Exercise 9.1

Question 1:

Identify the terms, their coefficients for each of the following expressions:

(i) $5xyz^2 - 3zy$ (ii) $1 + x + x^2$

(iii) $4x^2y^2 - 4x^2y^2z^2 + z^2$ (iv) $3 - pq + qr - rp$

(v) $\frac{x}{2} + \frac{y}{2} - xy$ (vi) $0.3a - 0.6ab + 0.5b$

Answer 1:

(i) Terms: $5xyz^2$ and $-3zy$

Coefficient in $5xyz^2$ is 5 and in $-3zy$ is -3.

(ii) Terms: $1, x$ and x^2.

Coefficient of x and coefficient of x^2 is 1.

(iii) Terms: $4x^2y^2, -4x^2y^2z^2$ and z^2.

Coefficient in $4x^2y^2$ is 4, coefficient of $-4x^2y^2z^2$ is -4 and coefficient of z^2 is 1.

(iv) Terms: $3, -pq, qr$ and $-rp$

Coefficient of $-pq$ is -1, coefficient of qr is 1 and coefficient of $-rp$ is -1.

(v) Terms: $\frac{x}{2}, \frac{y}{2}$ and $-xy$

Coefficient of $\frac{x}{2}$ is $\frac{1}{2}$, coefficient of $\frac{y}{2}$ is $\frac{1}{2}$ and coefficient of $-xy$ is -1.

(vi) Terms: $0.3a, -0.6ab$ and $0.5b$

Coefficient of $0.3a$ is 0.3, coefficient of $-0.6ab$ is -0.6 and coefficient of $0.5b$ is 0.5.

Question 2:

Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any of these three categories:

$x + y$, 1000, $x + x^2 + x^3 + x^4$, $7 + y + 5x$, $2y - 3y^2$, $2y - 3y^2 + 4y^3$, $5x - 4y + 3xy$, $4z - 15^{\ 2}$, , pqr, $p^2q + pq^2$, $2p + 2q$

Answer 2:

(i) Since $x+y$ contains two terms. Therefore it is binomial.

(ii) Since 1000 contains one terms. Therefore it is monomial.

(iii) Since $x+x^2+x^3+x^4$ contains four terms. Therefore it is a polynomial and it does not fit in above three categories.

(iv) Since $7+y+5x$ contains three terms. Therefore it is trinomial.

(v) Since $2y-3y^2$ contains two terms. Therefore it is binomial.

(vi) Since $2y-3y^2+4y^3$ contains three terms. Therefore it is trinomial.

(vii) Since $5x-4y+3xy$ contains three terms. Therefore it is trinomial.

(viii) Since $4x-15z^2$ contains two terms. Therefore it is binomial.

(ix) Since $ab+bc+cd+da$ contains four terms. Therefore it is a polynomial and it does not fit in above three categories.

(x) Since pqr contains one terms. Therefore it is monomial.

(xi) Since p^2q+pq^2 contains two terms. Therefore it is binomial.

(xii) Since $2p+2q$ contains two terms. Therefore it is binomial.

Question 3:

Add the following:

(i) $ab-bc, bc-ca, ca-ab$

(ii) $a-b+ab, b-c+bc, c-a+ac$

(iii) $2p^2q^2-3pq+4, 5+7pq-3p^2q^2$

(iv) $l^2+m^2, m^2+n^2, n^2+l^2+2lm+2mn+2nl$

Answer 3:

(i) $ab-bc, bc-ca, ca-ab$

$$\begin{array}{l} ab-bc \\ \quad +bc-ca \\ -ab \quad +ca \\ \hline \boxed{0+0+0} \end{array}$$

Hence the sum is 0.

(ii) $a-b+ab, b-c+bc, c-a+ac$

$$\begin{array}{l} a-b-ab \\ \quad +b \quad -c+bc \\ -a \quad +c \quad +ac \\ \hline \boxed{0+0+ab+0+bc+ac} \end{array}$$

Hence the sum is $ab+bc+ac$.

(iii) $2p^2q^2-3pq+4, 5+7pq-3p^2q^2$

$$\begin{array}{r} 2p^2q^2-3pq+4 \\ -3p^2q^2+7pq+5 \\ \hline \boxed{-p^2q^2+4pq+9} \end{array}$$

Hence the sum is $-p^2q^2+4pq+9$.

(iv) $l^2+m^2, m^2+n^2, n^2+l^2, 2lm+2mn+2nl$

$$\begin{array}{r} l^2+m^2 \\ +\quad m^2+n^2 \\ +l^2\quad +n^2 \\ +\qquad 2lm+2mn+2nl \\ \hline \boxed{2l^2+2m^2+2n^2+2lm+2mn+2nl} \end{array}$$

Hence the sum is $2\left(l^2+m^2+n^2+lm+mn+nl\right)$

Question 4:

(a) Subtract $4a-7ab+3b+12$ from $12a-9ab+5b-3$.

(b) Subtract $3xy+5yz-7zx$ from $5xy-2yz-2zx+10xyz$.

(c) Subtract $4p^2q-3pq+5pq^2-8p+7q-10$ from $18-3p-11q+5pq-2pq^2+5p^2q$.

