FIRST TERMINAL EXAMINATION 2016 - 2017

CLASS - XII

SUBJECT: MATHEMATICS

TIME: 3 HOURS M.M.: 100

Note :

1. Q1 – Q4 are of 1 mark each.
2. Q5 - Q12 are of 2 marks each.
3. Q13 - Q23 are of 4 marks each.
4. Q24 - Q29 are of 6 marks each.
5. All questions have to be done in order.

Section - A

Q1. Find α given that $A + A' = I$ if $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Q2. Evaluate : $\sec^{-1}(-\sqrt{2}) + \sin^{-1}(-\frac{\sqrt{3}}{2})$.

Q3. Find $\frac{dy}{dx}$ given that $x = ct$, $y = \frac{c}{t}$.

Q4. Evaluate : $\int_{-\pi}^{\pi} \frac{2x}{1+\cos x}\, dx$.

Section – B

Q5. Find x if $\begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x \\ 3 \end{bmatrix} = 0$.

Q6. Find k given that A (2, −3), B (k, −1) and C (0,4) are collinear.

Q7. Find gof given that $f(x) = \log(x+2)$ and $g(x) = \sqrt{x+5}$.

Q8. Prove : $\tan^{-1}\frac{63}{16} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$

OR

Solve : $\tan(\cos^{-1} x) = \sin(\tan^{-1} 2)$.

Q9. Find $\frac{dy}{dx}$ given that $\sin(xy) + \frac{x}{y} = x^2 - y$.

Q10. A spherical balloon is being inflated by pumping in 900 cm^3 of gas per second. Find the rate at which its surface area is increasing when the radius is 15 cm.

Q11. Evaluate : $\int e^x (\tan x - \log|\cos x|)\, dx$.

Q12. Evaluate : $\int x \tan x \sec^2 x\, dx$.

Section – C

Q13. For the matrix $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y so that $A^2 + xI = yA$.

Hence, find A^{-1}.

Q14. Using properties of determinants, prove that :

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ac)^3.$$

Q15. Let $f: R \to R$ be defined by $f(x) = 5x - 9$. Find f^{-1}, if it exists.

Q16. Differentiate $y = \tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right)$ w.r.t. $z = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$

Q17. If $y = \left[\log(x + \sqrt{x^2+1})\right]^2$, show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2$.

Q18. Verify Langrange's Mean Value Theorem for the function $f(x) = (x-3)(x-6)(x-9)$ in $[3, 5]$.

Q19. Find the intervals in which $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is strictly increasing and /or decreasing.

Q20. Evaluate : $\int (2x+1)\sqrt{4+6x-3x^2}\ dx$.

Q21. Evaluate : $\int Cosec^3x\ dx$.

Q22. Evaluate : $\int_{-1}^{5} (2x^2 - 3x + 7)\,dx$. as a limit of sums.

Q23. Using properties of definite integrals, evaluate $\int_0^{\frac{\pi}{2}} \frac{x\ dx}{\sin x + \cos x}$.

Section – D

Q24. If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1}.

Hence, solve the following system of linear equations :
$x - 2y = 10; \quad 2x + y + 3z = 8; \quad -2y + z = 7.$

Q25. (i) Let N be the set of all natural numbers and R be a relation on N × N defined by
(a, b) R (c, d) iff ad (b + c) = bc (a + d)
Check whether R is an equivalence relation.

(ii) A binary operation * is defined by : a * b = a + b + ab for all a, b ∈ R −{ −

- Check whether * is associative.
- Find the identity element, if it exists.
- Find the inverse of an element of R −{ − 1}.

Q26. Find $\frac{dy}{dx}$ given $y = (x\cot x)^{\sin x} + \frac{e^{3x}.\cos^2 x}{2(5x-4)}$

Q27. Find the volume of the largest right circular cylinder that can be inscribed in a sphere of radius R.

Q28. Evaluate : $\int \frac{\cot x + \cot^3 x}{1 + \cot^3 x}\ dx$.

Q29. Using integration, find the area bounded by the curves
$4x^2 + 4y^2 \le 9$ and $x^2 \le 4y$