

16. Prove that $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) = \pi$

17. Prove that the function $f: R \rightarrow R$ defined by $f(x) = \frac{x}{x^2+1} \forall x \in R$ is neither one-one nor onto.
 Modify the co-domain of $f(x)$ to make it surjective.

18. If $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$ and $A^{-1} = A'$ where A' denotes transpose of A , then find the value of α .

19. Show that the function, $f(x) = 2x + \cot^{-1} x + \log(\sqrt{1+x^2} - x)$ is increasing in R .

20. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{j} - \hat{k}$, find a vector \vec{c} such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$.

21. Let $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$. Find the angle between $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$. Also find a unit vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

22. Find the equation of the tangent to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where the curve cuts the x-axis.

23. Solve the Linear Programming Problem graphically:

Minimise $Z = x + 2y$

Subject to $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$

SECTION D

24. Let M be the set of all singular matrices of the form $\begin{bmatrix} x & x \\ x & x \end{bmatrix}$, $x \in R - \{0\}$. Let $*$ be an operation defined on M by $A*B = AB \forall A, B \in M$. Determine whether $*$ is binary, commutative and associative. Determine the identity element in M , if any. Find all invertible elements, if possible.

25. Find the values of x which satisfy the equation $\cos(\sin^{-1}x + \sin^{-1}(1-x)) = x$. Also find the values of y which satisfy the equation $\sin^{-1}(\sin(y)) = y = \cos^{-1}(\cos(y))$

26. For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$. Show that $A^3 - 6A^2 + 5A + 11I = 0$.

Hence find the values of A^4 and A^{-1} .

27. An amount of Rs.5000 is put into three investments at the rate of interest of 6%, 7% and 8% per annum respectively. The total annual income from interest is Rs.358. If the combined income from the first two investments is Rs.70 more than the income from third, find the amount of each investment using matrix method. According to you, are savings or investments important in one's life? Give reasons.

28. Show that the semi vertical angle of a right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

29. A manufacturing company makes two models A and B of a product. Each piece of a model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs.8000 on each piece of model A and Rs. 12000 on each piece of Model B. Formulate this problem as a L.P.P. and determine how many pieces of Model A and Model B should be manufactured per week to realise a maximum profit. What is the maximum profit per week?

GENERAL INSTRUCTIONS:

- (i) All questions are compulsory
(ii) The question paper consists of 29 questions divided in to 4 sections A, B, C and D.
Questions 1 - 4 in Section A are very short - answer type questions carrying 1 mark each.
Questions 5 - 12 in Section B are short - answer type questions carrying 2 mark each.
Questions 13 - 23 in Section C are long - answer-I type questions carrying 4 mark each.
Questions 24 - 29 in Section D are long - answer-II type questions carrying 6 mark each.
- (iii) All questions in section A are to be answered in one word / one sentence as per the exact requirement of the question.
- (iv) There is no overall choice. However, internal choice has been provided in some questions. You have to attempt only one of the alternatives in all such questions.
- (v) Use of calculators is not permitted.

SECTION A

1. Find $\frac{dy}{dx}$ if $y=x^x$.
2. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then find the value of $|A| |Adj A|$.
3. Find the values of λ if the angle between the vectors $\vec{a} = 2\lambda^2 \hat{i} + 4\lambda \hat{j} + \hat{k}$ and $\vec{b} = 7\hat{i} - 2\hat{j} + \lambda \hat{k}$ is obtuse.
4. Find the vector equation of a line passing through $(-2, 4, -5)$ and parallel to the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.

SECTION B

5. Find the equation of the line joining $A(1,3)$ and $B(0,0)$ using determinants. Also find k if $D(k,0)$ is a point such that area of $\triangle ABD$ is 3 sq. units.
6. If $e^y(x+1) = 1$, show that $\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 = 0$.
7. Give example of three matrices A, B and C to show that $AB = AC \Rightarrow B = C$.
8. Let $f, g: \mathbb{R} \rightarrow \mathbb{R}$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x$. find fog and gof.
9. Simplify $\tan(\cos^{-1} x)$ and hence find the value of $\tan(\cos^{-1} \frac{8}{17})$.
10. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - 4\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} + 3\hat{k}$, find the altitude of the parallelepiped determined by the vectors \vec{a} , \vec{b} and \vec{c} , where base is given by the parallelogram with \vec{a} and \vec{b} as the adjacent sides.
11. If x changes from 4 to 4.01, then find the approximate change in $\log_e x$.
12. Find the value of p so that lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.

SECTION C

13. Prove using properties of determinants

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

14. Find the values of a and b if $f(x) = \begin{cases} x^2 + 3x + a & x \leq 1 \\ bx + 2 & x > 1 \end{cases}$ is everywhere differentiable.
15. Prove that the derivative of $\tan^{-1} \left(\frac{\sqrt{1+x^2}-1}{x} \right)$ with respect to $\tan^{-1} x$ is independent of x .