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ANDHRA EDUCATION SOCIETY SCHOOLS
NEW DELHI
SUMMATIVE ASSESSMENT-I (2016-17)
CLASS - XII
SUBJECT - MATHEMATICS

Time : 3:00 Hrs.

Max Marks : 100

All questions are compulsory

- a) Section A questions carry 1 mark each.
- b) Section B questions carry 2 mark each.
- c) Section C questions carry 4 mark each.
- d) Section D questions carry 6 marks each.

SECTION - A

1. Find the values of $\tan(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2})$.
2. Prove that $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$.
3. If $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, then $A + A^T = I$, if the value of α .
4. Evaluate : $\int \frac{e^{2x}}{e^x+2} dx$.

SECTION -B

5. Find $\frac{dy}{dx}$ if $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$
6. Let $f : \{1, 3, 4\} \rightarrow \{1, 2, 5\}$ and $g : \{1, 2, 5\} \rightarrow \{1, 3\}$ be given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$. Write down $g \circ f$.
7. Show that that the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is symmetric but neither reflexive nor transitive.
8. Consider the binary operation
• on the set $\{1, 2, 3, 4, 5\}$ defined by $a * b = \min\{a, b\}$. Write the operation table of the operation.
9. Prove that $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$.
10. If $\sin(\sin^{-1}\frac{1}{5} + \cos^{-1}x) = 1$, then find the value of x .
11. Construct a 2×2 matrix, $A = [a_{ij}]$, whose elements are given by $a_{ij} = \frac{i}{j}$.
12. Find equation of line joining $(1, 2)$ and $(3, 6)$ using determinants.

SECTION-C

13. Show that the relation R defined by $(a,b) R (c,d)$ iff $a + d = b + c$ on the set $N \times N$ is an equivalence relation.

OR

Let $*$ be a binary operation on Q defined by $a * b = \frac{3ab}{5}$. Show that $*$ is commutative as well as associative. Find its identity, if exist.

14. Prove that : $\cot^{-1} \left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right] = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$.

15. Find inverse of given matrix by Elementary transformations: $\begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$.

16. Show that the function $f(x) = |x - 3|, x \in R$, is continuous but not differentiable at $x = 3$.

17. Differentiate the following with respect to x : $\sin^{-1} \left[\frac{2^{x+1} 3^x}{1+36^x} \right]$.

OR

Differentiate the following function w.r.t. x ; $(\log x)^x + x^{\log x}$.

18. Find the intervals in which the function f given by $f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

OR

Using differentials, find the approximate value of $f(2.01)$, where $f(x) = 4x^3 + 5x^2 + 2$.

19. Sand is pouring from a pipe at the rate of $12 \text{ cm}^3/\text{s}$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

20. Find : $\int \frac{1}{\sin x + \sin 2x} dx$.

21. Evaluate ; $\int \frac{\cos x}{(2 + \sin x)(3 + \sin x)} dx$. OR $\int \left(\frac{1 + \sin x}{1 + \cos x} \right) e^x dx$.

22. Evaluate ; $\int_0^{\frac{\pi}{2}} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx$. OR $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$.

23. Solve the L.L.P. minimum $Z = 2x + 3y$ subject to the constraints:

$$2x + 3y \geq 6, \quad x - y \geq 0, \quad 2x + y \leq 8, \quad x \geq 0, \quad y \geq 0$$

SECTION - D

24. A company produces soft drinks that has a contract which requires that a minimum of 80 units of chemical A AND 60 UNITS of the chemical B go into each bottle of drink. The chemicals are available in prepared mix packets from two different suppliers. Supplier S had a packet of mix of 4 units of A and 2 units of B that costs Rs. 10. The supplier T has a packet of mix of 1 unit of A and 1 unit of B that costs Rs. 4. How many packets from mixes of S and T should the company purchase to honour the contract requirement and yet minimise cost? Make a L.L.P. and solve graphically.

25. Evaluate; $\int_1^3 (x^2 + x) dx$ as the limit of sum.

OR

Prove that : $\int_0^{\frac{\pi}{4}} (\sqrt{\tan x} + \sqrt{\cot x}) dx = \sqrt{2} \frac{\pi}{2}$.

26. A window is in the form of a rectangle surmounted by a semicircle. The total perimeter of the window is 10. find the dimensions of rectangular part of the window to admit maximum light through it.

27. Show that the surface area of closed cuboid with the square base and given volume is minimum when it is cube.

OR

The sum of the perimeters of a square and circle is k , where k is some constant. Show that the sum of their areas is least when the side of the square is equal to diameter of circle.

28. If $x = \frac{\sin^2 t}{\sqrt{\cos 2t}}$; $y = \frac{\cos^2 t}{\sqrt{\cos 2t}}$, then find $\frac{dy}{dx}$.

29. If $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 3 & -1 \\ -2 & 1 & 1 \end{bmatrix}$, find A^{-1} . Using Elementary transformation method and solve the system of equations: $2x + y + 3z = 9$, $x + 3y - z = 2$, $-2x + y + z = 7$.