Unit 5(Understanding Quadrilaterals & Practical Geometry)

Multiple Choice Questions

Question. 1 If three angles of a quadrilateral are each equal to 75°, then, the fourth angle is(a) 150° (b) 135°

(c) 45° (d) 75°

Solution.

(b) Given, three angles of quadrilateral = 75°

Let the fourth angle be $x°$.

Then, according to the property, $75° + 75° + 75° + x° = 360°$, since sum of the angles of a quadrilateral is 360°.

So, $225° + x° = 360°$ or $x° = 360° - 225° = 135°$

Hence, the fourth angle is 135°.

Question. 2 For which of the following, diagonals bisect each other?

(a) Square (b) Kite

(c) Trapezium (d) Quadrilateral

Solution. (a) We know that, the diagonals of a square bisect each other but the diagonals of kite, trapezium and quadrilateral do not bisect each other.

Question. 3 In which of the following figures, all angles are equal?

(a) Rectangle (b) Kite

(c) Trapezium (d) Rhombus

Solution. (a) In a rectangle, all angles are equal, i.e. all equal to 90°.

Question. 4 For which of the following figures, diagonals are perpendicular to each other?

(a) Parallelogram (b) Kite

(c) Trapezium (d) Rectangle

Solution. (b) The diagonals of a kite are perpendicular to each other.

Question. 5 For which of the following figures, diagonals are equal?

(a) Trapezium (b) Rhombus

(c) Parallelogram (d) Rectangle

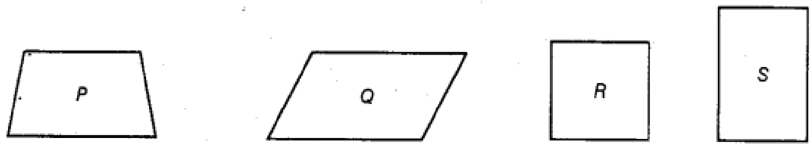
Solution. (d) By the property of a rectangle, we know that its diagonals are equal.

Question. 6 Which of the following figures satisfy the following properties?

All sides are congruent

All angles are right angles.

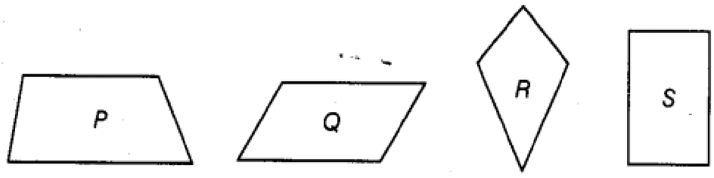
Opposite sides are parallel.



(a) *P* (b) Q (c) *R* (d) *S*

Solution. (c) We know that all the properties mentioned above are related to square and we can observe that figure R resembles a square.

Question. 7 Which of the following figures satisfy the following property?Has two pairs of congruent adjacent sides.

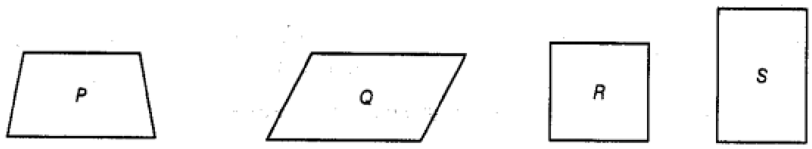


(a) *P* (b) Q (c) *R* (d) *S*

Solution. (c) We know that, a kite has two pairs of congruent adjacent sides and we can observe that figure R resembles a kite.

Question. 8 Which of the following figures satisfy the following property?

Only one pair of sides are parallel.



(a) *P* (b) Q (c) *R* (d) *S*

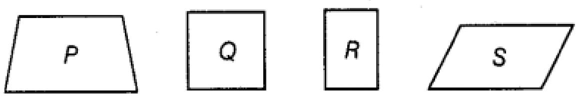
Solution. (a) We know that, in a trapezium, only one pair of sides are parallel and we can observe that figure P resembles a trapezium.

Question. 9 Which of the following figures do not satisfy any of the following properties?

All sides are equal.

All angles are right angles.

Opposite sides are parallel.



(a) *P* (b) Q (c) *R* (d) *S*

Solution. (a) On observing the above figures, we conclude that the figure P does not satisfy any of the given properties.

Question. 10 Which of the following properties describe a trapezium?

(a) A pair of opposite sides is parallel

(b) The diagonals bisect each other

(c) The diagonals are perpendicular to each other

(d) The diagonals are equal

Solution. (a) We know that, in a trapezium, a pair of opposite sides are parallel.

Question. 11 Which of the following is a propefay of a parallelogram?
(a) Opposite sides are parallel
(b) The diagonals bisect each other at right angles
(c) The diagonals are perpendicular to each other
(d) All angles are equal
Solution. (a) We,know that, in a parallelogram, opposite sides are parallel.

