ST.GEORGE'S SCHOOL,ALAKNANDA
MID TERM EXAMINATION 2016-17
SUBJECT: MATHEMATICS
CLASS:XII

DATE : 26.9.16.

MAX TIME :3 HRS

MAX. MARKS: 100

NO. OF PAGES:2

Section A: Q1 to Q4 one mark each , Section B: Q5 to Q12 two marks each
Section C: Q13 to Q23 four marks each, Section D: 24 to Q29 six marks each

SECTION - A

Q1. If $A = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 & 4 \end{bmatrix}$ then what is the order of AB ?

Q2. Evaluate $\int \dfrac{dx}{x + x \log x}$

Q3. Without expanding show that $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$.

Q4. Check if the operation $* : R \times R \to R$ given by $a * b = a + 2b$ is commutative or not?

SECTION - B

Q5. Find the value of 'x' for which $A = \begin{bmatrix} 3 & 5 \\ 7 & x \end{bmatrix}$ is a singular matrix.

Q6. Find the order and degree of differential equation: $\dfrac{d^4 y}{dx^4} + \sin\left(\dfrac{d^3 y}{dx^3}\right) = 0$

Q7. Show that the function $f(x) = |x - 3|$, $x \in \Re$ is not differentiable at $x=3$

Q8. Consider the binary operation $*$ on the set $\{1,2,3,4,5\}$ defined by a*b=HCF of a and b. Write the

operation table of the operation $*$

Q9. Differentiate $\sin^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right)$ with respect to 'x'.

Q10. Evaluate $\int \dfrac{dx}{\sqrt{3x+2} + \sqrt{3x+1}}$

Q11. If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find 'x' and 'y' such that $A^2 - xA + yI = 0$, where 'x' and 'y' are non zero scalars.

Q12. Evaluate $\int \dfrac{\log x \, dx}{x^2}$

SECTION – C

Q13. Differentiate the following function with respect to 'x' $f(x) = \tan^{-1}\left(\dfrac{1-x}{1+x}\right) - \tan^{-1}\left(\dfrac{x+2}{1-2x}\right)$

Q14. Evaluate : $\int (\sqrt{\cot x} + \sqrt{\tan x})dx$

Q15. If $\tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \pi$, then prove that $a + b + c = abc$

Q16. If $y = x^x$, then prove that $\dfrac{d^2 y}{dx^2} - \dfrac{1}{y}\left(\dfrac{dy}{dx}\right)^2 - \dfrac{y}{x} = 0$

Q17. Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \dfrac{\cos^2 x}{\cos^2 x + 4\sin^2 x}dx$.

Q18. Evaluate : $\displaystyle\int_0^{\frac{\pi}{2}} \log(\sin x)dx$

Q19. Prove the following : $\cos\left[\tan^{-1}\{\sin(\cot^{-1} x)\}\right] = \sqrt{\dfrac{1+x^2}{2+x^2}}$

Q20. If $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \ldots to \infty}}}$ then prove that $\dfrac{dy}{dx} = \dfrac{\sec^2 x}{2y-1}$.

Q21. Form the differential equation of the family of ellipses having foci on y-axis and center at the origin.

Q22. Verify LMVT for the function $f(x) = x + \dfrac{1}{x}$ in $[1,3]$.

Q23. Show that the relation R on the set
$A = \{x \in Z : 0 \le x \le 12\}$, given by $R = \{(a,b) : |a-b|$ is a multiple of $4\}$ is an equivalence relation.

SECTION - D

Q24. In a survey of 20 richest persons of three residential society A,B,C it is found that in society A, 5 believe in honesty, 10 in hardwork and 5 in unfair means while in B, 5 believe in honesty, 8 in hardwork and 7 in unfair means and in C, 6 believe in honesty, 8 in hardwork and 6 in unfair means. If the per day income of 20 richest persons of society A, B and C are Rs.32,500, R.30,500, Rs.31,000 respectively, then find the per day income of each type of people by matrix method. (i) Which type of person has more per day income ? (ii) According to you, which type of person is better for country ?

Q25. Evaluate : $\displaystyle\int_1^3 \left[|x-1| + |x-2| + |x-3|\right] dx$

Q26. If the length of three sides of trapezium other than base is equal to 10cm, then find the area of the trapezium when it is maximum.

Q27. Evaluate: $\displaystyle\int \dfrac{x+2}{(x^2 + 3x + 3)\sqrt{x+1}}dx$

Q28. If a,b, c are real numbers and $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$ show that either

$a + b + c = 0$ or $a = b = c$

Q29. Using integration, find the area enclosed between the two circles $x^2 + y^2 = 9$ and $(x-3)^2 + y^2 = 9$

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