Question 1:

Monochromatic light having a wavelength of 589nm from the air is incident on a water surface. Find the frequency, wavelength and speed of (i) reflected and (ii) refracted light? [1.33 is the Refractive index of water]

Solution:

Monochromatic light incident having wavelength, $\lambda = 589 \text{ nm} = 589 \text{ x} \cdot 10^{-9} \text{ m}$

Speed of light in air, c = 3 x 108 m s-1

Refractive index of water, μ = 1.33

(i) In the same medium through which incident ray passed the ray will be reflected back.

Therefore the wavelength, speed, and frequency of the reflected ray will be the same as that of the incident ray.

Frequency of light can be found from the relation:

$$v = \frac{c}{\lambda} = \frac{3 \times 10^8}{589 \times 10^{-9}} = 5.09 \times 10^{14} \text{ Hz}$$

Hence, $c = 3 \times 10^8 \text{ m s}^{-1}$, $5.09 \times 10^{14} \text{ Hz}$, and 589 nm are the speed, frequency, and wavelength of the reflected light.

(b) The frequency of light which is travelling never depends upon the property of the medium. Therefore, the frequency of the refracted ray in water will be equal to the frequency of the incident or reflected light in air.

Refracted frequency, $v = 5.09 \times 10^{14} Hz$

Following is the relation between the speed of light in water and the refractive index of the water:

$$v = \frac{c}{\mu} = v = \frac{3 \times 10^8}{1.33} = 2.26 \times 10^8 \,\mathrm{m \, s^{-1}}$$

Below is the relation for finding the wavelength of light in water:

$$\lambda = \frac{v}{V} = \frac{2.26 \times 10^8}{5.00 \times 10^{14}} = 444.007 \times 10^{-9} \,\mathrm{m} = 444.01 \,\mathrm{nm}$$

Therefore, 444.007×10^{-9} m, 444.01nm, and 5.09×10^{14} Hz are the speed, frequency, and the wavelength of the refracted light.

Question 2:

What is the shape of the wavefront in each of the following cases:

(i) Light diverging from a point source.

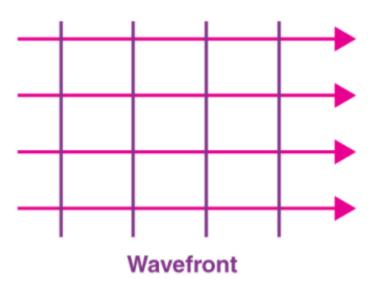
- (ii) Light emerging out of a convex lens when a point source is placed at its focus.
- (iii) The portion of the wavefront of the light from a distant star intercepted by the Earth.

Solution:

(i) When the light diverges from a point source, the shape of the wavefront is spherical. Following is the figure of the wavefront:



(ii) When the light is emerging from the convex lens, the shape of the wavefront is parallel odd. In this case, the point source is placed at its focus.



(iii) When the light is coming from a distant star that is intercepted by the earth, the shape of the wavefront is plane.

Question 3:

- (i) The refractive index of glass is 1.5. What is the speed of light in glass? Speed of light in a vacuum is ($3.0 \times 10^8 \text{ m s}^{-1}$)
- (ii) Is the speed of light in glass Independent of the colour of light? If not, which of the two colours red and violet travels slower in a glass prism?

Solution:

(i) Refractive Index of glass, $\mu = 1.5$

Speed of light, c = 3 x 108 ms⁻¹

The relation for the speed of light in a glass is: $v=rac{c}{u}$

=
$$\frac{3\times10^8}{1.5}$$
 = $2\times 10^8 m/s$

Hence, the speed of light in glass is 2 x 108 m s⁻¹

(ii) The speed of light is dependent on the colour of the light. For a white light, the refractive index of the violet component is greater than the refractive index of the red component. So, the speed of violet light is less than the speed of the red light in the glass. This will reduce the speed of violet light in glass prism when compared with red light.

