

Half Yearly Examination – 2016-17

No. of Printed Page : 6

Subject : Mathematics

Time : 3 hrs.

Roll No. 12221

Class : XII

Maximum Marks :

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General Instructions :

- (i) All questions are compulsory.
- (ii) This questions paper contain 29 questions.
- (iii) Question 1-4 in Section A are very short answer type questions carrying 1 mark each.
- (iv) Questions 5-12 in Section B are short answer type questions carrying 2 marks each.
- (v) Questions 13-23 in Section C are long answer-I type questions carrying 4 marks each.
- (vi) Questions 24-29 in Section D are long answer-II type questions carrying 6 marks each. There will be no overall choice in the question paper. However 30% internal choices will be given in 4 and 6 marks questions.

Section – A

1. Evaluate : $\sin\left[\frac{\pi}{6} - \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$

2. Find x if the matrix $\begin{pmatrix} 5-x & x+1 \\ 2 & 4 \end{pmatrix}$ is a singular matrix.

3. If A is invertible matrix of order 3 and $|A| = 5$, then find $|\text{adj } A|$.

4. If $f : \mathbb{R} - \left\{\frac{-3}{5}\right\} \rightarrow \mathbb{R}$ is a function defined by $f(x) = \frac{2x}{5x+3}$, then find f^{-1} .

$$\frac{x-2x}{6}$$

P.T.O.

Section - B

5. If $y = \tan^{-1}\left(\frac{5x}{1-6x^2}\right) - \frac{1}{16} < x < \frac{1}{\sqrt{6}}$, then prove that

$$\frac{dy}{dx} = \frac{2}{1+4x^2} + \frac{3}{1+9x^2}$$

6. Prove that $2 \tan^{-1} \sqrt{x} = \cos^{-1} \left(\frac{1-x}{1+x} \right)$.

7. If $A = \begin{pmatrix} 1 & 3 \\ 2 & +1 \\ 3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 1 & 2 \\ -1 & 0 \end{pmatrix}$, find matrix C such that $A + B + C$ is a
isomatrix. *zero matrix*

8. Without expanding the det, find the value of $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

9. If the radius of circle is increasing at the rate of 3 cm/sec, at what rate is its perimeter increasing when the radius is 10cm. *2πr*

10. Determine the point on the curve $y = x^3 - 3x^2 - 9x + 7$ at which the tangent is 11 to x-axis.

11. Find $\frac{dy}{dx}$ if $x = \tan \theta$ and $y = b \sec \theta$.

12. If $P(A) = 0.8$, $P(B) = 0.5$ and $P\left(\frac{B}{A}\right) = 0.4$ find $P(A \cup B)$.

Section - C

13. Prove that $\frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \left(\frac{1}{3} \right) = \frac{9}{4} \sin^{-1} \left(\frac{2\sqrt{2}}{3} \right)$

OR

If $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$, then find the value of x.

20. Find the intervals in which the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is strictly increasing and decreasing. Also find the point on which the tangents are parallel to x-axis.

OR

Find the equation of the normal to the curve $2y = x^2$ which passes through the point (2, 1).

21. A particle moves along the curve $y = \frac{2}{3}x^3 + 1$. Find the point on the curve at which the y coordinate is changing twice as fast as the x-coordinate.
22. (i) Find the approximate value of $\sqrt{0.037}$.
 (ii) Verify Rolle's Theorem for the function $f(x) = x^2 + 2x - 8$, $x \in [-4, 2]$
23. To raise money for an orphanage students, three Housing Societies A, B and C organise an exhibition in their locality, where they sold paperbags, Scrap books and pastel sheets at the rate of Rs. 20/-, Rs. 15/- and Rs. 10/- respectively per unit. School A sold 25 paper bags, 10 scrap books and 30 pastel sheets. School B sold 20 paper bags, 15 scrap books and 30 pastel sheets while school C sold 25 paper bags, 18 scrap books and 35 pastel sheets. Using matrices, find the total amount raised by each school. By such exhibition which values are inculcated in the students.

Section - D

24. If $p \neq 0$, $q \neq 0$ and $\begin{vmatrix} p & q & p\alpha + q \\ q & r & q\alpha + r \\ p\alpha + q & q\alpha + r & 0 \end{vmatrix} = 0$, then using the properties of det prove that the following statement is true.
- (a) p, q, r are in GP
 (b) α is a root of the equation $px^2 + 2qx + r = 0$

18/15
 500
 400
 2500
 225
 300
 500
 150
 300

OR

Using properties of determinants prove that

$$c^2 - a^2 - a^2 - c^2 + 2ac - 2a^2$$

$$\begin{vmatrix} (a+b)^2 & c & c \\ c & (b+c)^2 & a \\ a & b & (a+c)^2 \end{vmatrix} = 2(a+b+c)^3$$

$$\begin{array}{r} 400 \\ 2000 \\ \hline 5 \\ 385 \\ \hline 2 \end{array}$$

25. Find the equation of the line through the point (3, 4) which cuts from the first quadrant a triangle of minimum area. Also find its area.

26. A letter is known to have come either from TATANAGAR or KOLKATA. On the envelope, only the consecutive letters TA are visible. What is the probability that the letter has come from

- (i) KOLKATA (ii) TATANAGAR

OR

Colour balls are distributed in three bags as shown in the following table :

Bags	Colour of ball		
	White	Blue	Red
I	1	2	3
II	2	4	1
III	3	3	2

$$\begin{array}{r} 218 \\ 15 \\ \hline 90 \\ 180 \\ \hline 270 \end{array}$$

A bag is selected at random and then two balls are drawn from the selected bag. Find the probability that the balls drawn are white and red.

27. Show that the altitude of a right circular cone of maximum volume that can be inscribed in a sphere of radius R is $\frac{4R}{3}$ and find its volume.

$$\begin{array}{r} 15 \\ 850 \\ \hline 270 \end{array}$$

28. Let $f: N \rightarrow Y$ be a function defined by $f(x) = 4x^2 + 12x + 15$ where $y = \text{range of } f$. Show that f is invertible and find its inverse.

$$\frac{15}{9} \times \frac{1}{2} = \frac{15}{18} = \frac{5}{6}$$

$$3 \times \frac{1}{2} = \frac{3}{2}$$

GF-34-400/Mathematics/XII

◇ 5 (XII) ◇

$$\frac{400 - 30}{10000} = \frac{370}{10000} = 0.037$$

OR

Let $*$ be a binary operation defined on $Q \times Q$ by $(a, b) * (c, d) = (ac, b + ad)$ where Q is the set of rational nos. determine whether $*$ is commutative and associative. Find the identity elements for $*$ and the invertible elements of $Q \times Q$.

29. The dietician of certain organisation wishes to mix two types of food in such a way that vitamins contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food II contains 1 unit/kg of vitamin A and 2 units/kg vitamin C. It cost Rs. 50/ps kg to purchase food I and Rs. 70/kg to purchase food II. Formulate the problem as L.P.P. and solve it graphically.

$$(h-R)^2 + v^2 = v^2$$

$$\begin{aligned} x + 2y &= 10 \\ 2x + y &= 8 \end{aligned}$$
$$\begin{aligned} 2x + y &= 8 \\ -2x + 4y &= -10 \end{aligned}$$
$$5y = -2$$
$$y = -2/5$$
$$x + 2(-2/5) = 10$$
$$x - 4/5 = 10$$
$$x = 10 + 4/5 = 54/5$$