

Army
Public

HALF YEARLY EXAM (2016-17)
CLASS - XII
SUBJECT : MATHEMATICS

Time : 3 Hrs

M.M. : 100

General Instructions :

1. All questions are compulsory.
2. The question paper contains 29 questions.
3. Q.No 1 to 4 in Section A are 1 mark each.
4. Q.No 5 to 12 in Section B are 2 marks each.
5. Q.No. 13 to 23 in Section C are 4 marks each with four internal choices.
6. Q.No. 24 to 29 in Section D are 6 marks each with two internal choices.

SECTION A

1. Find $\tan^{-1}(\tan 9\pi/8)$
2. Evaluate $\int_1^2 [2x] dx$
3. If $f(x) = |\cos x|$, find $f'(3\pi/4)$
4. Find the value of c in Rolle's Theorem for the function $f(x) = x^3 - 3x$ in the interval $[0, \sqrt{3}]$

SECTION B

5. Show that the signum function is neither one- one nor onto.
6. Prove that $\sin^{-1}(3/5) = \frac{1}{2} \tan^{-1}(\frac{24}{7})$
7. If $A = \begin{bmatrix} x & 2 \\ 2 & x \end{bmatrix}$, $|A^4| = 625$. Find x
8. Show that the points $(a + 5, a - 4)$, $(a - 2, a + 3)$ and (a, a) do not lie on a straight line for any value of a
9. Evaluate: $\int \frac{dx}{e^x + e^{-x}}$
10. Solve: $\frac{dy}{dx} = 1 - x + y - xy$
11. Find x , if $[x \quad -5 \quad -1] \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$.
12. Find: $\int e^{2x} (-\sin x + 2 \cos x) dx$

SECTION C

13. Show that the curves $x = y^2$ and $xy = k$ cut orthogonally if $8k^2 = 1$.

P.T.O.

14. Water is dripping from a conical funnel at the rate of 5 cu.cm/sec. If the radius of the funnel is 10 cm and its height is 20 cm, find the rate at which the water level is dropping when it is 5 cm from the top.

15. Solve using properties of integrals:

$$\int_0^{\pi} \frac{x}{1+\cos^2 x} dx \quad \text{OR} \quad \int_0^{\pi/2} \frac{dx}{1+\sqrt{\cot x}}$$

16. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

17. Find the values of a for which the function 'f' defined as following is continuous

$$f(x) = \begin{cases} a \sin \frac{\pi}{2} (x+1), & x \leq 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$$

18. Separate the interval $[0, \pi/2]$ into sub intervals in which $f(x) = \sin^4 x + \cos^4 x$ is strictly increasing or decreasing.

19. Let I be the set of integers and R be a relation on I defined as

$R = \{(a, b) : a, b \in I \text{ and } (a-b) \text{ is divisible by } 5\}$. Prove that R is an equivalence relation.

$$20. \text{ Evaluate: } \int \frac{dx}{\sqrt[3]{x+\sqrt{x}}} \quad \text{OR} \quad \int \frac{dx}{\sqrt{7-6x-x^2}}$$

21. If the tangent to the curve $y = x^3 + ax + b$ at $P(1, -6)$ is parallel to the line $y - x = 5$, find the values of a and b

22. Solve the Differential equation: $y^2 dx + (x^2 - xy + y^2) dy = 0$
OR

$$(1+x^2) \frac{dy}{dx} + y = \tan^{-1} x$$

23. Solve for x: $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}x = -\frac{\pi}{2}$
OR

If $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$, then find x.

SECTION D

24. Given $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, find BA and use this to solve the system of equations: $y+2z=7$, $x-y=3$, $2x+3y+4z=17$

③

①

②

25 Evaluate: $\int_{-2}^2 |x \cos \pi x| dx$

OR

Evaluate the following as limit of sum: $\int_{-1}^2 (7x^2 - 5) dx$.

26. Show that the maximum volume of a cylinder which can be inscribed in a cone of height 'h' and semi-vertical angle α is $\frac{4}{27} \pi h^3 \tan^2 \alpha$.

27. Find the area of that part of the circle $x^2 + y^2 = 16$ which is exterior to the parabola $y^2 = 6x$.

28. Evaluate: $\int \sqrt{\tan x} \, dx$ OR $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} \, dx$

29. Show that the given differential equation is homogeneous, and hence solve it:

$$(3xy + y^2) \, dx + (x^2 + xy) \, dy = 0$$