x^2}\right\}, 0 < x < 1$ find $\frac{dy}{dx}$.

14. Evaluate : $\int \frac{\sqrt{x^2+1}\left\{\log(x^2+1) - 2\log x\right\}}{x^4}\,dx$

15. Find the equation of tangents to the curve $y = \cos(x + y), -2\pi \le x \le 2\pi$, parallel to the line $x + 2y = 0$.

16. Solve the equation :

$$\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}\,x = \frac{-\pi}{2}$$

17. In a ΔABC, if

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 + \sin A & 1 + \sin B & 1 + \sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix} = 0$$

then prove that ΔABC is an isosceles triangle.

18. Let $A = \begin{bmatrix} 0 & -\tan \alpha/2 \\ \tan \alpha/2 & 0 \end{bmatrix}$ and I be the identity matrix of order 2. Show that

$$I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

19. If $y = \dfrac{ax - b}{(x-1)(x-4)}$ has a turning point P (2, – 1), find the values of a and b and show that y is maximum at P.

OR

Find the difference between the greatest and least values of the function

$f(x) = \sin 2x - x$ on $\left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]$.

20. Find the intervals in which the function f given by $f(x) = \sin x + \cos x$, $0 \le x \le 2\pi$ is increasing or decreasing.

21. Water is running into a conical vessel, 15 cm deep and 5 cm radius at the rate of 0.1 cm^3/sec. When the water is 6 cm deep, find at what rate is the wet surface of the vessel increasing.

22. A function $f : R \to R$ satisfies the equation $f(x + y) = f(x) \cdot f(y)$ $\forall$ $x, y \in R$, $f(x) \neq 0$. Suppose the function is differentiable at $x = 0$ and $f'(0) = 2$, then prove that $f'(x) = 2 f(x)$.

23. Evaluate $\int_1^3 (x^2 + x)\, dx$ as limit of sums.

Section-D

24. Sketch the region common to the circle $x^2 + y^2 = 16$ and the parabola $x^2 = 6y$. Also, find the area of the region using integration.

25. Evaluate : $\int_0^{\pi/2} \dfrac{1}{\left(a^2 \cos^2 x + b^2 \sin^2 x\right)^2}\, dx$

26. Integrate any two :

(a) $\int \dfrac{1}{\sin x(2\cos^2 x - 1)}\, dx$

(b) $\int \sin x \sin 2x \sin 3x\, dx$

(c) $\int \sec^{-1}\sqrt{x}\, dx$

27. An amount of ₹ 5000 is put into three investments at the rate of interest of 6%, 7% and 8% per annum respectively. The total annual income is ₹ 358. If the combined income from the first two investments is ₹ 70 more than the income from the third, find the amount of each investment by matrix method. 'Disclosing income and paying income tax on time contribute to the development of the nation.' Comment.

28. If $I_1 = \int e^{ax} \cos bx \, dx$ and $I_2 = \int e^{ax} \sin bx \, dx$, prove that

$$\left(a^2 + b^2\right)\left(I_1^2 + I_2^2\right) = e^{2ax}$$

29. A point on the hypotenuse of a right triangle is at distances a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is $\left(a^{2/3} + b^{2/3}\right)^{3/2}$.

OR

Tangents to the circle $x^2 + y^2 = a^2$ at any point on it in the first quadrant makes intercepts OA and OB on x and y axes respectively, O being the centre of the circle. Find the minimum value of OA + OB.

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