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MANAVA BHARATI

INDIA INTERNATIONAL SCHOOL

MID-TERM EXAMINATION (2016-17)

Class XII – Mathematics

Time allowed: 3 hours **Maximum Marks : 100**

General Instructions

(i) Q.NO 1 to 4 carry 1 mark each
(ii) Q.NO 5 to 12 carry 2 marks each
(iii) Q.NO 13 to 23 carry 4 marks each
(iv) Q.NO 24 to 29 carry 6 marks each

1. If the matrix $\begin{bmatrix} 0 & 6-5x \\ x^2 & x+3 \end{bmatrix}$ is symmetric, find the values of x

2. If $A = \begin{bmatrix} \cos\propto & -\sin\propto \\ \sin\propto & \cos\propto \end{bmatrix}$ then $A + A' = I$, where I is the identity matrix of order 2. Find the value of $\propto$.

3. If $y = \sqrt{Sinx^2}$. Find $\frac{dy}{dx}$

4. Find the principle value of $Cot^{-1}(-\frac{1}{\sqrt{3}})$

5. Find the value $\tan(Sin^{-1}\frac{3}{5} + Cot^{-1}\frac{3}{2})$

6. Find $\frac{dy}{dx}$, if $Sin^2y + Cosxy = \pi$

7. Differentiate with respect to x, $Sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

8. Evaluate $\int \frac{dx}{\sqrt{5-4x-2x^2}}$

9. Evaluate $\int \frac{Cosx}{(1-sinx)(2-sinx)} dx$

10. Prove that the tangents to the curve $y = x^2 - 5x + 6$ at the points (2,0) and (3,0) are at right angle.

11. Verify Lagrange's Mean Value theorem for the function $f(x) = x^3 - 5x^2 - 3x$ in $[1, 3]$

12. If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its volume.

13. Using properties of determinants, prove that

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

14. Prove that $\text{Cot}^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\} = \frac{x}{2}$, $x \in (0, \frac{\pi}{4})$

OR

Simplify $\tan^{-1}\left\{\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right\}$, if $\frac{a}{b}\tan x > -1$

15. Express the matrix $A = \begin{bmatrix} 4 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$, as the sum of the symmetric and skew - symmetric matrix.

16. Find the value of a and b if the function defined by

$$f(x) = \begin{cases} 5 & , \text{if } x \le 2 \\ ax + b & , \text{if } 2 < x < 10 \\ 21 & , \text{if } x \ge 10 \end{cases}$$ is a continuous function

17. If $y = \log(x + \sqrt{x^2+1})$, prove that $(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

OR

Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ w.r.t. $\text{Cos}^{-1}(2x\sqrt{1-x^2})$ where $x \neq 0$

18. Two schools A and B decided to award the prizes to their students for three values honesty (x), punctuality (y) and obedience (z). School A decided to award a total of Rs 11000 for the three values to 5, 4 and 3 students respectively while school B decided to award a total of Rs.10700 for three values to 4, 5 and 3 students respectively . If all the three prizes together amount to Rs 2700 , then (i) represent the above situation by a matrix equation and form linear equations using matrix multiplication, (ii) is it possible to solve the system of equations so obtain using matrices ? (iii) which value you prefer to be rewarded most and why ?

19. Evaluate the integral $\int x \sin^{-1} x \, dx$

OR

Evaluate $\int (\sin^{-1} x)^2 \, dx$

20. Show that $y = \log(1+x) - \frac{2x}{2+x}$, $x > -1$, is an increasing function of x throughout its domain

21. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point (1,2)

or

Find the point on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with the axes.

22. Evaluate the integral $\int \frac{x^2+1}{x^2-5x+6}\,dx$

23. Evaluate the integral $\int \frac{dx}{1-\tan x}$

24. Show that $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$

25. Using properties of determinants prove that

$$\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$$

26. Evaluate the definite integral as limit of sums, $\int_1^4 (x^2 - x)dx$

OR

Evaluate the integral, $\int \frac{(3\sin\phi - 2)\cos\phi}{5 - \cos^2\phi - 4\sin\phi}\,d\phi$

27. (a) Prove that the function f given by $f(x) = |x-1|$, $x \in R$ is not differential at $x = 1$.

(b) Show that $f(x) = \begin{cases} \frac{x-|x|}{2}, & where\ x \neq 0 \\ 2, & where\ x = 0 \end{cases}$ is discontinuous at $x = 0$

28. (a) For a positive constant a find $\frac{dy}{dx}$, where $y = a^{t+1/t}$, $x = (t + 1/t)^a$

(b) Show that the function given by $f(x) = \frac{\log x}{x}$ has maximum at $x = e$

29. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere

OR

Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ its vertex at one end of the major axis.