Question 4:

In Young's double-slit experiment, 0.28mm separation between the slits and the screen is placed 1.4m away. 1.2cm is the distance between the central bright fringe and the fourth bright fringe. Determine the wavelength of light used in the experiment.

Solution:

Distance between the slits and the screen, D = 1.4 m

and the distance between the slits, $d = 0.28 \text{ mm} = 0.28 \text{ x } 10^{-3} \text{ m}$

Distance between the central fringe and the fourth (n = 4) fringe,

$$u = 1.2 cm = 1.2 \times 10^{-2} m$$

For constructive interference, the following is the relation for the distance between the two fringes:

$$u = n \lambda \frac{D}{d}$$

Where, n = order of fringes

= 4λ = Wavelength of light used

Rearranging the formula, we get

$$\lambda = \frac{ud}{nD}$$

$$= \frac{1.2 \times 10^{-2} \times 0.28 \times 10^{-3}}{4 \times 1.4}$$

$$= 6 \times 10^{-7} = 600$$
nm

600nm is the wavelength of the light.

Question 5:

In Young's double-slit experiment using the monochromatic light of wavelength λ , the intensity of light at a

point on the screen where path difference is λ , is K units. What is the intensity of light at a point where path

difference is $\frac{\lambda}{3}$?

Solution:

Let $\,I_1\,$ and $\,I_2\,$ be the intensity of the two light waves. Their resultant intensities can be obtained as:

$$I' = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

Where,

 $\phi\,$ = Phase difference between the two waves

For monochromatic light waves:

$$I_1 = I_2$$

Therefore
$$I'=I_1+I_2+2\sqrt{I_1\;I_2}\;cos\phi$$

=
$$2I_1 + 2I_1 \cos\phi$$

Phase difference =
$$\frac{2\pi}{\lambda}$$
 $imes$ $Path \ difference$

Since path difference = $\,\lambda$, Phase difference, $\,\phi=\,2\pi\,$ and I' = K [Given]

Therefore
$$I_1 = rac{K}{4} \cdot (\mathbf{i})$$

When path difference= $\frac{\lambda}{3}$

Phase difference, $\,\phi=\,rac{2\pi}{3}\,$

Hence, resultant intensity:

$$I'_g = I_1 + I_1 + 2\sqrt{I_1 I_1} \cos \frac{2\pi}{3}$$

$$= 2I_1 + 2I_1(-\frac{1}{2})$$

Using equation (i), we can write:

$$I_g = I_1 = \frac{K}{4}$$

Hence, the intensity of light at a point where the path difference is $\frac{\lambda}{3}$ is $\frac{K}{4}$ units.

Question 6:

A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes in a Young's double-slit experiment.

- (a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm.
- (b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

Solution:

Wavelength of the light beam, λ_1 = 650 nm

Wavelength of another light beam, $\,\lambda_2\,$ = 520 nm

Distance of the slits from the screen = D

Distance between the two slits = d

(i) Distance of the nth bright fringe on the screen from the central maximum is given by the relation,

$$x = n \lambda_1 \left(\frac{D}{d}\right)$$

For third bright fringe, n=3

Therefore x =
$$3 \times 650 \frac{D}{d} = 1950 \frac{D}{d} nm$$

(b) Let, the $\,n^{th}\,$ bright fringe due to wavelength $\,\lambda_2\,$ and $\,(n-1)^{th}\,$ bright fringe due to wavelength $\,\lambda_2\,$

coincide on the screen. The value of n can be obtained by equating the conditions for bright fringes:

$$n\lambda_2 = (n-1)\lambda_1$$

$$520n = 650n - 650$$

650=130n

Therefore n = 5

Hence, the least distance from the central maximum can be obtained by the relation:

x =
$$n \; \lambda_2 \; \frac{D}{d} \;$$
 = $5 \times \; 520 \frac{D}{d} = \; 2600 \frac{D}{d} \;$ nm

Note: The value of d and D are not given in the question.

