SADHU VASWANI

CLASXII-MATHEMATICS – TERMI SET I

MM :100

TIME:3hrs

GENERAL INSTRUCTIONS:

1)Questions 1to 6 carries 1 mark each ,questions 7 to 19 carries 4 marks each,

Questions 20 to 26 carries 6 marks each.

2All questions are compulsory.However internal choices have been provided in some questions,in which case only one of the alternatives should be answered.

SECTION-A

1)Write the value of x-y+z from the equation $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

2)If $R=\{(x,y): x+2y=8\}$ is a relation on N ,write the range of R.

3)Find the value of $\cot(\sec^{-1} x + \sin^{-1}\frac{1}{x})$.

4)Verify Rolle's theorem for the function f(x)=|x|,in [-1,1].

5)Radius of a variable circle is changing at the rate of 5cm/sec.What is the the radius of the circle at a time when its area is changing at the rate of $100cm^2$/s ?

6)If the radius of a sphere is measured as 9cm with an error of 0.03cm,then find the approximate error in calculating its surface area.

Section-B

7)A trust caring for handicapped children gets Rs.30,000 every month from its donors. The trust spends half of the funds received for medical and educational care of the children and for that it charges 2% of the spent amount from them and deposits the balance amount in a private bank to get the money multipled so that in future

the trust goes on functioning regularly.What percent of interest should the trust get from the bank to get a total interest of Rs.1800 every month.Using matrix method find the rate of interest.Do you think people should donate to such trusts?

8) *If* $F(x)=\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **,prove that** $[F(x)]^{-1}=F(-x)$.

9)Using properties of determinants prove that

$$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix}=(b^2-ac)(ax^2+2bxy+cy^2)$$

OR

If $A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ *show that* $A^2-5A+7I=O$*.Hence find* A^{-1}.

10)Prove that the relation R in the set A={5,6,7,8,9}given by R={(a,b):|a-b| is divisible by 2},is an equivalence relation.Find all elements related to the element 6.

11)Show that the function f:R→R defined by f(x) =$\frac{x}{x^2+1}$,for all x€R is neither one-one nor onto.

12)Show that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)=\frac{4-\sqrt{7}}{3}$.

13)Simplify: $\tan^{-1}\left\{\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right\}$ **, if** $\frac{a}{b}\tan x\geq -1$.

OR

Simplify :$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$ **,x€(0,$\frac{\pi}{4}$)**

14) Find the points of discontinuity of the function f given by f(x)= $\begin{cases} |x|+3, if\ x \leq -3 \\ -2x, if -3 < x \leq 3 \\ 6x+2, if\ x \geq 3 \end{cases}$

15) If y= $\left[log(x+\sqrt{x^2+1})\right]^2$,show that $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-2=0$.

16) x and y are the sides of two squares such that $y = x - x^2$.Find the rate of change of the area of second square with respect to the area of the first square.

OR

A water tank has a shape of an inverted right circular cone .Its semivertical angle is $\tan^{-1}(0.5)$.water is poured into it at a constant rate of 5 cu.m per min.Find the rate at which the level of the water is rising at the instant when the depth of the water in the tank is 10m.

17) Show that the minimum of Z occurs at more than two points.Minimise and maximize Z= x+2y subject to constraints x+2y≥100, 2x-y≤0 , 2x+y≤200, x,y≥0.

18) Find whether the function f(x)=cos(2x+$\frac{\pi}{4}$) , $\frac{3\pi}{8}$<x<$\frac{5\pi}{8}$ is increasing or decreasing.

OR

Find the intervals in which the function f given by f(x)=$2x^3$- $9x^2+12x+15$ is strictly increasing or decreasing.

19) Find the absolute maximum value and the absolute minimum value of the function

f (x)=4x - $\frac{1}{2}x^2$, *in the* given interval x∈[-2, $\frac{9}{2}$].

SECTION-C

20) Find the equation of the tangents to the curve y=x^3+2x+6 which are perpendicular

to the line $x+14y+4=0$.

21)Using elementary transformation ,find the inverse of the matrix: $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix}$

22)Let QxQ ,where Q is the set of rational nos,and * be a binary operation defined

On A by (a,b)*(c,d)=(ac,b+ad),for (a,b),(c,d)∈A.Then find i)identity element of * in A,

ii)invertible element of A ,and hence find the inverse of elements (5,3) and $(\frac{1}{2},4)$

23)Solve for x:$\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)+\frac{1}{2}\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\frac{2\pi}{3}$

OR

Solve for x:$\sin^{-1}x+\sin^{-1}2x=\frac{2\pi}{3}$,x>0.

24)Differentiate $y=(x\cos x)^x+(x\sin x)^{\frac{1}{x}}$,with respect to x.

OR

If x=asin2t(1+cos2t) and y=bcos2t(1-cos2t),show that $\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}}=\frac{b}{a}$.

25)Showthat the volume of the greatest cylinder which can be inscribed in a cone of

height h and semi vertical angle 30^0 is $\frac{4}{81}\pi h^3$.

26)Amanufacturer has 30and 17 units of workers(male and female),and Capital

respectively,which he uses to produce two types of goods A and B.To produce 1 unit

of A ,2 workers and 3 units of capital are required while 3 workers and 1 unit of

Capital is required to produce 1 unit of B.If A and B are priced at Rs100 and Rs120

per unit respectively ,how should he use his resources to maximize the total revenue?

Form the above as an LPP problem and solve graphically.