Question. 12 What is the maximum number of obtuse angles that a quadrilateral can have?
(a) 1 (b) 2
(c) 3 (d) 4
Solution. (c) We know that, the sum of all the angles of a quadrilateral is 360°.
Also, an obtuse angle is more than 90° and less than 180°.
Thus, all the angles of a quadrilateral cannot be obtuse.
Hence, almost 3 angles can be obtuse.

Question. 13 How many non-overlapping triangles can we make in a-n-gon (polygon having n sides), by joining the vertices?
(a)n-1 (b)n-2
(c) n – 3 (d) n – 4
Solution. (b) The number of non-overlapping triangles in a n-gon = n – 2, i.e. 2 less than the number of sides.

Question. 14 What is the sum of all the angles of a pentagon?
(a) 180° (b) 360° (c) 540° (d) 720°
Solution. (c) We know that, the sum of angles of a polygon is (n – 2) x 180°, where n is the number of sides of the polygon.
In pentagon, n = 5
Sum of the angles = (n – 2) x 180° = (5 – 2) x 180°
= 3 x 180°= 540°

Question. 15 What is the sum of all angles of a hexagon?
(a) 180° (b) 360° (c) 540° (d) 720°
Solution. (d) Sum of all angles of a n-gon is (n – 2) x 180°.
In hexagon, n = 6, therefore the required sum = (6 – 2) x 180° = 4 x 180° = 720°

Question. 16 If two adjacent angles of a parallelogram are (5x – 5) and (10x + 35), then the ratio of these angles is
(a) 1 : 3 (b) 2 : 3 (c) 1 : 4 (d) 1 : 2
Solution.

(a) We know that, adjacent angles of a parallelogram are supplementary, i.e. their sum equals 180°.

$\therefore (5x - 5) + (10x + 35) = 180^\circ$
$\Rightarrow \quad 15x + 30^\circ = 180^\circ$
$\Rightarrow \quad 15x = 150^\circ$
$\Rightarrow \quad x = 10^\circ$

Thus, the angles are $(5 \times 10 - 5)$ and $(10 \times 10 + 35)$, i.e. 45° and 135°.
Hence, the required ratio is 45° : 135°, i.e. 1 : 3.

Question. 17 A quadrilateral whose all sides are equal, opposite angles are equal and the diagonals bisect each other at-right angles is a .
(a) rhombus (b) parallelogram (c) square (d) rectangle
Solution. (a) We know that, in rhombus, all sides are equal, opposite angles are equal and

diagonals bisect each other at right angles.

Question. 18 A quadrilateral whose opposite sides and all the angles are equal is a
(a) rectangle (b) parallelogram (c) square (d) rhombus
Solution. (a) We know that, in a rectangle, opposite sides and all the angles are equal.

Question. 19 A quadrilateral whose all sides, diagonals and angles are equal is a
(a) square (b) trapezium (c) rectangle (d) rhombus
Solution. (a) These are the properties of a square, i.e. in a square, all sides, diagonals and angles are equal.

Question. 20 How many diagonals does a hexagon have?
(a) 9 (b) 8 (c) 2 (d) 6
Solution.

(a) We know that, the number of diagonals in a polygon of n sides is $\frac{n(n-3)}{2}$.

In hexagon, $n = 6$

$\therefore$ Number of diagonals in a hexagon $= \frac{6(6-3)}{2} = \frac{6 \times 3}{2} = 3 \times 3 = 9$

Question. 21 If the adjacent sides of a parallelogram are equal, then parallelogram is a
(a) rectangle (b) trapezium (c) rhombus (d) square
Solution. (c)We know that, in a parallelogram, opposite sides are equal.
But according to the question, adjacent sides are also equal.
Thus, the parallelogram in which all the sides are equal is known as rhombus.

Question. 22 If the diagonals of a quadrilateral are equal and bisect each other, then the quadrilateral is a
(a) rhombus (b) rectangle (c) square (d) parallelogram
Solution. (b) Since, diagonals are equal and bisect each other, therefore it will be a rectangle.

Question. 23 The sum of all exterior angles of a triangle is
(a) 180° (b) 360° (c) 540° (d) 720°
Solution. (b) We know that the sum of exterior angles, taken in order of any polygon is 360° and triangle is also a polygon.
Hence, the sum of all exterior angles of a triangle is 360°.

Question. 24 Which of the following is an equiangular and equilateral polygon?
(a) Square (b) Rectangle (c) Rhombus (d) Right triangle
Solution. (a) In a square, all the sides and all the angles are equal.
Hence, square is an equiangular and equilateral polygon.

Question. 25 Which one has all the properties of a kite and a parallelogram?
(a) Trapezium (b) Rhombus (c) Rectangle (d) Parallelogram
Solution. (b) In a kite
Two pairs of equal sides.
Diagonals bisect at 90°.
One pair of opposite angles are equal.
In a parallelogram Opposite sides are equal.
Opposite angles are equal.
Diagonals bisect each other.
So, from the given options, all these properties are satisfied by rhombus.

