First Terminal Examination 2016 - 2017

Class – XII

Subject – Mathematics

AMITY

Time : 3 Hours **Max. Marks : 100**

General Instructions :

(a) All questions are compulsory.

(b) The question paper consists of 29 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of 1 mark each, Section B comprises of 8 questions of 2 marks each, Section C comprises of 11 questions of 4 marks each and Section D comprises of 6 questions of 6 marks each.

(c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(d) Do all the questions, however internal choice is provided in 4 questions of 4 marks and 2 questions of 6 marks.

(e) Use of calculators is not permitted.

SECTION – 'A' (1×4=4)

1. Evaluate : $\int \frac{\sin 2x\,dx}{a^2\cos^2 x + b^2 \sin^2 x}$.

2. If $f(x) = [x]$ and $g(x) = |x|$, find $gof\left(\frac{5}{3}\right) - fog\left(\frac{5}{3}\right)$.

3. Differentiate $\log_8(\log x)$ with respect to x.

4. If A is a square matrix of order 2 and adj (adj A) = $\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$, find $|A|$.

SECTION – 'B' (2×8=16)

5. Given that the function $f : R \to R$ defined by $f(x) = \frac{2x-1}{3}$, $x \in R$ is one-one, prove it to be onto. Also, find the inverse of the function f.

6. Find the inverse of $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$.

7. Find the approximate change in the volume V of a cube of side x metres caused by increasing the side by 2%.

8. If $y = \log \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$, show that $\frac{dy}{dx} - \sec x = 0$.

9. A point source of light along a straight road is at a height of 6 metres. A boy 2 metres in height is walking away from the source of light. How fast is his shadow increasing if he is walking at the rate of 5 km/hr ?

10. Use Lagrange's Mean Value Theorem to determine the x-coordinate of the point P on the curve $y = \sqrt{x-2}$, where the tangent is parallel to the chord joining (2, 0) and (3, 1).

11. If $\int \frac{2^{1/x}}{x^2} dx = k2^{1/x} + c$, find k.

12. Anil wants to invest at most ₹ 12000 in Bonds A and B. According to the rules, he has to invest at least ₹ 2000 in Bond A and at least ₹ 4000 in Bond B. If the amounts invested in the two Bonds is ₹ x and ₹ y respectively, find the constraints in maximizing the profit ?

SECTION – 'C' (4×11=44)

13. Let A = Z × Z and '∗' be the binary operation on A defined by :

(a, b) ∗ (c, d) = (a + c, b + d)

Show that '∗' is commutative and associative. Also, find inverse and identity (if exists) in '∗'.

OR

Let f, g : R → R be two functions defined as f(x) = |x| + x and g (x) = |x| – x ∀ x ∈ R. Then, find fog and gof.

14. Find greatest and least values of $\left(\sin^{-1} x\right)^2 + \left(\cos^{-1} x\right)^2$.

15. If $\tan^{-1}\left(\frac{\sqrt{1+x^n}+\sqrt{1-x^n}}{\sqrt{1+x^n}-\sqrt{1-x^n}}\right) = \alpha$; $-1 < x < 1$, prove that $\sin 2\alpha = x^n$.

16. Using properties of definite integrals, evaluate $\int_1^4 \{|x-1|+|x-2|+|x-3|\} dx$

OR

Evaluate $\int_{-a}^{+a} \sqrt{\frac{a-x}{a+x}} dx$

17. Evaluate : $\int \frac{2x}{(1+x^2)(3+x^2)}\,dx.$

18. Evaluate : $\int e^{2x}\,\frac{\sin 4x - 2}{1-\cos 4x}\,dx.$

19. Using integration, find the area bounded by the lines $y = 4x + 5$, $y = 5 - x$ and $4y = x + 5$.

20. To raise money for orphanage, students of three schools A, B and C organized an exhibition in their locality, where they sold paper bags, scrap-books and pastel sheets made by them using recycled paper at the rate of ₹ 20/-, ₹ 15/- and ₹ 10/- per unit respectively. School A sold 25 paper bags, 10 scrap-books and 30 pastel sheets. School B sold 20 paper bags, 15 scrap-books and 30 pastel sheets. School C sold 25 paper bags, 18 scrap-books and 35 pastel sheets. Using matrices, find the total amount raised by each school. By such exhibition, which values are inculcated in the students ?

OR

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the following system of equations :

$2x - 3y + 5z = 16$; $3x + 2y - 4z = -4$; $x + y - 2z = -3$.

21. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x - y)$, prove that $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$.

22. If $x^p y^q = (x + y)^{p+q}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

23. Find the equation of the tangent and normal to the curve $y(x - 2)(x - 3) - x + 7 = 0$ at the point where it cuts the axis of x.

OR

Find the intervals in which the function given by $f(x) = \frac{4\sin x - 2x - x\cos x}{2 + \cos x}$, $0 < x < 2\pi$, is strictly increasing or strictly decreasing.

SECTION – 'D' (6×6=36)

24. Find the equation of the line through the point P (3, 4) which cuts from the first quadrant a triangle of minimum area. Also find the area of the triangle.

25. Show that the volume of the greatest cylinder which can be inscribed in a cone of height 'h' and semi-vertical angle 45° is $\frac{4}{27}\pi h^3$.

26. Using properties of determinants, prove that :

$$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = (b^2-ac)(ax^2+2bxy+cy^2).$$

27. Evaluate, using properties of definite integrals : $\int_0^1 \frac{\log(1+x)}{1+x^2}$.

OR

Evaluate : $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1+\cos x}}{(1-\cos x)^2\sqrt{1-\cos x}}\,dx$

28. Using integration, find the area of the region enclosed between the circles $x^2 + y^2 = 1$ and $x^2 + (y - 1)^2 = 1$.

OR

Using integration, find the area of the region $\{(x, y) : y^2 \le 4x, 4x^2 + 4y^2 \le 9\}$.

29. A company produces two types of belts A and B. Profits on these types are ₹ 2 and ₹ 1.50 on each belt respectively. A belt of type A requires twice as much time as a belt of type B. The company can produce at the most 1000 belts of type B per day. Materials for 800 belts per day is available. At the most 400 buckles of type A and 700 of those of type B are available per day. How many belts of each type should the company produce so as to maximise the profit ? Formulate as LPP and solve graphically.

****