CODE – 041 (S) B

Time allowed: 3 hrs M.M.: 100

General Instruction:-

(i) All questions are compulsory.

(ii) Please check that this question paper contains 29 questions.

(iii) Question 1 to 4 in Section A are very short type questions carrying 1 mark each.

(iv) Question 5 to 12 in Section B are short answer type questions carrying 2 marks each.

(v) Question 13 to 23 in Section C are long answer I type questions carrying 4 marks each.

(vi) Question 24 to 29 in Section D are long answer II type questions carrying 6 marks each.

(vii) Calculator and any electronic device / Gadgets/ Mobile phones is not permitted.

Section A

(Question 1 to 4 carry 1 mark each)

1. If A and B are two matrices such that AB = A and BA = B, then prove that $B^2=B$.

2. Find the principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

3. Integrate $\int a^{cx+p}dx$

4. The distance travelled by a car in **t** seconds after the brakes are applied is 's' feet, where, $s = 22t - 12t^2$ prove that the distance travelled by the car before it stops is 10.08 feet.

Section B

(Question 5 to 12 carry 2 marks each)

5. Solve $\left(1+e^{2x}\right)dy+\left(1+y^2\right)e^x dx=0$ where x=0, y=1

6. If $\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3}$ then prove that $\cos^{-1}x+\cos^{-1}y=\frac{\pi}{3}$

7. Prove that $f(x)=x^3-3x^2+3x-100$ is strictly increasing on R.

8. Integrate $\int_{-\pi/2}^{\pi/2}\frac{dx}{1+e^{\sin x}}$

9. Form differential equation of the family of curves y = a sin (bx + c), where a and c are arbitrary constants.

10. Integrate $\int \tan x.\tan 2x.\tan 3x dx$

11. If $f'(x) = \sqrt{2x^2 - 1}$ and $y = f(x^2)$ then prove that $\frac{dy}{dx}$ at x = 1 is 2

12. Find the value of 'k' so that the function f defined by

$$f(x) = \begin{cases} k(x^2 - 2x) & if \quad x \le 0 \\ 4x + 1 & if \quad x > 0 \end{cases}$$ is continuous at x = 0?

Also check whether the function is continuous at x=1

Section C

(Question 13 to 23 carry 4 marks each)

13. Verify Rolle's theorem

$f(x) = x^3 - 6x^2 + 11x - 6$ on [1, 3]

14. If $5f(x) + 3f\left(\frac{1}{x}\right) = x + 2$ and $y = xf(x)$ then prove that $\frac{dy}{dx} = \frac{7}{8}$ at x=1

OR

Using approximation method find the value of $\sqrt{0.037}$ upto 3 decimal places.

15. Discuss the continuity of the function f, where f is defined by :

$$f(x) = \begin{cases} 2x & if \quad x < 0 \\ 0 & if \quad 0 \le x \le 1 \\ 4x & if \quad x > 1 \end{cases}$$

16. Integrate

(i) $\int \sqrt{1 + 2\tan x(\tan x + \sec x)}\, dx$ (ii) $\int \frac{dx}{x^2\left(x^4 + 1\right)^{\frac{3}{4}}}$

17. Find the equation of all lines having slope -1 and are tangents to the curve $y = \frac{1}{x-1}, x \neq 1$

18. Solve the differential equation $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$

OR

Integrate $\int \frac{dx}{\sqrt{\sin^3 x \sin(x + \alpha)}}$

19. Show that $f: N \to N$ given by $f(x) = \begin{cases} x+1 & \text{if } x \text{ is odd} \\ x-1 & \text{if } x \text{ is even} \end{cases}$

is both one- one and onto

20. If $\left(\sin^{-1} x\right)^2 + \left(\cos^{-1} x\right)^2 = \frac{5\pi^2}{8}$ then prove that $x = -\frac{1}{\sqrt{2}}$

21. Evaluate $\int_1^3 \left(2x^2 + 5x\right) dx$ limits as a sum.

22. If $A = \begin{bmatrix} 8 & -4 & 1 \\ 10 & 0 & 6 \\ 8 & 1 & 6 \end{bmatrix}$, find A^{-1},

hence solve the system of equations: $8x - 4y + z = 5; 10x + 6z = 4; 8x + y + 6z = \frac{5}{2}$

OR

Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school A wants to award Rs x each, Rs y each and Rs z each for the three respective values to 3, 2 and 1 students respectively with a total award money of Rs 1600. School B wants to spend Rs 2300 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three value as before). If the total amount for one prize on each value is Rs 900, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for award.

23. Prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

Section D

(Question 24 to 29 carry 6 marks each)

24. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2m and volume is $8m^3$. If building of tank costs Rs 70 per sq. metre for the base and Rs 45 per sq. metre for sides, what is the cost of least expensive tank?

25. Prove that

$$\sqrt{1+x^2}\left[\left\{x\cos(\cot^{-1}x)+\sin(\cot^{-1}x)\right\}^2-1\right]^{\frac{1}{2}}=1,\ \text{for}\ 0<x<1$$

OR

Integrate $\int\frac{2\sin 2\theta-\cos\theta}{6-\cos^2\theta-4\sin\theta}\,d\theta$

26. If the given matrix $A=\begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix}$ then find A^{-1} using elementary row transformation method.

27. Find the area of the region $\left\{(x,y):x^2+y^2\le 2ax,\ y^2\ge ax,\ x\ge 0,\ y\ge 0\right\}$

28. Integrate $\int\frac{\sqrt{x^2+1}\left[\log\left(x^2+1\right)-2\log x\right]}{x^4}dx$

OR

Integrate $\int\frac{x^2}{\left(x\sin x+\cos x\right)^2}dx$

29. Let $f(x)=3x^2+4xg'(1)+g''(2)$ and $g(x)=2x^2+3xf'(2)+f'(3)$ for $x\in R$,

then prove that $f'(3)+g''(2)=10$

OR

(i) Show that all elements of the main diagonal of skew - symmetric matrix are zeros.
(ii) Show that any square matrix can be written as the sum of symmetric and skew - symmetric matrices.