Question. 26 The angles of a quadrilateral are in the ratio 1 : 2 : 3 : 4. The smallest angle is

(a) 72° (b) 144° (c) 36° (d) 18°
Solution.
(c) Let the angles of the given quadrilateral be $x°$, $2x°$, $3x°$ and $4x°$.

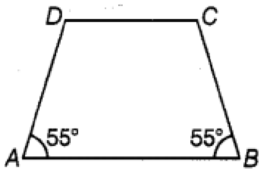
$\therefore x° + 2x° + 3x° + 4x° = 360°$ [$\because$ sum of the angles of a quadrilateral is 360°]

$\Rightarrow 10x = 360°$

$\Rightarrow x = \frac{360°}{10} = 36°$

Hence, the smallest angle = 36°

Question. 27 In the trapezium ABCD, the measure of $\angle D$ is
(a) 55° (b) 115° (c)135° (d) 125°



Solution.
(d) We know that, in a trapezium, the angles on either sides of base are supplementary angle.

In trapezium *ABCD*,

$\therefore \angle A + \angle D = 180°$

$\Rightarrow 55° + \angle D = 180°$

$\Rightarrow \angle D = 180° - 55°$

$\Rightarrow \angle D = 125°$

Question. 28 A quadrilateral has three acute angles. If each measures 80°, then the measure of the fourth angle is
(a) 150° (b) 120° (c) 105° (d) 140°
Solution.
(b) Let the fourth angle be x, then $80° + 80° + 80° + x = 360°$

[$\because$ sum of all the angles of quadrilateral is 360°]

$\Rightarrow 240° + x = 360°$

$\Rightarrow x = 360° - 240°$

$\Rightarrow x = 120°$

Question. 29 The number of sides of a regular polygon where each exterior angle has a measure of 45° is
(a) 8 (b) 10 (c) 4 (d) 6
Solution.
(a) We know that, the sum of exterior angles taken in an order of a polygon is 360°.

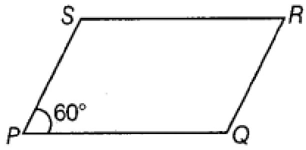
Since, each exterior angle measures 45°, therefore the number of sides

$= \frac{\text{Sum of exterior angles}}{\text{Measure of an exterior angle}}$

$= \frac{360°}{45°} = 8$

Question. 30 In a parallelogram PQRS, if $\angle P$ = 60°, then other three angles are
(a) 45°, 135°, 120° (b) 60°, 120°, 120°
(c) 60°, 135°, 135° (d) 45°, 135°, 135°
Solution.

(b) Given, $\angle P = 60°$



Since, in a parallelogram, adjacent angles are supplementary,

$\angle P + \angle Q = 180° \Rightarrow 60° + \angle Q = 180° \quad \Rightarrow \quad \angle Q = 120°$

Also, opposite angles are equal in a parallelogram.

Therefore, $\angle R = \angle P = 60°$, $\angle S = \angle Q = 120°$

Hence, other three angles are 60°, 120°, 120°.

Question. 31 If two adjacent angles of a parallelogram are in the ratio 2 : 3, then the measure of angles are

(a) 72°, 108° (b) 36°, 54° (c) 80°, 120° (d) 96°, 144°

Solution. (a) Let the angles be 2x and 3x.

Then, 2x + 3x = 180° [adjacent angles of a parallelogram are supplementary]

=> 5x = 180°

=> x = 36°

Hence, the measures of angles are 2x = 2 x 36°= 72° and 3x = 3×36°= 108°

Question. 32 IfPQRS is a parallelogram then $\angle P - \angle R$ is equal to

(a) 60° (b) 90° (c) 80° (d) 0°

Solution. (d) Since, in a parallelogram, opposite angles are equal. Therefore, $\angle P - \angle R$ = 0, as $\angle P$ and $\angle R$ are opposite angles.

Question. 33 The sum of adjacent angles of a parallelogram is

(a) 180° (b) 120° (c) 360° (d) 90°

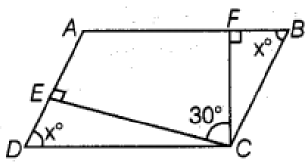
Solution. (a) By property of the parallelogram, we know that, the sum of adjacent angles of a parallelogram is 180°.

Question. 34 The angle between the two altitudes of a parallelogram through the same vertex of an obtuse angle of the parallelogram is 30°. The measure of the obtuse angle is

(a) 100° (b) 150° (c) 105° (d) 120°

Solution.

(b) Let EC and FC be altitudes and $\angle ECF = 30°$.