Question 7:

In a double-slit experiment, 0.2° is found to be the angular width of a fringe on a screen placed 1 m away. The wavelength of light used is 600 nm. What will be the angular width of the fringe if the entire experimental

apparatus is immersed in water? Take refractive index of water to be $\, rac{4}{3} \, .$

Solution:

Distance of the screen from the slits, D=1m

Wavelength of light used, $\lambda_1 = 600 \text{ nm}$

Angular width of the fringe in air θ_1 = 0.2°

Angular width of the fringe in water= θ_2

Refractive index of water, $\mu=rac{4}{3}$

 $\mu=rac{ heta_1}{ heta_2}$ is the relation between the refractive index and the angular width

$$\theta_2 = \frac{3}{4}\theta_1$$

$$= \frac{3}{4} \times 0.2 = 0.15$$

Therefore, 0.15° is the reduction in the angular width of the fringe in water.

Question 8:

What is the Brewster angle for air to glass transition? (Refractive index of glass=1.5.)

Solution:

Refractive index of glass, $\,\mu=\,1.5\,$

Consider Brewster angle = θ

Following is the relation between the Brewster angle and the refractive index:

$$tan\theta = \mu$$

$$\theta = tan^{-1}(1.5) = 56.31^{\circ}$$

Therefore, the Brewster angle for air to glass transition is 56.31°

Question 9:

Light of wavelength 5000 Armstrong falls on a plane reflecting surface. What are the wavelength and frequency of the reflected light? For what angle of incidence is the reflected ray normal to the incident ray?

Solution:

Wavelength of incident light, $[\lambda] = 5000$ Armstrong = 5000 x 10⁻¹⁰ m

Speed of light, c =3 x 108 m

Following is the relation for the frequency of incident light:

$$v = \frac{c}{\lambda} = \frac{3 \times 10^8}{5000 \times 10^{-10}} = 6 \times 10^{14}$$

The wavelength and frequency of incident light is equal to the reflected ray. Therefore, 5000 Armstrong and 6 x 10^{14} Hz is the wavelength and frequency of the reflected light. When reflected ray is normal to incident ray, the sum of the angle of incidence, \angle i and angle of reflection, \angle r is 90°.

From laws of reflection we know that the angle of incidence is always equal to the angle of reflection

$$\angle i + \angle r = 90^{\circ}$$

i.e.
$$\angle i + \angle i = 90^{\circ}$$

Hence,
$$\angle i = \frac{90}{2}$$
 = 45°

Therefore, 45° is the angle of incidence.

Question 10:

Estimate the distance for which ray optics is a good approximation for an aperture of 4 mm and wavelength 400 nm.

Solution:

Fresnel's distance (Z_F) is the distance which is used in ray optics for a good approximation. Following is the

relation,

$$Z_F = \frac{a^2}{\lambda}$$

Where,

Aperture width, $a = 4 \text{ mm} = 4 \times 10^{-3} \text{ m}$

Wavelength of light, λ = 400 nm = 400 × 10⁻⁹ m

$$Z_F = rac{(4 imes 10^{-3})^2}{400 imes 10^{-9}}$$
 = 40m

Therefore, 40m is the distance for which the ray optics is a good approximation.

Question 11:

The 6563 Å H α line emitted by hydrogen in a star is found to be red-shifted by 15 Å. Estimate the speed with which the star is receding from the Earth.

Solution:

$$\lambda = 6563 \text{ Å}$$

$$\Delta \lambda = 15 \text{ Å}$$

Since the star is receding, the velocity (v) is negative.

$$\Delta \lambda = - v \lambda / c$$

$$V = - c\Delta \lambda/\lambda$$

$$= - (3 \times 10^8) \times (15 \text{ Å}/ 6563 \text{ Å})$$

$$= -6.86 \times 10^5 \text{ m/s}$$

Question 12:

Explain how Corpuscular theory predicts the speed of light in a medium, say, water, to be greater than the speed of light in vacuum. Is the prediction confirmed by experimental determination of the speed of light in water? If not, which alternative picture of light is consistent with experiment?

Solution:

According to Newton's Corpuscular theory, velocity of light in the denser medium (water) is greater than the velocity of light in the rarer medium (vacuum). This was experimentally wrong.

