Exercise 13.1

Question 1:

Following are the car parking charges near a railway station up to:

4 hours	₹60
8 hours	₹100
12 hours	₹140
24 hours	₹180

Check if the parking charges are in direct proportion to the parking time.

Answer 1:

Charges per hour:

$$C_1 = \frac{60}{4} = ₹15$$

$$C_2 = \frac{100}{8} = ₹12.50$$

$$C_3 = \frac{140}{12} = ₹11.67$$

$$C_4 = \frac{180}{24} = ₹7.50$$

Here, the charges per hour are not same, i.e., $C_1 \neq C_2 \neq C_3 \neq C_4$

Therefore, the parking charges are not in direct proportion to the parking time.

Question 2:

A mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of base. In the following table, find the parts of base that need to be added.

Parts of red pigment	**1**	**4**	**7**	**12**	**20**
Parts of base	8	----	----	----	----

Answer 2:

Let the ratio of parts of red pigment and parts of base be $\frac{a}{b}$.

Here $\quad a_1 = 1, b_1 = 8 \quad \Rightarrow \quad \frac{a_1}{b_1} = \frac{1}{8} = k$ (say)

When $\quad a_2 = 4, b_2 = ?$

$$k = \frac{a_2}{b_2} \qquad \Rightarrow \qquad b_2 = \frac{a_2}{k} = \frac{4}{\frac{1}{8}} = 4 \times 8 = 32$$

When $\quad a_3 = 7, b_3 = ?$

$$k = \frac{a_3}{b_3} \qquad \Rightarrow \qquad b_3 = \frac{a_3}{k} = \frac{7}{\frac{1}{8}} = 7 \times 8 = 56$$

When $\quad a_4 = 12, b_4 = ?$

$$k = \frac{a_4}{b_4} \qquad \Rightarrow \qquad b_4 = \frac{a_4}{k} = \frac{12}{\frac{1}{8}} = 12 \times 8 = 96$$

When $\quad a_5 = 20, b_5 = ?$

$$k = \frac{a_5}{b_5} \qquad \Rightarrow \qquad b_5 = \frac{a_5}{k} = \frac{20}{\frac{1}{8}} = 20 \times 8 = 160$$

Parts of red pigment	**1**	**4**	**7**	**12**	**20**
Parts of base	8	32	56	96	160

Question 3:

In Question 2 above, if 1 part of a red pigment requires 75 mL of base, how much red pigment should we mix with 1800 mL of base?

Answer 3:

Let the parts of red pigment mix with 1800 mL base be x.

Parts of red pigment	**1**	x
Parts of base	75	1800

Since it is in direct proportion.

$$\therefore \qquad \frac{1}{75} = \frac{x}{1800}$$

$$\Rightarrow \qquad 75 \times x = 1 \times 1800$$

$$\Rightarrow \qquad x = \frac{1 \times 1800}{75} = 24 \text{ parts}$$

Hence, with base 1800 mL, 24 parts red pigment should be mixed.

Question 4:

A machine in a soft drink factory fills 840 bottles in six hours. How many bottles will it fill in five hours?

Answer 4:

Let the number of bottles filled in five hours be x.

Hours	**1**	x
Bottles	75	1800

Here ratio of hours and bottles are in direct proportion.

$$\therefore \qquad \frac{6}{840} = \frac{5}{x}$$

$$\Rightarrow \qquad 6 \times x = 5 \times 840$$

$$\Rightarrow \qquad x = \frac{5 \times 840}{6} = 700 \text{ bottles}$$

Hence, machine will fill 700 bottles in five hours.

Question 5:

A photograph of a bacteria enlarged 50,000 times attains a length of 5 cm as shown in the diagram. What is the *actual* length of the bacteria? If the photograph is enlarged 20,000 times only, what would be its enlarged length?

Answer 5:

Let enlarged length of bacteria be x.

Actual length of bacteria = $\frac{5}{50000} = \frac{1}{10000}$ cm = 10^{-4} cm

Length	**5**	x
Enlarged length	50,000	20,000

Here length and enlarged length of bacteria are in direct proportion.

$\therefore \qquad \frac{5}{50000} = \frac{x}{20000}$

$\Rightarrow \qquad x \times 50000 = 5 \times 20000$

$\Rightarrow \qquad x = \frac{5 \times 20000}{50000}$ = 2 cm

Hence, the enlarged length of bacteria is 2 cm.