Let $\angle EDC = x = \angle FBC$

So, $\angle ECD = 90 - x°$ and $\angle BCF = 90° - x$

So, by property of the parallelogram,

$$\angle ADC + \angle DCB = 180°$$

$$\angle ADC + (\angle ECD + \angle ECF + \angle BCF) = 180°$$

$\Rightarrow \quad x + 90° - x + 30° + 90° - x = 180°$

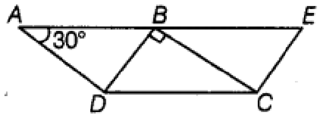
$\Rightarrow \quad -x = 180° - 210° = -30°$

$\Rightarrow \quad x = 30°$

Hence, $\quad \angle DCB = 30° + 60° + 60° = 150°$

Question. 35 In the given figure, ABCD and BDCE are parallelograms with common base DC. If $BC \perp BD$, then $\angle BEC$ is equal to

(a) 60° (b) 30° (c) 150° (d) 120°



Solution.

(a) $\angle BAD = 30°$ [given]

$\therefore$ $\angle BCD = 30°$ [$\because$ opposite angles of a parallelogram are equal]

In ΔCBD, by angle sum property of a triangle, we have

$\angle DBC + \angle BCD + \angle CDB = 180°$

$\Rightarrow$ $90° + 30° + \angle CDB = 180°$

$\Rightarrow$ $\angle CDB = 180° - 120° = 60°$

$\therefore$ $\angle BEC = 60°$ [$\because$ opposite angles of a parallelogram are equal]

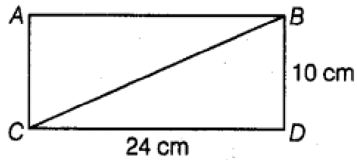
Question. 36 Length of one of the diagonals of a rectangle whose sides are 10 cm and 24 cm is

(a) 25 cm (b) 20 cm (c) 26 cm (d) 3.5 cm

Solution.

(c) In ΔBCD,

$\angle BDC = 90°$



$\therefore$ Using Pythagoras theorem,

we have, $BC^2 = BD^2 + CD^2$

$\Rightarrow$ $BC^2 = 10^2 + 24^2 = 100 + 576$

$\Rightarrow$ $BC^2 = 676$

$\Rightarrow$ $BC = \sqrt{676}$

$\Rightarrow$ $BC = 26$ cm

Question. 37 If the adjacent angles of a parallelogram are equal, then the parallelogram is a (a) rectangle (b) trapezium (c) rhombus (d) None of these

Solution. (a) We know that, the adjacent angles of a parallelogram are supplementary, i.e. their sum equals 180° and given that both the angles are same. Therefore, each angle will be of measure 90°. .

Hence, the parallelogram is a rectangle.

Question. 38 Which of the following can be four interior angles of a quadrilateral?

(a) 140°, 40°, 20°, 160° (b) 270°, 150°, 30°, 20°

(c) 40°, 70°, 90°, 60° (d) 110°, 40°, 30°, 180°

Solution. (a) We know that, the sum of interior angles of a quadrilateral is 360°.

Thus, the angles in option (a) can be four interior angles of a quadrilateral as their sum is 360°.

Question. 39 The sum of angles of a concave quadrilateral is

(a) more than 360° (b) less than 360°

(c) equal to 360° (d) twice of 360°

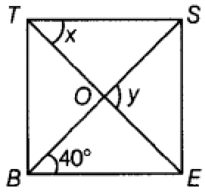
Solution. (c) We know that, the sum of interior angles of any polygon (convex or concave) having n sides is(n -2) x 180°.

.-.The sum of angles of a concave quadrilateral is (4 – 2) x 180°, i.e. 360°

Question. 40 Which of the following can never be the measure of exterior angle of a regular polygon? (a) 22° (b) 36° (c)45° (d) 30°

Solution. (a) Since, we know that, the sum of measures of exterior angles of a polygon is 360°, i.e. measure of each exterior angle =360°/n ,where n is the number of sides/angles.
Thus, measure of each exterior angle will always divide 360° completely.
Hence, 22° can never be the measure of exterior angle of a regular polygon.

Question. 41 In the figure, BEST is a rhombus, then the value of y – x is
(a) 40° (b) 50° (c) 20° (d) 10°



Solution.

(a) Given, a rhombus *BEST*.

$\because$ $TS \parallel BE$ and BS is transversal.

$\therefore\ \angle SBE = \angle TSB = 40°$ [alternate interior angles]

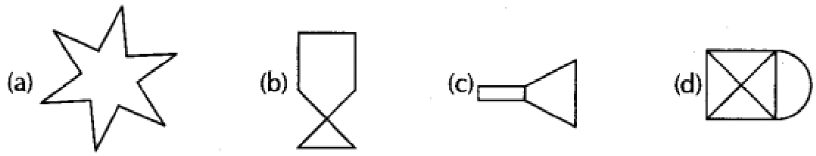
Also, $\angle y = 90°$ [diagonals bisect at 90°]

In ΔTSO, $\angle STO + \angle TSO = \angle SOE$

$x + 40° = 90° \Rightarrow x = 50°$ [exterior angle property of triangle]

$\therefore\ y - x = 90° - 50° = 40°$

Question. 42 The closed curve which is also a polygon, is



Solution. (a) Figure (a) is polygon as no two line segments intersect each other.

