Ansh
C5

GYAN BHARATI SCHOOL

Second Terminal Examination (2016 – 17)

Class – SS2

Subject – Mathematics

Time Allowed – 3 Hour **MM – 100**

General Instructions:

(i) Write your name and Roll number on the question paper as soon as you get it. No rough work is to be done on the question paper.
(ii) There are 4 printed pages in this paper.
(ii) There are 29 questions in all and all questions are compulsory.
(iii) Marks for each question are indicated against it.
(iv) Calculators are not allowed. However you may ask for log tables, if required.
(v) There is no overall choice. However internal choice is provided in 3 questions of four marks each and 3 questions of six marks each.
(vi) Steps are required in all one marker questions.

SECTION A (1 MARK QUESTIONS)

(Steps are required in 1 markers)

Q1 If $f(x) = \frac{1}{x-1}$, then find the points of discontinuity of $f(f(x))$. **[1]**

Q2 How many symmetric relations can be defined on $A = \{1,2,3,4\}$. **[1]**

Q3 Find $|adj.A|$ if $A = \begin{bmatrix} 2 & 5 & 3 \\ 3 & 4 & 1 \\ 1 & 6 & 3 \end{bmatrix}$. **[1]**

Q4 Discuss the applicability of Rolle's theorem for $y = \sec x$ on $[\pi/4, 7\pi/4]$. **[1]**

SECTION B (2 MARK QUESTIONS)

Q5 Find least value of 'a' for which $2x^2 - 3ax + 7$ is decreasing function on [1,2]. **[2]**

Q6 Find the value of $|adj.(adj.A)|$ if $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix}$. **[2]**

Q7 If $y = \frac{1}{1 + x^{n-m} + x^{p-m}} + \frac{1}{1 + x^{m-n} + x^{p-n}} + \frac{1}{1 + x^{m-p} + x^{n-p}}$, then find $\frac{dy}{dx}$ at $x = e^{m^{n^p}}$. **[2]**

Q8 Evaluate : $\sin^{-1}(\sin 10)$. **[2]**

Q9 If $x, y, z \in [-1, 1]$, such that $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$, then find the value of $x^2y^2 + y^2z^2 + z^2x^2$. **[2]**

Q10 Let $f : R \to R$ be defined as $f(x) = 10x + 7$. Find the function $g : R \to R$ such that $gof = fog = I_R$. **[2]**

Q11 Check the continuity of $f(x) = \begin{cases} \frac{[x]-1}{x-1} & \text{when } x \neq 1 \\ -1 & \text{when } x = 1 \end{cases}$, at $x = 1$, where $[.]$ represents greatest integer function. **[2]**

Q12 Without expanding at any stage, prove that $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix} = 0.$ **[2]**

SECTION C (4 MARK QUESTIONS)

Q13 A spherical balloon is used for spreading the message "SAVE TREES". The radius of the balloon is increasing at the rate of π cm/s. At what rate is its volume increasing when its diameter is 4cm. Why it is important to save trees ? **[4]**

Q14 Find the intervals in which the function $f(x) = -2x^3 + 9x^2 - 12x + 18$, is (i) Increasing, (ii) Decreasing. **[4]**

Q15 If $f(x) = 4x^3 + 5x^2 + 2$, find approximate value of $f(2.01)$, using differentials. **[4]**

OR

Verify Rolle's theorem for $f(x) = (x - a)^m (x - b)^n$ on $[a, b]$ where $m, n \in N$.

Q16 Using graphical method, solve the following linear programming problem : **[4]**
Maximize $Z = 4x + 5y$ subject to the constraints :

$$2x + y \leq 30 \; ; \; x + 2y \leq 24 \; ; \; x \geq 3 \; ; \; y \leq 9 \; ; \; y \geq 0$$

Q17 Prove that the product of following matrices (in any order) is a null matrix, where θ and ϕ differ by an odd multiple of $\pi/2$: $A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$, $B = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$. **[4]**

Q18 If $y = \cos^{-1}\left(\frac{3 + 5\cos x}{5 + 3\cos x}\right)$, then prove that $\cos x = \frac{4 - 5\frac{dy}{dx}}{3\frac{dy}{dx}}$. **[4]**

Q19 If $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$ and $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$, then prove that $\frac{dy}{dx} = -\cot(3t)$. [4]

Q20 If $y = \left(x + \sqrt{x^2+1}\right)^m$; prove that $(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2 y = 0$. [4]

OR

If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$, prove that $\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b.