Question 6:

In a model of a ship, the mast is 9 cm high, while the mast of the actual ship is 12 m high. If the length if the ship is 28 m, how long is the model ship?

Answer 6:

Let the length of model ship be x.

Length of actual ship (in m)	**12**	**28**
Length of model ship (in cm)	9	x

Here length of mast and actual length of ship are in direct proportion.

$\therefore \qquad \frac{12}{9} = \frac{28}{x}$

$\Rightarrow \qquad x \times 12 = 28 \times 9$

$\Rightarrow \qquad x = \frac{28 \times 9}{12}$ = 21 cm

Hence, the length of the model ship is 21 cm.

Question 7:

Suppose 2 kg of sugar contains 9 x 10^6 crystals. How many sugar crystals are there in (i) 5 kg of sugar? (ii) 1.2 kg of sugar?

Answer 7:

(i) Let sugar crystals be x.

Weight of sugar (in kg)	**2**	**5**
No. of crystals	9×10^6	x

Here weight of sugar and number of crystals are in direct proportion.

$$\therefore \quad \frac{2}{9\times10^6}=\frac{5}{x}$$

$$\Rightarrow \quad x\times2=5\times9\times10^6$$

$$\Rightarrow \quad x=\frac{5\times9\times10^6}{2}$$

$$= 22.5\times10^6=2.25\times10^7$$

Hence, the number of sugar crystals is 2.25×10^7.

(ii) Let sugar crystals be x.

Weight of sugar (in kg)	**2**	**1.2**
No. of crystals	9×10^6	x

Here weight of sugar and number of crystals are in direct proportion.

$$\therefore \quad \frac{2}{9\times10^6}=\frac{1.2}{x}$$

$$\Rightarrow \quad x\times2=1.2\times9\times10^6$$

$$\Rightarrow \quad x=\frac{1.2\times9\times10^6}{2}$$

$$= 0.6\times9\times10^6=5.4\times10^6$$

Hence, the number of sugar crystals is 5.4×10^6.

Question 8:

Rashmi has a road map with a scale of 1 cm representing 18 km. She drives on a road for 72 km. What would be her distance covered in the map?

Answer 8:

Let distance covered in the map be x.

Actual distance (in km)	**18**	**72**
Distance covered in map (in cm)	1	x

Here actual distance and distance covered in the map are in direct proportion.

$$\therefore \quad \frac{18}{1} = \frac{72}{x}$$

$$\Rightarrow \quad x \times 18 = 72 \times 1$$

$$\Rightarrow \quad x = \frac{72 \times 1}{18} = 4 \text{ cm}$$

Hence, the distance covered in the map is 4 cm.

Question 9:

A 5 m 60 cm high vertical pole casts a shadow 3 m 20 cm long. Find at the same time (i) the length of the shadow cast by another pole 10 m 50 cm high (ii) the height of a pole which casts a shadow 5 m long.

Answer 9:

Here height of the pole and length of the shadow are in direct proportion.

And 1 m = 100 cm

5 m 60 cm = 5 x 100 + 60 = 560 cm

3 m 20 cm = 3 x 100 + 20 = 320 cm

10 m 50 cm = 10 x 100 + 50 = 1050 cm

5 m = 5 x 100 = 500 cm

(i). Let the length of the shadow of another pole be x.

Height of pole (in cm)	**560**	**1050**
Length of shadow (in cm)	320	x

$$\therefore \quad \frac{560}{320} = \frac{1050}{x}$$

$$\Rightarrow \quad x \times 560 = 1050 \times 320$$

$$\Rightarrow \quad x = \frac{1050 \times 320}{560}$$

= 600 cm = 6 m

Hence, the length of the shadow of another pole is 6 m.

(ii). Let the height of the pole be x.

Height of pole (in cm)	**560**	x
Length of shadow (in cm)	320	500

$$\therefore \quad \frac{560}{320} = \frac{x}{500}$$

$$\Rightarrow \quad x \times 320 = 560 \times 500$$

$$\Rightarrow \quad x = \frac{560 \times 500}{320}$$

= 875 cm = 8 m 75 cm

Hence, the height of the pole is 8 m 75 cm.

Question 10:

A loaded truck travels 14 km in 25 minutes. If the speed remains the same, how far can it travel in 5 hours?

Answer 10:

Let distance covered in 5 hours be x km.