Question. 43 Which of the following is not true for an exterior angle of a regular polygon with n sides?

(a) Each exterior angle $= \dfrac{360°}{n}$ (b) Exterior angle $= 180° -$ Interior angle

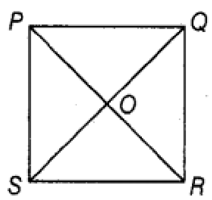
(c) $n = \dfrac{360°}{\text{Exterior angle}}$ (d) Each exterior angle $= \dfrac{(n-2) \times 180°}{n}$

Solution. (d) We know that, (a) and (b) are the formulae to find the measure of each exterior angle, when number of sides and measure of an interior angle respectively are given and (c) is the formula to find number of sides of polygon when exterior angle is given.
Hence, the formula given in option (d) is not true for an exterior angle of a regular polygon with n sides.

Question. 44 PQRS is a square. PR and SQ intersect at 0. Then, $\angle POQ$ is a (a) right angle (b) straight angle (c) reflex angle (d) complete angle

Solution.

(a)

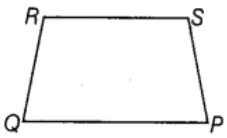


We know that, the diagonals of a square intersect each other at right angle.
Hence, $\angle POQ = 90°$, i.e. right angle.

Question. 45 Two adjacent angles of a parallelogram are in the ratio 1 : 5. Then, all the angles of the parallelogram are
(a) 30°, 150°, 30°, 150° (b) 85°, 95°, 85°, 95° .
(c) 45°, 135°, 45°, 135° (d) 30°, 180°, 30°, 180°
Solution. (a) Let the adjacent angles of a parallelogram be x and 5x, respectively.
Then, x + 5x = 180° [adjacent angles of a parallelogram are supplementary] => 6x = 180°
=> x = 30°
The adjacent angles are 30° and 150°.
Hence, the angles are 30°, 150°, 30°, 150°

Question. 46 A parallelogram PQRS is constructed with sides QR = 6 cm, PQ = 4 cm and $\angle PQR$ = 90°. Then, PQRS is a
(a) square (b) rectangle (c) rhombus (d) trapezium
Solution. (b) We know that, if in a parallelogram one angle is of 90°, then all angles will be of 90° and a parallelogram with all angles equal to 90° is called a rectangle.

Question. 47 The angles P, Q, R and 5 of a quadrilateral are in the ratio 1:3 :7:9. Then, PQRS is a
(a) parallelogram (b) trapezium with PQ \\ RS
(c) trapezium with QR \\PS (d) kite
Solution.
(b)



Let the angles be x, $3x$, $7x$ and $9x$, then

$x + 3x + 7x + 9x = 360°$ [$\because$ sum of angles in any quadrilateral is 360°]

$\Rightarrow \quad 20x = 360°$

$\Rightarrow \quad x = \dfrac{360°}{20}$

$\Rightarrow \quad x = 18°$

Then, the angles P, Q, R and S are 18°, 54°, 126° and 162°, respectively.
Since, $\angle P + \angle S = 18° + 162° = 18C^{\circ}$ and $\angle Q + \angle R = 54° + 126° = 180°$
$\therefore$ The quadrilateral $PQRS$ is a trapezium with $PQ \parallel RS$.

Question. 48 PQRS is a trapezium in which PQ || SR and ZP = 130°, $\angle Q$ = 110°. Then, $\angle R$ is equal to.
(a) 70° (b) 50° (c)65° (d) 55°
Solution.
(a) Since, $PQRS$ is a trapezium and $PQ \parallel SR$.

$\therefore \angle Q + \angle R = 180°$ [$\because$ angles between the pair of parallel sides are supplementary]

$\Rightarrow \quad \angle R = 180° - 110° = 70°$

Question. 49 The number of sides of a regular polygon whose each interior angle is of 135° is (a) 6 (b) 7 (c) 8 (d) 9
Solution.
(c) We know that, the measures of each exterior angle of a polygon having n sides is given by $\dfrac{360°}{n}$.

$\therefore$ The number of sides, $n = \dfrac{360°}{\text{Exterior angle}} = \dfrac{360°}{180° - 135°}$

[$\because$ exterior angle + interior angle = 180°]

$= \dfrac{360°}{45°} = 8$

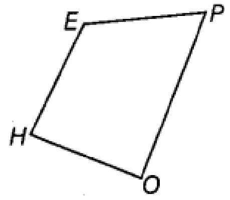
Question. 50 If a diagonal of a quadrilateral bisects both the angles, then it is a
(a) kite (b) parallelogram (c) rhombus (d) rectangle
Solution. (c) If a diagonal of a quadrilateral bisects both the angles, then it is a rhombus.