Answer 4:

(a)

$$\begin{array}{r} 12a-9ab+5b-3 \\ 4a-7ab+3b+12 \\ (-)\quad (+)\quad (-)(-) \\ \hline \boxed{8a-2ab+2b-15} \end{array}$$

(b)

$$\begin{array}{r} 5xy-2yz-2zx+10xyz \\ 3xy+5yz-7zx \\ (-)\quad (-)\quad (+) \\ \hline \boxed{2xy-7yz+5zx+10xyz} \end{array}$$

(c)

$$\begin{array}{r} 5p^2q-2pq^2+5pq-11q-3p+18 \\ 4p^2q+5pq^2-3pq+7q\ -8p-10 \\ (-)\quad (-)\quad (+)\quad (-)\ (+)\ (+) \\ \hline \boxed{p^2q-7pq^2+8pq-18q+5p+28} \end{array}$$

Exercise 9.2

Question 1:

Find the product of the following pairs of monomials:

(i) $4, 7p$ (ii) $-4p, 7p$ (iii) $-4p, 7pq$

(iv) $4p^3, -3p$ (iv) $4p, 0$

Answer 1:

(i) $4 \times 7p = 4 \times 7 \times p = 28p$

(ii) $-4p \times 7p = (-4 \times 7) \times (p \times p) = -28p^2$

(iii) $-4p \times 7pq = (-4 \times 7)(p \times pq) = -28p^2q$

(iv) $4p^3 \times -3p = (4 \times -3)(p^3 \times p) = -12p^4$

(v) $4p \times 0 = (4 \times 0)(p) = 0$

Question 2:

Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively:

(p, q); $(10m, 5n)$; $(20x^2, 5y^2)$; $(4x, 3x^2)$; $(3mn, 4np)$

Answer 2:

(i) Area of rectangle = length x breadth
= $p \times q = pq$ sq. units

(ii) Area of rectangle = length x breadth
= $10m \times 5n = (10 \times 5)(m \times n) = 50mn$ sq. units

(iii) Area of rectangle = length x breadth
= $20x^2 \times 5y^2 = (20 \times 5)(x^2 \times y^2) = 100x^2y^2$ sq. units

(iv) Area of rectangle = length x breadth
= $4x \times 3x^2 = (4 \times 3)(x \times x^2) = 12x^3$ sq. units

(v) Area of rectangle = length x breadth
= $3mn \times 4np = (3 \times 4)(mn \times np) = 12mn^2p$ sq. units

Question 3:

Complete the table of products:

First monomial ⟶ Second monomial ↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	.....	.....	.....	.....	.....
$-5y$	.....	.....	$-15x^2y$	.....	.....	.....
$3x^2$	.....	.....	.....	.....	.....	.....
$-4xy$	.....	.....	.....	.....	.....	.....
$7x^2y$	.....	.....	.....	.....	.....	.....
$-9x^2y^2$	.....	.....	.....	.....	.....	.....

Answer 3:

First monomial ⟶ Second monomial ↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	$-10xy$	$6x^3$	$-8x^2y$	$14x^3y$	$-18x^3y^2$
$-5y$	$-10xy$	$25y^2$	$-15x^2y$	$20xy^2$	$-35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$-15x^2y$	$9x^4$	$-12x^3y$	$21x^4y$	$-27x^4y^2$
$-4xy$	$8x^2y$	$20xy^2$	$-12x^3y$	$16x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$-35x^2y^2$	$21x^4y$	$-28x^3y^2$	$49x^4y^2$	$-63x^4y^3$
$-9x^2y^2$	$-18x^3y^2$	$45x^2y^3$	$-27x^4y^2$	$36x^3y^3$	$-63x^4y^3$	$81x^4y^4$

Question 4:

Obtain the volume of rectangular boxes with the following length, breadth and height respectively:

(i) $5a, 3a^2 7a^4$ (ii) $2p, 4q, 8r$

(iii) $xy, 2x^2y, 2xy^2$ (iv) $a, 2b, 3c$

Answer 4:

(i) Volume of rectangular box = length x breadth x height

$= 5a \times 3a^2 \times 7a^4 = (5 \times 3 \times 7)(a \times a^2 \times a^4)$

$= 105a^7$ cubic units

(ii) Volume of rectangular box = length x breadth x height

$= 2p \times 4q \times 8r = (2 \times 4 \times 8)(p \times q \times r)$

$= 64pqr$ cubic units

(iii) Volume of rectangular box = length x breadth x height

$= xy \times 2x^2y \times 2xy^2 = (1 \times 2 \times 2)(x \times x^2 \times x \times y \times y \times y^2)$

$= 4x^4y^4$ cubic units

(iv) Volume of rectangular box = length x breadth x height

$= a \times 2b \times 3c = (1 \times 2 \times 3)(a \times b \times c) = 6abc$ cubic units

Question 5:

Obtain the product of:

(i) xy, yz, zx

(ii) $a, -a^2, a^3$

(iii) $2, 4y, 8y^2, 16y^3$

(iv) $a, 2b, 3c, 6abc$

(v) $m, -mn, mnp$

Answer 5:

(i) $xy \times yz \times zx = x \times x \times y \times y \times z \times z = x^2y^2z^2$

(ii) $a \times (-a^2) \times a^3 = (-1)(a \times a^2 \times a^3) = -a^6$

(iii) $2 \times 4y \times 8y^2 \times 16y^3 = (2 \times 4 \times 8 \times 16)(y \times y^2 \times y^3) = 1024y^6$

(iv) $a \times 2b \times 3c \times 6abc = (1 \times 2 \times 3 \times 6)(a \times b \times c \times abc) = 36a^2b^2c^2$