At the angle of incidence (i) of the light of velocity v, the angle of refraction is r.

Due to the change in the medium, the change in the velocity of light in water is v

Using Snells law,

$$c \sin i = v \sin r$$
 ——(1)

The relation between the velocities and the refractive index is

$$v/c = \mu$$
 ———(2)

$$v/c = \sin i/\sin r = \mu - (3)$$

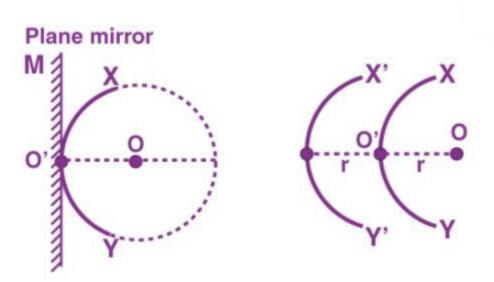
But μ > 1 so v > c is not possible

Huygens wave theory is consistent with the experiment.

Question 13:

You have learnt in the text how Huygens' principle leads to the laws of reflection and refraction. Use the same principle to deduce directly that a point object placed in front of a plane mirror produces a virtual image whose distance from the mirror is equal to the object distance from the mirror.

Solution:



Consider an object O placed in front of the plane mirror MO' at a distance r. The object is taken as the centre point O and a circle is drawn such that it just touches the plane mirror at point O'.

According to Huygens' Principle, XY is the wavefront of the incident light. If the mirror was not present, then a similar wavefront X'Y' (as XY) would form behind O' at distance r. X'Y' can be considered as a

virtual reflected ray for the plane mirror. Therefore, a point object placed in front of the plane mirror produces a virtual image whose distance from the mirror is equal to the object distance (r).

Question 14:

Let us list some of the factors, which could possibly influence the speed of wave propagation:

- (i) nature of the source.
- (ii) the direction of propagation.
- (iii) the motion of the source and/or observer.
- (iv) wavelength.
- (v) the intensity of the wave. On which of these factors, if any, does
- (a) the speed of light in a vacuum,
- (b) the speed of light in a medium (say, glass or water), depend?

Solution:

- (a) The speed of light in the vacuum does not depend on any of the factors listed.
- (b) The speed of light in the medium depends on the wavelength of the light in that medium

Question 15:

For sound waves, the Doppler formula for frequency shift differs slightly between the two situations: (i) source at rest; observer moving, and (ii) source moving; observer at rest. The exact Doppler formulas for the case of light waves in vacuum are, however, strictly identical for these situations. Explain why this should be so. Would you expect the formulas to be strictly identical for the two situations in the case of light travelling in a medium?

Solution:

The Doppler formula differs slightly between the two situations because the sound waves can travel only through the medium. The motion of the observer relative to the medium is different in both cases. Hence, the doppler formula is different.

Light waves can propagate in vacuum. In the vacuum, the speed of light does not depend on the motion of the observer and the source.

When light travels in the medium, the doppler formula for the two cases will be different.

Question 16:

In a double-slit experiment using the light of wavelength 600 nm, the angular width of a fringe formed on a distant screen is 0.1°. What is the spacing between the two slits?

Solution:

Wavelength of the light, $\lambda = 600 \text{ nm}$

The angular width of the fringe formed, θ = 0.1° = 0.1 π /180

Spacing between the slits, $d = \lambda/\theta = (600 \times 10^{-9} \times 180)/(0.1 \times 3.14)$

= 108000 x 10⁻⁹/0.314 d =3.44 x 10⁻⁴ m

Question 17:

Answer the following questions:

- (a) In a single slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band?
- (b) In what way is diffraction from each slit related to the interference pattern in a double-slit experiment?
- (c) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the shadow of the obstacle. Explain why?
- (d) Two students are separated by a 7 m partition wall in a room 10 m high. If both light and sound waves can bend around obstacles, how is it that the students are unable to see each other even though they can converse easily?
- (e) Ray optics is based on the assumption that light travels in a straight line. Diffraction effects (observed when light propagates through small apertures/slits or around small obstacles) disprove this assumption. Yet the ray optics assumption is so commonly used in an understanding location and several other properties of images in optical instruments. What is the justification?