Question. 51 To construct a unique parallelogram, the minimum number of measurements required is (a) 2 (b) 3 (c) 4 (d) 5
Solution. (b) We know that, in a parallelogram, opposite sides are equal and parallel. Also, opposite angles are equal.
So, to construct a parallelogram uniquely, we require the measure of any two non-parallel sides and the measure of an angle.
Hence, the minimum number of measurements required to draw a unique parallelogram is 3.

Question. 52 To construct a unique rectangle, the minimum number of measurements required is (a) 4 (b) 3 (0 2 (d) 1
Solution. (c) Since, in a rectangle, opposite sides are equal and parallel, so we need the measurement of only two adjacent sides, i.e. length and breadth. Also, each angle measures 90°.
Hence, we require only two measurements to construct a unique rectangle.

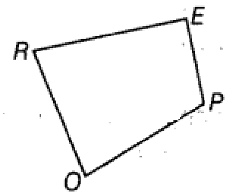
Fill in the Blanks

In questions 53 to 91, fill in the blanks to make the statements true.
Question. 53 In quadrilateral HOPE, the pairs of opposite sides are—————.
Solution.



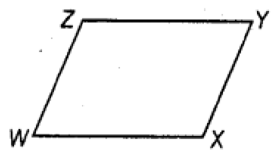
***EH*, *PO* and *HO*, *EP* are pairs of opposite sides.**

Question. 54 In quadrilateral ROPE, the pairs of adjacent angles are—————-.
Solution .



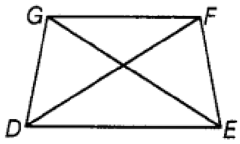
The pairs of adjacent angles are $\angle R$, $\angle O$; $\angle O$, $\angle P$; $\angle P$, $\angle E$; $\angle E$, $\angle R$

Question. 55 In quadrilateral WXYZ, the pairs of opposite angles are—————.
Solution.



The pairs of opposite angles are $\angle W$, $\angle Y$; $\angle X$, $\angle Z$.

Question . 56 The diagonals of the quadrilateral DEFG are————and—————.
Solution.



The diagonals are **GE** and **FD**.

Question. 57 The sum of all——— of a quadrilateral is 360°.
Solution. angles
We know that, the sum of all angles of a quadrilateral is 360°.

Question. 58 The measure of each exterior angle of a regular pentagon is————— .
Solution.
72°
Measure of exterior angle $= \dfrac{360°}{\text{Number of sides}} = \dfrac{360°}{5} = 72°$
[$\because$ in pentagon, number of sides, $n = 5$]

Question. 59 Sum of the angles of a hexagon is———————-.
Solution.
720°
Since, the sum of angles of an n-gon $= (n - 2) \times 180°$
$\therefore$ Sum of the angles of a hexagon $= (6 - 2) \times 180° = 4 \times 180° = 720°$
[$\because$ in hexagon, number of sides, $n = 6$]

Question. 60 The measure of each exterior angle of a regular polygon of 18 sides is———.
Solution.
20°
We know that, measure of each exterior angle $= \dfrac{360°}{\text{Number of sides}} = \dfrac{360°}{18} = 20°$

Question. 61 The number of sides of a regular polygon, where each exterior angle has a measure of 36°, is—————-.
Solution.
10°
We know that, the sum of exterior angles of a regular polygon is 360°.
Further, since each exterior angle is of 36°, therefore number of sides $= \dfrac{360°}{\text{Exterior angle}}$
$= \dfrac{360°}{36°} = 10°$

Question. 62

is a closed curve entirely made up of line segments. The another name for this shape is ______ .
Solution. concave polygon
As one interior angle is of greater than 180°.

Question. 63 A quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure is——————.
Solution. kite
By the property of a kite, we know that, it has two opposite angles of equal measure.

Question. 64 The measure of each angle of a regular pentagon is—————.
Solution.

We know that, the sum of interior angles of a polygon = $(n-2) \times 180°$
$= (5-2) \times 180° = 540°$
Since, it is a regular pentagon. [$\because$ in pentagon, number of sides, $n = 5$]

$\therefore$ Measure of each interior angle $= \dfrac{\text{Sum of interior angles}}{\text{Number of sides}} = \dfrac{540°}{5} = \mathbf{108°}$

Question. 65 The name of three-sided regular polygon is—————-.
Solution. equilateral triangle, as polygon is regular, i.e. length of each side is same.

Question. 66 The number of diagonals in a hexagon is—————-.
Solution.

9, We know that, number of diagonals of a n-gon $= \dfrac{n(n-3)}{2}$

Here, $n = 6$, therefore the number of diagonals $= \dfrac{6(6-3)}{2} = \dfrac{6 \times 3}{2} = \mathbf{9}$

Question. 67 A polygon is a simple closed curve made up of only————.
Solution. line segments ,
Since a simple closed curve made up of only line segments is called a polygon.

Question. 68 A regular polygon is a polygon whose all sides are equal and all———are equal.
Solution. angles
In a regular polygon, all sides are equal and all angles are equal.