(v) $m \times -mn \times mnp = (1)(m \times m \times m \times n \times n \times p) = -m^3n^2p$

Exercise 9.3

Question 1:

Carry out the multiplication of the expressions in each of the following pairs:

(i) $4p, q+r$ (ii) $ab, a-b$

(iii) $a+b, 7a^2b^2$ (iv) $a^2-9, 4a$

(v) $pq+qr+rp, 0$

Answer 1:

(i) $4p\times(q+r) = 4p\times q + 4p\times r$

$= 4pq + 4pr$

(ii) $ab\times(a-b) = ab\times a - ab\times b$

$= a^2b - ab^2$

(iii) $(a+b)\times 7a^2b^2 = a\times 7a^2b^2 + b\times 7a^2b^2$

$= 7a^3b^2 + 7a^2b^3$

(iv) $(a^2-9)\times 4a = a^2\times 4a - 4a\times 9$

$= 4a^3 - 36a$

(v) $(pq+qr+rp)\times 0 = pq\times 0 + qr\times 0 + rp\times 0$

$= 0 + 0 + 0 = 0$

Question 2:

Complete the table:

	First expression	Second expression	Product
(i)	a	$b+c+d$	.....
(ii)	$x+y-5$	$5xy$	.....
(iii)	p	$6p^2-7p+5$	.....
(iv)	$4p^2q^2$	p^2-q^2	.....
(v)	$a+b+c$	abc	.....

Answer 2:

	First expression	Second expression	Product
(i)	a	$b+c+d$	$a(b+c+d)$ $= a\times b+a\times c+a\times d$ $= ab+ac+ad$
(ii)	$x+y-5$	$5xy$	$5xy(x+y-5)$ $= 5xy\times x+5xy\times y-5xy\times 5$ $= 5x^2y+5xy^2-25xy$
(iii)	p	$6p^2-7p+5$	$p(6p^2-7p+5)$ $= p\times 6p^2-p\times 7p+p\times 5$ $= 6p^3-7p^2+5p$
(iv)	$4p^2q^2$	p^2-q^2	$4p^2q^2(p^2-q^2)$ $= 4p^2q^2\times p^2-4p^2q^2\times q^2$ $= 4p^4q^2-4p^2q^4$
(v)	$a+b+c$	abc	$abc(a+b+c)$ $= abc\times a+abc\times b+abc\times c$ $= a^2bc+ab^2c+abc^2$

Question 3:

Find the product:

(i) $\left(a^2\right)\times\left(2a^{22}\right)\times\left(4a^{26}\right)$

(ii) $\left(\frac{2}{3}xy\right)\times\left(\frac{-9}{10}x^2y^2\right)$

(iii) $\left(\frac{-10}{3}pq^3\right)\times\left(\frac{6}{5}p^3q\right)$

(iv) $x\times x^2\times x^3\times x^4$

Answer 3:

(i) $\left(a^2\right)\times\left(2a^{22}\right)\times\left(4a^{26}\right)=(2\times4)\left(a^2\times a^{22}\times a^{26}\right)$

$= 8\times a^{2+22+26}=8a^{50}$

(ii) $\left(\frac{2}{3}xy\right)\times\left(\frac{-9}{10}x^2y^2\right)=\left(\frac{2}{3}\times\frac{-9}{10}\right)\left(x\times x^2\times y\times y^2\right)$

$=\frac{-3}{5}x^3y^3$

(iii) $\left(\frac{-10}{3}pq^3\right)\left(\frac{6}{5}p^3q\right)=\left(\frac{-10}{3}\times\frac{6}{5}\right)\left(p\times p^3\times q^3\times q\right)$

$=-4p^4q^4$

(iv) $x\times x^2\times x^3\times x^4=x^{1+2+3+4}=x^{10}$

Question 4:

(a) Simplify: $3x\left(4x-5\right)+3$ and find values for (i) $x=3$ (ii) $x=\frac{1}{2}$.

(b) Simplify: $a\left(a^2+a+1\right)+5$ and find its value for (i) $a=0$ (ii) $a=1$ (iii) $a=-1$.

Answer 4:

(a) $3x\left(4x-5\right)+3=3x\times4x-3x\times5+3=12x^2-15x+3$

(i) For $x=3$, $12x^2+15x+3=12\left(3\right)^2-15\times3+3$

$= 12 \text{ x } 9 - 45 + 3 = 108 - 45 + 3 = 66$

(ii) For $x=\frac{1}{2}$, $12x^2-15x+3=12\left(\frac{1}{2}\right)^2-15\times\frac{1}{2}+3=12\times\frac{1}{4}-\frac{15}{2}+3$

$=6-\frac{15}{2}=\frac{12-15}{2}=\frac{-3}{2}$

(b) $a\left(a^2+a+1\right)+5=a\times a^2+a\times a+a\times1+5=a^3+a^2+a+5$

(i) For $a=0$, $a^3+a^2+a+5=\left(0\right)^3+\left(0\right)^2+\left(0\right)+5$

$= 0 + 0 + 0 + 5 = 5$

(ii) For $a=1$, $a^3+a^2+a+5=\left(1\right)^3+\left(1\right)^2+\left(1\right)+5$

$= 1 + 1 + 1 + 5 = 8$

(iii) For $a=-1$, $a^3+a^2+a+5 = (-1)^3+(-1)^2+(-1)+5$

$= -1+1-1+5 = -2+6 = 4$

Question 5:

(a) Add: $p(p-q), q(q-r)$ and $r(r-p)$.