Solution:

- (a) In a single slit diffraction experiment, if the width of the slit is made double the original width, the size of the central diffraction band reduces to half and the intensity of the band increases four times.
- (b) If the width of each slit is comparable to the wavelength of light used, the interference pattern thus obtained in the double-slit experiment is modified by diffraction from each of the two slits.
- (c) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the shadow of the obstacle. The bright spot is formed due to the constructive interference of the light waves that gets diffracted from the edges of the circular obstacle.
- (d) The obstacle bends the waves by a large angle if the wavelength of the wave is comparable with the size of the obstacle. The wavelength of the light wave is much smaller than the size of the wall. Therefore, the diffraction angle is also very small. As a result, the students will not be able to see each other. On the other hand, the size of the wall and the wavelength of the sound wave is comparable. Hence, the diffraction angle is large. Therefore, students can hear each other.
- (e) The size of the aperture in the optical instruments are much larger than the wavelength of light. Therefore, the diffraction effect of light is negligible in these instruments. Thus the assumption of light travelling in a straight line can be used in these instruments.

Question 18:

Two towers on top of two hills are 40 km apart. The line joining them passes 50 m above a hill halfway between the towers. What is the longest wavelength of radio waves, which can be sent between the towers without appreciable diffraction effects?

Solution:

Distance between the towers = 40 km

Height of the line joining the hills, d = 50 m

The radial spread of the radio waves must not exceed 50 m

Aperture a = d = 50 m

The hill is located halfway between the towers. Therefore, Fresnel's distance is $Z_p = 20 \text{ km}$

Fresnel's distance can be given by the equation, $Z_p = a^2/\lambda$

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\lambda = a^2/Z_p
= (50)^2/(20 \times 10^3)
= 250/20 = 12.5 \times 10^{-3} \text{ m} = 12.5 \text{ cm}
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Question 19:

A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1 m away. It is observed that the first minimum is at a distance of 2.5 mm from the centre of the screen. Find the width of the slit.

Solution:

Wavelength of the beam of light, $\lambda = 500 \text{ nm}$

Distance between the slit and the screen, D= 1 m

Distance of the first minimum from the centre of the screen, $x = 2.5 \text{ mm} = 2.5 \text{ x } 10^{-3} \text{ m}$

First minima, n = 1

Consider the equation, $n\lambda = xd/D$

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\Rightarrow d = nλD/x = (1 x 500 x 10<sup>-9</sup> x 1) /(2.5 x 10<sup>-3</sup>)
= 200 x 10<sup>-6</sup> m
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Question 20:

Answer the following questions:

- (a) When a low flying aircraft passes overhead, we sometimes notice a slight shaking of the picture on our TV screen. Suggest a possible explanation.
- (b) As you have learnt in the text, the principle of linear superposition of wave displacement is basic to understanding intensity distributions in diffraction and interference patterns. What is the justification of this principle?

Solution:

- (a) The weak radar signals from the aircraft interfere with the TV signal received by the antenna.
- (b) This is because superposition follows from the linear character of a differential equation that governs wave motion.

Question 21:

In deriving the single slit diffraction pattern, it was stated that the intensity is zero at angles of $n\lambda/a$. Justify this by suitably dividing the slit to bring out the cancellation.

Solution:

Let "a" be the width of a single slit. The single slit is further divided into n smaller slits of width a'.

$$a' = a/n$$

Each of the smaller slits should produce zero intensity for the single slit to produce zero intensity.

For this to happen, the angle of diffraction, $\theta = \lambda/a'$

$$\Rightarrow \theta = \lambda/(a/n)$$

or,
$$\theta$$
 = n λ /a

Therefore, in deriving the single slit diffraction pattern, it was stated that the intensity is zero at angles of $n\lambda/a$.