

CAMBRIDGE SCHOOL SRINIWASPURI NEWDELHI
HALF YEARLY EXAMINATION - 2017-2018
MATHEMATICS
CLASS XII (SET B)

MAX.MARKS:100

TIME: 3HRS.

GENERAL INSTRUCTIONS:

1. All questions are compulsory.
2. Q.1 to 4 in Section A carries 1 mark each.
3. Q.5 to Q.12 in Section B carries 2 marks each.
4. Q.13 to Q.23 in Section C carries 4 marks each.
5. Q.24 to Q.29 in Section D carries 6 marks each.

SECTION - A

1. Evaluate : $\int (e^{\log a^x} + e^{\log x^a} + e^{\log a^a}) dx$.

2. $\int \frac{1}{\sin^2 x \cos^2 x} dx$

3. Tell graphically the difference b/w Rolle's theorem & L.M.V. theorem.

4. For what value of x is $\begin{pmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 1 \end{pmatrix}$ singular ?.

SECTION -B

5. If $A = \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix}$ & $B = (1 \ 3 \ -6)$ then verify $(AB)^t = B^t A^t$

6. Evaluate : $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$.

7. The surface area of a spherical bubble is increasing at the rate of $2\text{cm}^2/\text{sec}$. Find the rate at which the volume of the bubble is increasing at the instant if its radius is 6cm.

8. Using differential approximations, find the value of $f(2.1)$ if $f(x) = 4x^2 + 5x + 2$.

9. Find the absolute max. value & min. value of f is given by $\sin x + \cos x$ for all x in $[0, \pi]$.

10. Integrate : $\int_0^{\pi} \frac{x \tan x}{\sec x \operatorname{cosec} x} . dx$

11. Integrate : $\int \sqrt{\frac{a+x}{a-x}} dx$.

12. Integrate : $\int x \sin^{-1} x dx$.

SECTION - C

13. Let $A = \{1, 2, 3\}$ find the number of relations on A containing $(1, 2)$ & $(1, 3)$ which are reflexive & symmetric but not transitive.

14. Let $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$ & f, g are functions defined by $f(x) = x^2 - x$ for all x in A & $g(x) = 2|x - 1/2|$, for all x in B , find if $f = g$.

15. Find a & b such that f is continuous if

$$f(x) = \begin{cases} 5; & x \leq 2 \\ ax + b; & 2 < x < 10 \\ 21; & x \geq 10 \end{cases}$$

OR

Show that $|x|$ is continuous but not differentiable at $x = 0$.

16. Prove : $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$ OR $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$.

17. If $y = 3 \cos(\log x) + 4 \sin(\log x)$, then prove that $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$.

18. If $x^{13} y^7 = (x + y)^{20}$, then prove that $dy/dx = y/x$ in $[0, 3]$

19. Find the point on $x^3 - 11x + 5$ at which the equations of the tangent is $y = x + 11$.

20. Verify Rolle's theorem for $f(x) = (x-1)^{2/3}$ in $[0, 3]$

21. Integrate : $\int \frac{5x+3}{\sqrt{x^2+4x+10}} dx$

22. Integrate : $\int \frac{x^2+1}{(x+3)(x-1)^2} dx$

OR

$$\int \frac{(x^2+2)}{(x^2+1)(x^2+4)} dx$$

23. Evaluate : $\int_1^5 |x-1| + |x-2| + |x-3| dx$.

SECTION - D

24. Let $*$ be a binary operation defined on Q . Find which of the following are binary operations which of them are commutative & find the identity element if it exists.....

(i) $a*b = a-b$ for all a, b in Q .

(ii) $a*b = a + ab$ for all a, b in Q

(2)

OR

Let * be defined on a set $A = \{0, 1, 2, 3\}$ by $a * b = ab \pmod{4}$. Write the composition table of the operation *. Also answer the following

(i) is * a binary operation (ii) is * commutative? (iii) what is the identity element.

25. Given $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ find BA & use it to solve $y + 2z = 7; x - y = 3;$
 $2x + 3y + 4z = 17$. with respect to x.

OR

Find the inverse of $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ by elementary row transformations.

26. Find the interval in which $x^4 - 8x^3 + 22x^2 - 24x + 21$ is (i) increasing (ii) decreasing.

27. If $x = e^\theta a (\sin \theta - \cos \theta)$ & $y = e^\theta a (\sin \theta + \cos \theta)$ then find the second derivative of y with respect to x.

28. Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6\sqrt{3}r$.

OR

Show that the semi vertical angle of a right circular cone of given surface area & maximum volume is $\sin^{-1}(1/3)$.

29. Prove : (i) $\int_0^a f(x) dx = \int_0^a f(a-x) dx$ (ii) Find $\int_0^{\pi/2} \frac{\sin x}{\cos x + \sin x} dx$.