(b) Add: $2x(z-x-y)$ and $2y(z-y-zx)$.

(c) Subtract: $3l(l-4m+5n)$ from $4l(10n-3m+2l)$.

(d) Subtract: $3a(a+b+c)-2b(a-b+c)$ from $4c(-a+b+c)$.

Answer 5:

(a) $p(p-q)+q(q-r)+r(r-p) = p^2-pq+q^2-qr+r^2-rp$

$= p^2+q^2+r^2-pq-qr-rp$

(b) $2x(z-x-y)+2y(z-y-x) = 2xz-2x^2-2xy+2yz-2y^2-2xy$

$= 2xz-2xy-2xy+2yz-2x^2-2y^2$

$= -2x^2-2y^2-4xy+2yz+2zx$

(c) $4l(10n-3m+2l)-3l(l-4m+5n) = 40ln-12lm+8l^2-3l^2+12lm-15ln$

$= 8l^2-3l^2-12lm+12lm+40ln-15ln$

$= 5l^2+25ln$

(d) $4c(-a+b+c)-[3a(a+b+c)-2b(a-b+c)]$

$= -4ac+4bc+4c^2-[3a^2+3ab+3ac-2ab+2b^2-2bc]$

$= -4ac+4bc+4c^2-[3a^2+2b^2+3ab-2bc+3ac-2ab]$

$= -4ac+4bc+4c^2-[3a^2+2b^2+ab+3ac-2bc]$

$= -4ac+4bc+4c^2-3a^2-2b^2-ab-3ac+2bc$

$= -3a^2-2b^2+4c^2-ab+4bc+2bc-4ac-3ac$

$= -3a^2-2b^2+4c^2-ab+6bc-7ac$

Exercise 9.4

Question 1:

Multiply the binomials:

(i) $(2x+5)$ and $(4x-3)$

(ii) $(y-8)$ and $(3y-4)$

(iii) $(2.5l-0.5m)$ and $(2.5l+0.5m)$

(iv) $(a+3b)$ and $(x+5)$

(v) $(2pq+3q^2)$ and $(3pq-2q^2)$

(vi) $\left(\frac{3}{4}a^2+3b^2\right)$ and $4\left(a^2-\frac{2}{3}b^2\right)$

Answer 1:

(i) $(2x+5)\times(4x-3)=2x(4x-3)+5(4x-3)$

$= 2x\times4x-2x\times3+5\times4x-5\times3$

$= 8x^2-6x+20x-15$

$= 8x^2+14x-15$

(ii) $(y-8)\times(3y-4)=y(3y-4)-8(3y-4)$

$= y\times3y-y\times4-8\times3y-8\times-4$

$= 3y^2-4y-24y+12$

$= 3y^2-28y+12$

(iii) $(2.5l-0.5m)\times(2.5l+0.5m)=2.5l\times(2.5l+0.5m)-0.5m\times(2.5l+0.5m)$

$= 2.5l\times2.5l+0.5l\times0.5m-0.5m\times2.5l-0.5m\times0.5m$

$= 6.25l^2+1.25lm-1.25lm-0.25m^2$

$= 6.25l^2-0.25m^2$

(iv) $(a+3b)\times(x+5)=a(x+5)+3b(x+5)$

$= a\times x+a\times5+3b\times x+3b\times5$

$= ax+5a+3bx+15b$

(v) $(2pq+3q^2)(3pq-2q^2) = 2pq\times(3pq-2q^2)+3q^2(3pq-2q^2)$

$= 2pq\times3pq-2pq\times2q^2+3q^2\times3pq-3q^2\times2q^2$

$= 6p^2q^2-4pq^3+9pq^3-6q^4$

$= 6p^2q^2+5pq^3-6q^4$

(vi) $\left(\frac{3}{4}a^2+3b^2\right)\times 4\left(a^2-\frac{2}{3}b^2\right)=\left(\frac{3}{4}a^2+3b^2\right)\times\left(4a^2-\frac{8}{3}b^2\right)$

$=\frac{3}{4}a^2\times\left(4a^2-\frac{8}{3}b^2\right)+3b^2\times\left(4a^2-\frac{8}{3}b^2\right)$

$=\frac{3}{4}a^2\times 4a^2-\frac{3}{4}a^2\times\frac{8}{3}b^2+3b^2\times 4a^2-3b^2\times\frac{8}{3}b^2$

$=3a^4-2a^2b^2+12a^2b^2-8b^4$

$=3a^4+10a^2b^2-8b^4$

Question 2:

Find the product:

(i) $(5-2x)(3+x)$ (ii) $(x+7y)(7x-y)$

(iii) $(a^2+b)(a+b^2)$ (iv) $(p^2-q^2)(2p+q)$

Answer 2:

(i) $(5-2x)(3+x)=5\times(3+x)-2x(3+x)=5\times 3+5\times x-2x\times 3-2x\times x$

$=15+5x-6x-2x^2=15-x-2x^2$

(ii) $(x+7y)(7x-y)=x(7x-y)+7y\times(7x-y)$

$=x\times 7x-x\times y+7y\times 7x-7y\times y$

$=7x^2-xy+49xy-7y^2$

$=7x^2+48xy-7y^2$

(iii) $(a^2+b)(a+b^2)=a^2\times(a+b^2)+b\times(a+b^2)$

$=a^2\times a+a^2\times b^2+b\times a+b\times b^2$

$=a^3+a^2b^2+ab+b^3$

(iv) $(p^2-q^2)(2p+q)=p^2\times(2p+q)-q^2(2p+q)$

$=p^2\times 2p+p^2\times q-q^2\times 2p-q^2\times q$

$=2p^3+p^2q-2pq^2-q^3$

Question 3:

Simplify:

(i) $(x^2-5)(x+5)+25$

(ii) $(a^2+5)(b^2+3)+5$

(iii) $(t+s^2)(t^2-s)$

(iv) $(a+b)(c-d)+(a-b)(c+d)+2(ac+bd)$

(v) $(x+y)(2x+y)+(x+2y)(x-y)$

(vi) $(x+y)(x^2-xy+y^2)$

(vii) $(1.5x-4y)(1.5x+4y+3)-4.5x+12y$

(viii) $(a+b+c)(a+b-c)$

Answer 3:

(i) $(x^2-5)(x+5)+25 = x^2(x+5)-5(x+5)+25$

$= x^2\times x + x^2\times 5 - 5\times x - 5\times 5 + 25$

$= x^3 + 5x^2 - 5x - 25 + 25$

$= x^3 + 5x^2 - 5x$

(ii) $(a^2+5)(b^3+3)+5 = a^2(b^3+3)+5(b^3+3)+5$

$= a^2\times b^3 + a^2\times 3 + 5\times b^3 + 5\times 3 + 5$

$= a^2b^3 + 3a^2 + 5b^3 + 15 + 5$

$= a^2b^3 + 3a^2 + 5b^3 + 20$

(iii) $(t+s^2)(t^2-s) = t(t^2-s)+s^2(t^2-s)$

$= t\times t^2 - t\times s + s^2\times t^2 - s^2\times s$

$= t^3 - st + s^2t^2 - s^3$

(iv) $(a+b)(c-d)+(a-b)(c+d)+2(ac+bd)$

$= a(c-d)+b(c-d)+a(c+d)-b(c+d)+2ac+2bd$

$= ac-ad+bc-bd+ac+ad-bc-bd+2ac+2bd$

$= ac+ac-ad+ad+bc-bc-bd-bd+2ac+2bd$

$= 2ac-2bd+2ac+2bd$

$= 4ac$

(v) $(x+y)(2x+y)+(x+2y)(x-y)=x(2x+y)+y(2x+y)+x(x-y)+2y(x-y)$

$= 2x^2+xy+2xy+y^2+x^2-xy+2xy-2y^2$

$= 2x^2+x^2+xy+2xy-xy+2xy+y^2-2y^2$

$= 3x^2+4xy-y^2$

(vi) $(x+y)(x^2-xy+y^2)=x(x^2-xy+y^2)+y(x^2-xy+y^2)$

$= x^3-x^2y+xy^2+x^2y-xy^2+y^3$

$= x^3-x^2y+x^2y+xy^2-xy^2+y^3$

$= x^3+y^3$

(vii) $(1.5x-4y)(1.5x+4y+3)-4.5x+12y$

$=1.5x(1.5x+4y+3)-4y(1.5x+4y+3)-4.5x+12y$

$= 2.25x^2+6.0xy+4.5x-6.0xy-16y^2-12y-4.5x+12y$

$= 2.25x^2+6.0xy-6.0xy+4.5x-4.5x-16y^2-12y+12y$

$= 2.25x^2-16y^2$

(viii) $(a+b+c)(a+b-c)=a(a+b-c)+b(a+b-c)+c(a+b-c)$

$= a^2+ab-ac+ab+b^2-bc+ac+bc-c^2$

$= a^2+ab+ab-ac+ac-bc+bc+b^2-c^2$

$= a^2+b^2-c^2+2ab$

Exercise 9.5

Question 1:

Use a suitable identity to get each of the following products:

(i) $(x+3)(x+3)$

(ii) $(2y+5)(2y+5)$

(iii) $(2a-7)(2a-7)$

(iv) $\left(3a-\frac{1}{2}\right)\left(3a-\frac{1}{2}\right)$

(v) $(1.1m-0.4)(1.1m+0.4)$

(vi) $\left(a^2+b^2\right)\left(-a^2+b^2\right)$

(vii) $(6x-7)(6x+7)$

(viii) $(-a+c)(-a+c)$

(ix) $\left(\frac{x}{2}+\frac{3y}{4}\right)\left(\frac{x}{2}+\frac{3y}{4}\right)$

(x) $(7a-9b)(7a-9b)$

Answer 1:

(i) $(x+3)(x+3)=(x+3)^2$

$=(x)^2+2\times x\times 3+(3)^2$ [Using identity $(a+b)^2=a^2+2ab+b^2$]

$= x^2+6x+9$

(ii) $(2y+5)(2y+5)=(2y+5)^2$

$= (2y)^2+2\times 2y\times 5+(5)^2$ [Using identity $(a+b)^2=a^2+2ab+b^2$]

$= 4y^2+20y+25$

(iii) $(2a-7)(2a-7)=(2a-7)^2$

$= (2a)^2-2\times 2a\times 7+(7)^2$ [Using identity $(a-b)^2=a^2-2ab+b^2$]

$= 4a^2-28a+49$

(iv) $\left(3a-\frac{1}{2}\right)\left(3a-\frac{1}{2}\right)=\left(3a-\frac{1}{2}\right)^2$