$\because$ 1 hour = 60 minutes

$\therefore$ 5 hours = 5 x 60 = 300 minutes

Distance (in km)	**14**	x
Time (in minutes)	25	300

Here distance covered and time in direct proportion.

$$\therefore \quad \frac{14}{25} = \frac{x}{300}$$

$$\Rightarrow \qquad x \times 25 = 14 \times 300$$

$$\Rightarrow \qquad x = \frac{14 \times 300}{25} = 168 \text{ km}$$

Hence, the distance covered in 5 hours is 168 km.

Exercise 13.2

Question 1:

Which of the following are in inverse proportion:

(i) The number of workers on a job and the time to complete the job.

(ii) The time taken for a journey and the distance travelled in a uniform speed.

(iii) Area of cultivated land and the crop harvested.

(iv) The time taken for a fixed journey and the speed of the vehicle.

(v) The population of a country and the area of land per person.

Answer 1:

(i) The number of workers and the time to complete the job is in inverse proportion because less workers will take more time to complete a work and more workers will take less time to complete the same work.

(ii) Time and distance covered in direct proportion.

(iii) It is a direct proportion because more are of cultivated land will yield more crops.

(iv) Time and speed are inverse proportion because if time is less, speed is more.

(v) It is a inverse proportion. If the population of a country increases, the area of land per person decreases.

Question 2:

In a Television game show, the prize money of ₹1,00,000 is to be divided equally amongst the winners. Complete the following table and find whether the prize money given to an individual winner is directly or inversely proportional to the number of winners:

No. of winners	**1**	**2**	**4**	**5**	**8**	**10**	**20**
Prize for each winner (in ₹)	1,00,000	50,000	----	----	----	----	----

Answer 2:

Here number of winners and prize money are in inverse proportion because winners are increasing, prize money is decreasing.

When the number of winners are 4, each winner will get = $\frac{100000}{4}$ = ₹25,000

When the number of winners are 5, each winner will get = $\frac{100000}{5}$ = ₹20,000

When the number of winners are 8, each winner will get = $\frac{100000}{8}$ = ₹12,500

When the number of winners are 10, each winner will get = $\frac{100000}{10}$ = ₹10,000

When the number of winners are 20, each winner will get = $\frac{100000}{20}$ = ₹5,000

Question 3:

Rehman is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spokes are equal. Help him by completing the following table:

No. of spokes	**4**	**6**	**8**	**10**	**12**
Angle between a pair of consecutive spokes	90°	60°	----	----	----

(i) Are the number of spokes and the angles formed between the pairs of consecutive spokes in inverse proportion?

(ii) Calculate the angle between a pair of consecutive spokes on a wheel with 15 spokes.

(iii) How many spokes would be needed, if the angle between a pair of consecutive spokes is 40° ?

Answer 3:

Here the number of spokes are increasing and the angle between a pair of consecutive spokes is decreasing. So, it is a inverse proportion and angle at the centre of a circle is 360°.

When the number of spokes is 8,

then angle between a pair of consecutive spokes = $\frac{360^\circ}{8} = 45^\circ$

When the number of spokes is 10,

then angle between a pair of consecutive spokes= $\frac{360^\circ}{10} = 36^\circ$

When the number of spokes is 12,

then angle between a pair of consecutive spokes= $\frac{360^\circ}{12} = 30^\circ$

No. of spokes	**4**	**6**	**8**	**10**	**12**
Angle between a pair of consecutive spokes	90°	60°	45°	36°	30°

(i) Yes, the number of spokes and the angles formed between a pair of consecutive spokes is in inverse proportion.

(ii) When the number of spokes is 15, then angle between a pair of consecutive spokes = $\frac{360^\circ}{15} = 24^\circ$.

(iii) The number of spokes would be needed = $\frac{360^\circ}{40^\circ} = 9$

Question 4:

If a box of sweets is divided among 24 children, they will get 5 sweets each. How many would each get, if the number of the children is reduced by 4?

Answer 4:

$\because$ Each child gets = 5 sweets

$\therefore$ 24 children will get 24 x 5 = 120 sweets

Total number of sweets = 120

If the number of children is reduced by 4, then children left = 24 – 4 = 20

Now each child will get sweets = $\frac{120}{20}$ = 6 sweets

Question 5:

A farmer has enough food to feed 20 animals in his cattle for 6 days. How long would the food last if there were 10 more animals in his cattle?

Answer 5:

Let the number of days be x.