Q21 Find maximum and minimum values of $(\sin^{-1} x)^3 + (\cos^{-1} x)^3$. [4]

Q22 Show that the following relation defined on N x N is equivalence relation: [4]

$(a, b)\ R\ (c, d) \Leftrightarrow ad(b + c) = bc(a + d)$

Q23 If $a_1, a_2, a_3, \ldots, a_n$ are in A.P. with common difference 'd', then prove that : [4]

$\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \tan^{-1}\left(\frac{d}{1+a_3a_4}\right) + \text{---}$ upto n terms. $= \tan^{-1}\left(\frac{nd}{1+a_1 a_{n+1}}\right)$

OR

Solve : $\sin^{-1}\left[\sin\left(\frac{2x^2+4}{1+x^2}\right)\right] < \pi - 3$

SECTION D (6 MARK QUESTIONS)

Q24 A tall electric pole is to be kept in vertical position by a stretched straight wire from the pole to the ground. The wire has to clear a wall 6m high and 4m from the pole. What is the least length of the wire that can be used between the pole and the ground. $2\times\left(2^{2/3} + 3^{2/3}\right)^{3/2}$ [6]

OR

The fuel charges for running a train are proportional to the square of the speed generated in km per hour and costs Rs.48 per hour at 16 km/hr. What is the most economical speed if the fixed charges i.e., salaries etc. amount to Rs.300 per hour.

Q25 A company manufactures two types of sign boards made of ply wood, displaying – SAVE ENVIORNMENT and BE COURTEOUS. Each sign board of type A requires 5 minutes for cutting and 10 minutes for assembling. Each sign board of type B requires 8 minutes for cutting and 8 minutes for assembling. There are 3 hours 20 minutes available for cutting and 4 hours available for assembling. The profit is 50 paisa for each of type A and 60 paisa for each of type B sign boards. How many sign boards of each type should the company manufacture in order to maximize the profit ? What is the maximum profit ? Give your views about values involved in the question. [6]

Q26 Show that the normal at any point θ to the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ is at a constant distance from the origin. [6]

OR

If $x\cos\alpha + y\sin\alpha = p$ may be a tangent to the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then prove that $a^2\cos^2\alpha + b^2\sin^2\alpha = p^2$.

Q27 The sum of surface areas of a rectangular parallelopiped with sides x, 2x, x/3 and a sphere of radius r is given to be constant. Prove that the sum of their volumes is least if x = 3r. Also find the minimum value of the sum of volumes. [6]

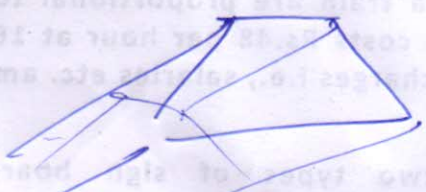
Q28 Find the product A B, where $A = \begin{pmatrix} 1 & 4 & 0 \\ -1 & 2 & 2 \\ 0 & 0 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 4 & -8 & 8 \\ 2 & 2 & -2 \\ 0 & 0 & 6 \end{pmatrix}$. [6]

Hence solve : $x - y = 0$; $2x + y = 3$; $y + z = 2$

Q29 If x, y, z are all different and $\begin{vmatrix} x & x^3 & x^4-1 \\ y & y^3 & y^4-1 \\ z & z^3 & z^4-1 \end{vmatrix} = 0$, then show that $xyz(xy + yz + zx) = x + y + z$. [6]

OR

Prove that : $\begin{vmatrix} a & b-c & b+c \\ c+a & b & c-a \\ a-b & a+b & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$.



$n^2 \quad 9n^2 \quad n^2 = \frac{n^2}{9}$

$4\pi\frac{n^2}{9} + 2$ $\quad 4\pi a^2 + 2(15n^2)$

$= k$

$\Rightarrow k = a^2(4\pi + 30)$

$k^2 = a^4($