Mukul Dahiya
XII - B

AMITY, PV

First Terminal Examination 2016 - 2017

Class – XII

Subject – Mathematics

Time : 3 Hours **Max. Marks : 100**

General Instructions :

1. All questions are compulsory.
2. The question paper consists of 29 questions.
3. Questions 1 to 4 in Section A are very short answer type questions of 1 mark each.
4. Questions 5 to 12 in Section B are short answer type questions of 2 marks each.
5. Questions 13 to 23 in Section C are long answer type I questions of 4 marks each.
6. Questions 24 to 29 in Section D are long answer type II questions of 6 marks each.

SECTION – 'A' (1×4=4)

1. If $f(x) = x + 7$ and $g(x) = x - 7$, $x \in R$, find $(fog)(7)$.

2. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, find α satisfying $0 < \alpha < \frac{\pi}{2}$, when $A + A^T = \sqrt{2} I_2$; where A^T is transpose of A.

3. If $A = \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}$ is written as P + Q, where P is a symmetric matrix and Q is a skew symmetric matrix, then write the matrix P.

4. Evaluate : $\sin^{-1}\left(\frac{-1}{2}\right) + 2\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

SECTION – 'B' (2×8=16)

5. If $y = \tan^{-1}\left(\frac{5x}{1-6x^2}\right)$, $\frac{-1}{\sqrt{6}} < x < \frac{1}{\sqrt{6}}$, then prove that $\frac{dy}{dx} = \frac{2}{1+4x^2} + \frac{3}{1+9x^2}$.

6. If $\int (e^{ax} + bx)\,dx = 4.e^{ax} + \frac{3}{2}x^2$, find the value of a and b.

7. Solve for x :

$$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x, \quad x > 0$$

8. If $\begin{bmatrix} 2x & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ 8 \end{bmatrix} = 0$, find the value of x.

9. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm then, find the approximate error in calculating its volume.

10. Using elementary transformation, find the inverse of $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$.

11. Differentiate $\sqrt{\tan\sqrt{x}}$ w.r.t x.

12. Find the condition for the curves $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; $xy = c^2$ to intersect orthogonally.

SECTION – 'C' (4×11=44)

13. By using the properties of determinants show that :

$$\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz\,(x+y+z)^3$$

14. If $f(x) = \begin{cases} \dfrac{\sin(a+1)x + 2\sin x}{x} & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ \dfrac{\sqrt{1+bx} - 1}{x} & \text{if } x > 0 \end{cases}$

is continuous at x = 0, then, find the values of a and b.

15. Solve for x : $\tan^{-1}(x-1) + \tan^{-1}(x) + \tan^{-1}(x+1) = \tan^{-1}3x$

OR

Prove that : $\tan^{-1}\left(\frac{6x - 8x^3}{1 - 12x^2}\right) - \tan^{-1}\left(\frac{4x}{1 - 4x^2}\right) = \tan^{-1}2x; \quad |2x| < \frac{1}{\sqrt{3}}$

16. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic metre per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4 m.

17. Find : $\int \frac{(2x-5)\, e^{2x}}{(2x-3)^3}\, dx$

OR

Find : $\int \frac{x^2+x+1}{(x^2+1)(x+2)}\, dx$

18. Solve the equation $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}x = \frac{-\pi}{2}$.

19. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$, where $x \in \frac{1}{\sqrt{2}}, 1$.

20. Find the area lying above x-axis and included between the curves $x^2 + y^2 = 8x$ and inside the curve $y^2 = 4x$.

21. On the set $R - \{-1\}$ a binary operation '$*$' is defined by $a * b = a + b + ab$ for all $a, b \in R - \{-1\}$, prove that '$*$' is commutative as well as associative on $R - \{-1\}$. Find the identify element also.

OR

Let $f : N \longrightarrow R$ be a function defined as $f(x) = 4x^2 + 12x + 15$.

Show that $f : N \longrightarrow S$, where S is the range of f, is invertible. Also find the inverse of f.

22. On her birthday, Garima decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got ₹ 10 more. However, if there were 16 children more, everyone would have got ₹ 10 less. Using matrix method, find the number of children and the amount distributed by Garima. What values are reflected by Garima's decision ?

23. Find the intervals in which the function given by $f(x) = \frac{4\sin x - 2x - x\cos x}{2 + \cos x}$ is

(i) increasing (ii) decreasing.

SECTION – 'D' (6×6=36)

24. Evaluate the following definite integral as limit of sum :

$$\int_0^4 (x + e^{2x})\, dx$$

25. Using Integration, find the area bounded by the tangent to the curve $4y = x^2$ at the point (2, 1) and the lines, whose equations are $x = 2y$ and $x = 3y - 3$.

26. A given rectangular area is to be fenced off in a field, whose length lies along a straight river. If no fencing is needed along the river, show that least length of fencing will be required, when length of the field is twice its breadth.

OR

If the sum of the lengths of the hypotenuse and a side of a right triangle is given, show that the area of the triangle is maximum, when the angle between them is $\frac{\pi}{3}$.

27. Show that the relation R in the Set A = {1, 2, 3, 4, 5} given by

$$R = \left\{(a,\ b) : |a - b| \text{ is even}\right\}$$

is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But, no element of {1, 3, 5} is related to any element of {2, 4}.

28. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$,

verify that $A^3 - 6A^2 + 9A - 4I = 0$ and hence, find A^{-1}.

29. Using the properties of definite integrals evaluate :

$$\int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x}\, dx$$

OR

$$\int_0^{\pi} \frac{x\, dx}{a^2\cos^2 x + b^2\sin^2 x}$$

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