Total number of animals = 20 + 10 = 30

Animals	**20**	**30**
Days	6	x

Here, the number of animals and the number of days are in inverse proportion.

$\therefore \quad \frac{20}{30} = \frac{x}{6}$

$\Rightarrow \quad 30 \times x = 20 \times 6$

$\Rightarrow \quad x = \frac{20 \times 6}{30}$ = 4

Hence, the food will last for four days.

Question 6:

A contractor estimates that 3 persons could rewire Jasminder's house in 4 days. If, he uses 4 persons instead of three, how long should they take to complete the job?

Answer 6:

Let time taken to complete the job be x.

Persons	**3**	**4**
Days	4	x

Here the number of persons and the number of days are in inverse proportion.

$\therefore \quad \frac{3}{4} = \frac{x}{4}$

$\Rightarrow \quad 4 \times x = 3 \times 4$

$\Rightarrow \qquad x = \frac{3 \times 4}{4}$ = 3 days

Hence, they will complete the job in 3 days.

Question 7:

A batch of bottles was packed in 25 boxes with 12 bottles in each box. If the same batch is packed using 20 bottles in each box, how many boxes would be filled?

Answer 7:

Let the number of boxes be x.

No. of bottles in each box	**12**	**20**
Boxes	25	x

Here the number of bottles and the number of boxes are in inverse proportion.

$\therefore \qquad \frac{12}{20} = \frac{x}{25}$

$\Rightarrow \qquad x \times 20 = 12 \times 25$

$\Rightarrow \qquad x = \frac{12 \times 25}{20}$ = 15

Hence, 15 boxes would be filled.

Question 8:

A factory requires 42 machines to produce a given number of articles in 63 days. How many machines would be required to produce the same number of articles in 54 days?

Answer 8:

Let the number of machines required be x.

Days	63	54
Machines	42	x

Here, the number of machines and the number of days are in inverse proportion.

$$\therefore \quad \frac{63}{54} = \frac{x}{42}$$

$$\Rightarrow \quad x \times 54 = 63 \times 42$$

$$\Rightarrow \quad x = \frac{63 \times 42}{54} = 49$$

Hence, 49 machines would be required.

Question 9:

A car takes 2 hours to reach a destination by travelling at the speed of 60 km/hr. How long will it take when the car travels at the speed of 80 km/hr?

Answer 9:

Let the number of hours be x.

Speed (in km/hr)	60	80
Time (in hours)	2	x

Here, the speed of car and time are in inverse proportion.

$$\therefore \quad \frac{60}{80} = \frac{x}{2}$$

$$\Rightarrow \quad x \times 80 = 60 \times 2$$

$$\Rightarrow \quad x = \frac{60 \times 2}{80} = \frac{3}{2} = 1\frac{1}{2} \text{ hrs.}$$

Hence, the car will take $1\frac{1}{2}$ hours to reach its destination.

Question 10:

Two persons could fit new windows in a house in 3 days.

(i) One of the persons fell ill before the work started. How long would the job take now?

(ii) How many persons would be needed to fit the windows in one day?

Answer 10:

(i) Let the number of days be x.

Persons	**2**	**1**
Days	3	x

Here, the number of persons and the number of days are in inverse proportion.

$$\therefore \quad \frac{2}{1}=\frac{x}{3}$$

$$\Rightarrow \quad x\times 1=2\times 3$$

$$\Rightarrow \quad x=\frac{2\times 3}{1} = 6 \text{ days}$$

(ii) Let the number of persons be x.

Persons	**2**	x
Days	3	1

Here, the number of persons and the number of days are in inverse proportion.

$$\therefore \quad \frac{2}{x}=\frac{1}{3}$$

$$\Rightarrow \quad x\times 1=2\times 3$$

$$\Rightarrow \quad x=\frac{2\times 3}{1} = 6 \text{ persons}$$

Question 11:

A school has 8 periods a day each of 45 minutes duration. How long would each period be, if the school has 9 periods a day, assuming the number of school hours to be the same?

Answer 11:

Let the duration of each period be x.

Period	**8**	**9**
Duration of period (in minutes)	45	x

Here the number of periods and the duration of periods are in inverse proportion.

$\therefore \quad \frac{8}{9} = \frac{x}{45}$

$\Rightarrow \quad x \times 9 = 8 \times 45$

$\Rightarrow \quad x = \frac{8 \times 45}{9}$ = 40 minutes

Hence, the duration of each period would be 40 minutes.