Question. 69 The sum of interior angles of a polygon of n sides is———- right angles.
Solution.

$\mathbf{(2n-4)}$
By the formula, sum of interior angles of a polygon of n sides $= (n-2) \times 180°$
$= (2n-4) \times 90°$

Question. 70 The sum of all exterior angles of a polygon is————.
Solution. 360°
As the sum of all exterior angles of a polygon is 360°.

Question. 71 ————-is a regular quadrilateral.
Solution. Square
Since in square, all the sides are of equal length and all angles are equal.

Question. 72 A quadrilateral in which a pair of opposite sides is parallel is————-.
Solution. trapezium
We know that, in a trapezium, one pair of sides is parallel.

Question. 73 If all sides of a quadrilateral are equal, it is a—————.
Solution. rhombus or square
As in both the quadrilaterals all sides are of equal length.

Question. 74 In a rhombus, diagonals intersect at———— angles.
Solution. right
The diagonals of a rhombus intersect at right angles.

Question. 75 ———measurements can determine a quadrilateral uniquely.
Solution. 5
To construct a unique quadrilateral, we require 5 measurements, i.e. four sides and one angle or three sides and two included angles or two adjacent sides and three angles are given.

Question. 76 A quadrilateral can be constructed uniquely, if its three sides and————angles are given.
Solution. two included
We cap determine a quadrilateral uniquely, if three sides and two included angles are given.

Question. 77 A rhombus is a parallelogram in which————sides are equal.
Solution. all
As length of each side is same in a rhombus.

Question. 78 The measure of——— angle of concave quadrilateral is more than 180°.
Solution. one
Concave polygon is a polygon in which at least one interior angle is more than 180°.

Question. 79 A diagonal of a quadrilateral is a line segment that joins two——— vertices of the quadrilateral.
Solution. opposite
Since the line segment connecting two opposite vertices is called diagonal.

Question. 80 The number of sides in a regular polygon having measure of an exterior angle as 72° is————— .
Solution. 5
We know that,the sum of exterior angles of any polygon is 360°.

$$\text{The number of sides in a regular polygon} = \frac{360^\circ}{\text{Exterior angle}}$$

$$\therefore \text{ The number of sides in given polygon} = \frac{360^\circ}{72^\circ} = 5$$

Question. 81 If the diagonals of a quadrilateral bisect each other, it is a————.
Solution. parallelogram
Since in a parallelogram, the diagonals bisect each other.

Question. 82 The adjacent sides of a parallelogram are 5 cm and 9 cm. Its perimeter is——.
Solution. 28 cm
Perimeter of a parallelogram = 2 (Sum of lengths of adjacent sides)
=2(5+ 9) = 2 x 14=28cm

Question. 83 A nonagon has————sides.
Solution. 9
Nonagon is a polygon which has 9 sides.

Question. 84 Diagonals of a rectangle are————.
Solution. equal
We know that, in a rectangle, both the diagonals are of equal length.

Question. 85 A polygon having 10 sides is known as————.
Solution. decagon
A polygon with 10 sides is called decagon.

Question. 86 A rectangle whose adjacent sides are equal becomes a ————.
Solution. square
If in a rectangle, adjacent sides are equal, then it is called a square.

Question. 87 If one diagonal of a rectangle is 6 cm long, length of the other diagonal is——.
Solution. 6 cm

Since both the diagonals of a rectangle are equal. Therefore, length of other diagonal is also 6 cm.

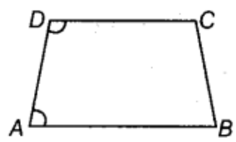
Question. 88 Adjacent angles of a parallelogram are————.
Solution. supplementary
By property of a parallelogram, we know that, the adjacent angles of a parallelogram are supplementary.

Question. 89 If only one diagonal of a quadrilateral bisects the other, then the quadrilateral is known as————.
Solution. kite
This is a property of kite, i.e. only one diagonal bisects the other.

Question. 90 In trapezium ABCD with AB || CD, if $\angle A$= 100°, then $\angle D$ =————.
Solution.

80°

In a trapezium, we know that, the angles on either side of the base are supplementary.
So, in trapezium *ABCD*, given *AB*||*CD*



$$\therefore \quad \angle A + \angle D = 180°$$
$$\Rightarrow \quad 100° + \angle D = 180°$$
$$\Rightarrow \quad \angle D = 180° - 100°$$
$$\therefore \quad \angle D = 80°$$

Question. 91 The polygon in which sum of all exterior angles is equal to the sum of interior angles is called————.
Solution. quadrilateral
We know that, the sum of exterior angles of a polygon is 360° and in a quadrilateral, sum of interior angles is also 360°. Therefore, a quadrilateral is a polygon in which the sum of both interior and exterior angles are equal.

True/False

In questions 92 to 131, state whether the statements are True or False.
Question. 92 All angles of a trapezium are equal.
Solution. False
As all angles of a trapezium are not equal.