$= (3a)^2-2\times 3a\times\frac{1}{2}+\left(\frac{1}{2}\right)^2$ [Using identity $(a-b)^2=a^2-2ab+b^2$]

$= 9a^2-3a+\frac{1}{4}$

(v) $(1.1m-0.4)(1.1m+0.4)=(1.1m)^2-(0.4)^2$

Using identity $(a-b)(a+b)=a^2-b^2$]

$= 1.21m^2 - 0.16$

(vi) $(a^2+b^2)(-a^2+b^2) = (b^2+a^2)(b^2-a^2)$

$= (b^2)^2 - (a^2)^2$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

$= b^4 - a^4$

(vii) $(6x-7)(6x+7) = (6x)^2 - (7)^2$ [Using identity $(a-b)(a+b) = a^2 - b^2$]

$= 36x^2 - 49$

(viii) $(-a+c)(-a+c) = (c-a)(c-a) = (c-a)^2$

$= (c)^2 - 2 \times c \times a + (a)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= c^2 - 2ca + a^2$

(ix) $\left(\frac{x}{2}+\frac{3y}{4}\right)\left(\frac{x}{2}+\frac{3y}{4}\right) = \left(\frac{x}{2}+\frac{3y}{4}\right)^2$

$= \left(\frac{x}{2}\right)^2 + 2 \times \frac{x}{2} \times \frac{3y}{4} + \left(\frac{3y}{4}\right)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= \frac{x^2}{4} + \frac{3}{4}xy + \frac{9}{16}y^2$

(x) $(7a-9b)(7a-9b) = (7a-9b)^2$

$= (7a)^2 - 2 \times 7a \times 9b + (9b)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= 49a^2 - 126ab + 81b^2$

Question 2:

Use the identity $(x+a)(x+b)=x^2+(a+b)x+ab$ to find the following products:

(i) $(x+3)(x+7)$ (ii) $(4x+5)(4x+1)$

(iii) $(4x-5)(4x-1)$ (iv) $(4x+5)(4x-1)$

(v) $(2x+5y)(2x+3y)$ (vi) $(2a^2+9)(2a^2+5)$

(vii) $(xyz-4)(xyz-2)$

Answer 2:

(i) $(x+3)(x+7)=(x)^2+(3+7)x+3\times 7$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= x^2+10x+21$

(ii) $(4x+5)(4x+1)=(4x)^2+(5+1)4x+5\times 1$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= 16x^2+6\times 4x+5=16x^2+24x+5$

(iii) $(4x-5)(4x-1) = (4x)^2+(-5-1)4x+(-5)\times(-1)$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= 16x^2+(-6)\times 4x+5=16x^2-24x+5$

(iv) $(4x+5)(4x-1)=(4x)^2+\{5\times(-1)\}\times 4x+5\times(-1)$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= 16x^2+(5-1)\times 4x-5$

$= 16x^2+4\times 4x-5$

$= 16x^2+16x-5$

(v) $(2x+5y)(2x+3y)=(2x)^2+(5y+3y)\times 2x+5y\times 3y$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= 4x^2+8y\times 2x+15y^2$

$= 4x^2+16xy+15y^2$

(vi) $(2a^2+9)(2a^2+5)=(2a^2)^2+(9+5)\times 2a^2+9\times 5$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= 4a^4+14\times 2a^2+45$

$= 4a^4+28a^2+45$

(vii) $(xyz-4)(xyz-2)=(xyz)^2+(-4-2)\times xyz+(-4)\times(-2)$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= x^2y^2z^2-6xyz+8$

Question 3:

Find the following squares by using identities:

(i) $(b-7)^2$ (ii) $(xy+3z)^2$ (iii) $(6x^2-5y)^2$

(iv) $\left(\frac{2}{3}m+\frac{3}{2}n\right)^2$ (v) $(0.4p-0.5q)^2$ (vi) $(2xy+5y)^2$

Answer 3:

(i) $(b-7)^2=(b)^2-2\times b\times 7+(7)^2$ [Using identity $(a-b)^2=a^2-2ab+b^2$]

$= b^2-14b+49$

(ii) $(xy+3z)^2=(xy)^2+2\times xy\times 3z+(3z)^2$ [Using identity $(a+b)^2=a^2+2ab+b^2$]

$= x^2y^2+6xyz+9z^2$

(iii) $(6x^2-5y)^2=(6x^2)^2-2\times 6x^2\times 5y+(5y)^2$

[Using identity $(a-b)^2=a^2-2ab+b^2$]

$= 36x^4-60x^2y+25y^2$

(iv) $\left(\frac{2}{3}m+\frac{3}{2}n\right)^2=\left(\frac{2}{3}m\right)^2+2\times\frac{2}{3}m\times\frac{3}{2}n+\left(\frac{3}{2}n\right)^2$

[Using identity $(a+b)^2=a^2+2ab+b^2$]

$= \frac{4}{9}m^2 + 2mn + \frac{9}{4}n^2$

(v) $(0.4p - 0.5q)^2 = (0.4p)^2 - 2 \times 0.4p \times 0.5q + (0.5q)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= 0.16p^2 - 0.40pq + 0.25q^2$

(vi) $(2xy + 5y)^2 = (2xy)^2 + 2 \times 2xy \times 5y + (5y)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= 4x^2y^2 + 20xy^2 + 25y^2$

Question 4:

Simplify:

(i) $(a^2 - b^2)^2$

(ii) $(2x+5)^2 - (2x-5)^2$

(iii) $(7m - 8n)^2 + (7m + 8n)^2$

(iv) $(4m + 5n)^2 + (5m + 4n)^2$

(v) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$

(vi) $(ab + bc)^2 - 2ab^2c$

(vii) $(m^2 - n^2m)^2 + 2m^3n^2$

Answer 4:

(i) $(a^2 - b^2)^2 = (a^2)^2 - 2 \times a^2 \times b^2 + (b^2)^2$ [Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= a^4 - 2a^2b^2 + b^4$

(ii) $(2x+5)^2 - (2x-5)^2 = (2x)^2 + 2 \times 2x \times 5 + (5)^2 - \left[(2x)^2 - 2 \times 2x \times 5 + (5)^2\right]$

[Using identities $(a+b)^2 = a^2 + 2ab + b^2$ and $(a-b)^2 = a^2 - 2ab + b^2$]

$= 4x^2 + 20x + 25 - \left[4x^2 - 20x + 25\right]$

$= 4x^2 + 20x + 25 - 4x^2 + 20x - 25$

$= 40x$

(iii)

$(7m-8n)^2 + (7m+8n)^2 = (7m)^2 - 2\times 7m\times 8n + (8n)^2 + \left[(7m)^2 + 2\times 7m\times 8n + (8n)^2\right]$

[Using identities $(a+b)^2 = a^2 + 2ab + b^2$ and $(a-b)^2 = a^2 - 2ab + b^2$]

$= 49m^2 - 112mn + 64n^2 + \left[49m^2 + 112mn + 64n^2\right]$

$= 49m^2 - 112mn + 64n^2 + 49m^2 + 112mn + 64n^2$

$= 98m^2 + 128n^2$

(iv) $(4m+5n)^2 + (5m+4n)^2 = (4m)^2 + 2\times 4m\times 5n + (5n)^2 + (5m)^2 + 2\times 5m\times 4n + (4n)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= 16m^2 + 40mn + 25n^2 + 25m^2 + 40mn + 16n^2$

$= 16m^2 + 25m^2 + 40mn + 40mn + 25n^2 + 16n^2$

$= 41m^2 + 80mn + 41n^2$

(v) $(2.5p-1.5q)^2 - (1.5p-2.5q)^2$

$= (2.5p)^2 - 2\times 2.5p\times 1.5q + (1.5q)^2 - \left[(1.5p)^2 - 2\times 1.5p\times 2.5q + (2.5q)^2\right]$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= 6.25p^2 - 7.50pq + 2.25q^2 - \left[2.25p^2 - 7.50pq + 6.25q^2\right]$

$= 6.25p^2 - 7.50pq + 2.25q^2 - 2.25p^2 + 7.50pq - 6.25q^2$

$= 4p^2 - 4q^2$

(vi) $(ab+bc)^2 - 2ab^2c = (ab)^2 + 2\times ab\times bc + (bc)^2 - 2ab^2c$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= a^2b^2 + 2ab^2c + b^2c^2 - 2ab^2c$

$= a^2b^2 + b^2c^2$

(vii) $\left(m^2-n^2m\right)^2+2m^3n^2 = \left(m^2\right)^2-2\times m^2\times n^2m+\left(n^2m\right)^2+2m^3n^2$

[Using identity $\left(a-b\right)^2=a^2-2ab+b^2$]

$= m^4-2m^3n^2+n^4m^2+2m^3n^2$

$= m^4+n^4m^2$

Question 5:

Show that:

(i) $\left(3x+7\right)^2-84x=\left(3x-7\right)^2$

(ii) $\left(9p-5q\right)^2+180pq=\left(9p+5q\right)^2$

(iii) $\left(\frac{4}{3}m-\frac{3}{4}n\right)^2+2mn=\frac{16}{9}m^2+\frac{9}{16}n^2$

(iv) $\left(4pq+3q\right)^2-\left(4pq-3q\right)^2=48pq^2$

(v) $\left(a-b\right)\left(a+b\right)+\left(b-c\right)\left(b+c\right)+\left(c-a\right)\left(c+a\right)=0$

Answer 5:

(i) L.H.S. $= \left(3x+7\right)^2-84x=\left(3x\right)^2+2\times3x\times7+\left(7\right)^2-84x$

[Using identity $\left(a+b\right)^2=a^2+2ab+b^2$]

$= 9x^2+42x+49-84x$

$= 9x^2-42x+49$

$= \left(3x-7\right)^2$ [$\because$ $\left(a-b\right)^2=a^2-2ab+b^2$]

$=$ R.H.S.

(ii) L.H.S. $= \left(9p-5q\right)^2+180pq=\left(9p\right)^2-2\times9p\times5q+\left(5q\right)^2+180pq$

[Using identity $\left(a-b\right)^2=a^2-2ab+b^2$]

$= 81p^2-90pq+25q^2+180pq$

$= 81p^2+90pq+25q^2$

$= \left(9p+5q\right)^2$ [$\because$ $\left(a+b\right)^2=a^2+2ab+b^2$]

(iii) $\text{L.H.S.} = \left(\frac{4}{3}m-\frac{3}{4}n\right)^2+2mn=\left(\frac{4}{3}m\right)^2-2\times\frac{4}{3}m\times\frac{3}{4}n+\left(\frac{3}{4}n\right)^2+2mn$

[Using identity $(a-b)^2=a^2-2ab+b^2$]

$=\frac{16}{9}m^2-2mn+\frac{9}{16}n^2+2mn$

$=\frac{16}{9}m^2+\frac{9}{16}n^2$

= R.H.S.

