APEEJAY SCHOOL ,SAKET
2017-2018
SUBJECT-MATHEMATICS
Class XII

M.Marks- 100 Time – 3 hours

General Instructions

- All questions are compulsory.
- This question paper contains 29 questions.
- Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
- Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
- Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
- Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

SECTION A

Q.No.1 Write the principal value of $\tan^{-1}(1) + \cos^{-1}(-1/2)$

Q.No.2 Evaluate : $\int_2^4 \frac{x\,dx}{1+x^2}$

Q.No.3 If A is a square matrix such that $A^2 = A$, then write the value of $7A - (I + A)^3$, where I is an identity matrix.

Q.No.4 Differentiate the following with respect to x: $\log(\cos e^x)$

SECTION B

Q.No.5 Using differentials find the approximate value of $\sqrt{49.5}$

Q.No.6 Differentiate the following function with respect to x:
$Y = (\sin x)^x + \sin^{-1}\sqrt{x}$

Q.No.7 Verify the applicability of LMV theorem for the function $\frac{1}{x+1}$ in [1,3]

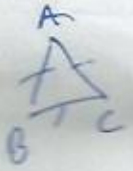
Q.No.8 If $f(x) = \sqrt{x^2+1}$; $g(x) = \frac{x+1}{x^2+1}$ and $h(x) = 2x-3$, then find $f'[h'\{g'(x)\}]$

Q.No.9 Find all points of discontinuity of the function
$f(t) = 1/(t^2 + t - 2)$, where $t = 1/(x-1)$

Q.No.10 Show that the relation R on R defined as $R = \{(a,b): a \leq b\}$, is reflexive and transitive but not symmetric.

Q.No.11 Give an example of two functions f:N→N and g: N→N such that gof is onto but f is not onto.

Q.No.12 Prove that $2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}$



SECTION C

Q.No.13 In a triangle ABC ,if

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix} = 0,$$

then prove that triangle ABC is an isosceles triangle.

Q.No.14 Evaluate $\int \frac{2x}{(3+x^2)(4-x^2)^2}\,dx$

Q.No.15 Evaluate $\int_0^4 (x + e^{2x})dx$ as a limit of the sum

Q.No.16 Solve the following LPP:

Minimise $Z = 5x + 7y$

Subject to the constraints: $2x+y \geq 8$,

$x + 2y \geq 10$

$x, y \geq 0$

Q.No.17 Show that the normal at any point 't' to the curve x =a cost + atsint , y = a sint –at cost is at a constant distance from the origin

Q.No.18 Prove that $\tan^{-1}\left[\frac{(1+x)^{1/2} - (1-x)^{1/2}}{(1+x)^{1/2} + (1-x)^{1/2}}\right] = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$, $-1/\sqrt{2} \leq x \leq 1$

Q.No.19 Show that the function f given by f: R-{4/3} → R-{4/3} , defined as $f(x) = \frac{4x+3}{3x+4}$ is bijective. Hence find f^{-1}

Q.No.20 Find the adjoint of the matrix $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence , show that $A(\text{adj}A) = |A|I_3$

Q.No.21 Evaluate $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}\,dx$

Q.No.22 Evaluate $\int_0^1 \cot^{-1}(1 - x + x^2)\,dx$

Q.No.23 Find the values of a and b , if the function f defined by

$$f(x)=\begin{cases} x^2+3x+a, & x\le 1 \\ bx+2, & x>1 \end{cases}$$

is differentiable at x=1.

SECTION D

Q.No.24 A trust invested some money in two types of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received Rs. 2,800 as interest. However, if trust had interchanged money in bonds, they would have gotRs.100 less as interest. Using matrix method , find the amount invested by the trust. Interest received on this amount will be given to Helpage India as donation. Which value is reflected in this question?

Q.No.25 Evaluate $\int (3x-2)\sqrt{x^2+x+1}\,dx$

Q.No.26 Obtain the inverse of the following matrix using elementary transformations.

$$A=\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

hence solve the system of equations:

8x+4y +3z= 19

2x +y +z = 5

x+2y +2z = 7

Q.No.27 Let A = {1,2,.....,9} and R be the relation in AXA defined by (a,b) R (c,d) if a+d = b+c for a,b,c,d ∈ A. Prove that R is an equivalence relation. Also obtain the equivalence class {(2,5)}

OR

Let f : [0,1)→R be a function defined by f (x) =$9x^2$+ 6x -5. Prove that f is not invertible. Modify, only the codomain of f to make f invertible and then find its inverse.

Q.No.28 Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius R is 4R/3. Also show that the maximum volume of the cone is 8/27 of the volume of the sphere.

Q.No.29 A manufacturing company makes two types of teaching aids A and B of Economics for class XII. Each type of A requires 9 labour hours of fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours of fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of Rs. 80 on each piece of type A and Rs.120 on each piece of type B. How many pieces of type A and type B should be manufactured per week to get a maximum profit? Make it as a LPP and solve graphically . What is the maximum profit per week?