(iv) $\text{L.H.S.} = (4pq+3q)^2-(4pq-3q)^2$

$=(4pq)^2+2\times4pq\times3q+(3q)^2-\left[(4pq)^2-2\times4pq\times3q+(3q)^2\right]$

[Using identities $(a+b)^2=a^2+2ab+b^2$ and $(a-b)^2=a^2-2ab+b^2$]

$=16p^2q^2+24pq^2+9q^2-\left[16p^2q^2-24pq^2+9q^2\right]$

$=16p^2q^2+24pq^2+9q^2-16p^2q^2+24pq^2-9q^2$

$=48pq^2$

= R.H.S.

(v) $\text{L.H.S.} = (a-b)(a+b)+(b-c)(b+c)+(c-a)(c+a)$

$=a^2-b^2+b^2-c^2+c^2-a^2$ [Using identity $(a-b)(a+b)=a^2-b^2$]

= 0

= R.H.S.

Question 6:

Using identities, evaluate:

(i)	71^2	(ii)	99^2	(iii)	102^2
(iv)	998^2	(v)	5.2^2	(vi)	297×303
(vii)	78×82	(viii)	8.9^2	(ix)	1.05×9.5

Answer 6:

(i) $71^2 = (70+1)^2 = (70)^2 + 2\times 70\times 1 + (1)^2$

[Using identity $(a+b)^2 = a^2 - 2ab + b^2$]

= 4900 + 140 + 1 = 5041

(ii) $99^2 = (100-1)^2 = (100)^2 - 2\times 100\times 1 + (1)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

= 10000 – 200 + 1 = 9801

(iii) $102^2 = (100+2)^2 = (100)^2 + 2\times 100\times 2 + (2)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

= 10000 + 400 + 4 = 10404

(iv) $998^2 = (1000-2)^2 = (1000)^2 - 2\times 1000\times 2 + (2)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

= 1000000 – 4000 + 4 = 996004

(v) $5.2^2 = (5+0.2)^2 = (5)^2 + 2\times 5\times 0.2 + (0.2)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

= 25 + 2.0 + 0.04 = 27.04

(vi) 297 x 303 = (300 – 3) x (300 + 3) = $(300)^2 - (3)^2$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

= 90000 – 9 = 89991

(vii) 78 x 82 = (80 – 2) x (80 + 2) = $(80)^2 - (2)^2$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

= 6400 – 4 = 6396

(viii) $8.9^2 = (8+0.9)^2 = (8)^2 + 2\times 8\times 0.9 + (0.9)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

= 64 + 14.4 + 0.81 = 79.21

(ix) 1.05 x 9.5 = (10 + 0.5) x (10 – 0.5) = $(10)^2 - (0.5)^2$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

Question 7:

Using $a^2 - b^2 = (a+b)(a-b)$, find

(i) $51^2 - 49^2$ (ii) $(1.02)^2 - (0.98)^2$

(iii) $153^2 - 147^2$ (iv) $12.1^2 - 7.9^2$

Answer 7:

(i) $51^2 - 49^2 = (51+49)(51-49)$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

= 100 x 2 = 200

(ii) $(1.02)^2 - (0.98)^2 = (1.02+0.98)(1.02-0.98)$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

= 2.00 x 0.04 = 0.08

(iii) $153^2 - 147^2 = (153+147)(153-147)$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

= 300 x 6 = 1800

(iv) $12.1^2 - 7.9^2 = (12.1+7.9)(12.1-7.9)$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

= 20.0 x 4.2 = 84.0 = 84

Question 8:

Using $(x+a)(x+b) = x^2 + (a+b)x + ab$, find

(i) 103 x 104
(ii) 5.1 x 5.2
(iii) 103 x 98
(iv) 9.7 x 9.8

Answer 8:

(i) 103 x 104 = (100 + 3) x (100 + 4) = $(100)^2 + (3+4)\times 100 + 3\times 4$

[Using identity $(x+a)(x+b) = x^2 + (a+b)x + ab$]

= 10000 + 7 x 100 + 12
= 10000 + 700 + 12 = 10712

(ii) 5.1 x 5.2 = (5 + 0.1) x (5 + 0.2) = $(5)^2 + (0.1+0.2)\times 5 + 0.1\times 0.2$

[Using identity $(x+a)(x+b) = x^2 + (a+b)x + ab$]

= 25 + 0.3 x 5 + 0.02
= 25 + 1.5 + 0.02 = 26.52

(iii) 103 x 98 = (100 + 3) x (100 – 2) = $(100)^2 + (3-2)\times 100 + 3\times(-2)$

[Using identity $(x+a)(x+b) = x^2 + (a+b)x + ab$]

= 10000 + (3 – 2) x 100 – 6
= 10000 + 100 – 6 = 10094

(iv) 9.7 x 9.8 = (10 – 0.3) x (10 – 0.2)

= $(10)^2 + \{(-0.3)+(-0.2)\}\times 10 + (-0.3)\times(-0.2)$

[Using identity $(x+a)(x+b) = x^2 + (a+b)x + ab$]

= $100 + \{-0.3-0.2\}\times 10 + 0.06$

= 100 – 0.5 x 10 + 0.06
= 100 – 5 + 0.06 = 95.06