Question. 93 All squares are rectangles.
Solution. True
Since squares possess all the properties of rectangles. Therefore, we can say that, all squares are rectangles but vice-versa is not true.

Question. 94 All kites are squares.
Solution. False
As kites do not satisfy all the properties of a square.
e.g. In square, all the angles are of 90° but in kite, it is not the case.

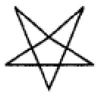
Question. 95 All rectangles are parallelograms.
Solution. True
Since rectangles satisfy all ”the”properties” of parallelograms. Therefore, we can say that, all rectangles are parallelograms but vice-versa is not true.

Question. 96 All rhombuses are square.
Solution. False
As in a rhombus, each angle is not a right angle, so rhombuses are not squares.

Question. 97 Sum of all the angles of a quadrilateral is 180°.
Solution. False
Since sum of all the angles of a quadrilateral is 360°.

Question. 98 A quadrilateral has two diagonals.
Solution. True
A quadrilateral has two diagonals.

Question. 99 Triangle is a polygon whose sum of exterior angles is double the sum of interior angles.
Solution. True
As the sum of interior angles of a triangle is 180° and the sum of exterior angles is 360°, i.e. double the sum of interior angles.

Question. 100
 **is a polygon.**

Solution. False
Because it is not a simple closed curve as it intersects with itself more than once.

Question. 101 A kite is not a convex quadrilateral.
Solution. False
A kite is a convex quadrilateral as the line segment joining any two opposite vertices inside it, lies completely inside it.

Question. 102 The sum of interior angles and the sum of exterior angles taken in an order are equal in case of quadrilaterals only.
Solution. True
Since the sum of interior angles as well as of exterior angles of a quadrilateral are 360°.

Question. 103 If the sum of interior angles is double the sum of exterior angles taken in an order of a polygon, then it is a hexagon.
Solution. True
Since the sum of exterior angles of a hexagon is 360° and the sum of interior angles of a hexagon is 720°, i.e. double the sum of exterior angles.

Question. 104 A polygon is regular, if all of its sides are equal.
Solution. False
By definition of a regular polygon, we know that, a polygon is regular, if all sides and all angles are equal.

Question. 105 Rectangle is a regular quadrilateral.
Solution. False
As its all sides are not equal.

Question. 106 If diagonals of a quadrilateral are equal, it must be a rectangle.
Solution. True
If diagonals are equal, then it is definitely a rectangle. –

Question. 107 If opposite angles of a quadrilateral are equal, it must be a parallelogram.
Solution. True
If opposite angles are equal, it has to be a parallelogram.

Question. 108 The interior angles of a triangle are in the ratio 1:2:3, then the ratio of its

exterior angles is 3 : 2 : 1.
Solution.

False

Given, ratio of interior angles = 1 : 2 : 3

Let the interior angles be x, $2x$ and $3x$.

So, $x + 2x + 3x = 180°$ [angle sum property of triangle]

$\Rightarrow \quad 6x = 180°$

$\Rightarrow \quad x = \frac{180°}{6} = 30°$

$\therefore$ The interior angles are 30°, 60° and 90°.

Now, the exterior angles will be (180°– 30°), (180°– 60°) and (180°– 90°),

i.e. 150°, 120° and 90°.

The ratio of exterior angles = 150° : 120° : 90° = 15 : 12 : 9 = 5 : 4 : 3

Question. 109

 is a concave pentagon.

Solution. False

As it has 6 sides, therefore it is a concave hexagon.

Question. 110 Diagonals of a rhombus are equal and perpendicular to each other.

Solution. False

As diagonals of a rhombus are perpendicular to each other but not equal.

Question. 111 Diagonals of a rectangle are equal.

Solution. True

The diagonals of a rectangle are equal.

Question. 112 Diagonals of rectangle bisect each other at right angles.

Solution. False

Diagonals of a rectangle does not bisect each other.

Question. 113 Every kite is a parallelogram.

Solution. False

Kite is not a parallelogram as its opposite sides are not equal and parallel.

Question. 114 Every trapezium is a parallelogram.

Solution. False

Since in a trapezium, only one pair of sides is parallel.

Question. 115 Every parallelogram is a rectangle.

Solution. False

As in a parallelogram, all angles are not right angles, while in a rectangle, all angles are equal and are right angles.

Question. 116 Every trapezium is a rectangle.

Solution. False

Since in a rectangle, opposite sides are equal and parallel but in a trapezium, it is not so.

Question. 117 Every rectangle is a trapezium.

Solution. True

As a rectangle satisfies all the properties of a trapezium. So, we can say that, every rectangle is a trapezium but vice-versa is not true.

Question. 118 Every square is a rhombus.
Solution. True
As a square possesses all the properties of a rhombus. So, we can say that, every square is a rhombus but vice-versa is not true.

Question. 119 Every square is a parallelogram.
Solution. True
Every square is also a parallelogram as it has all the properties of a parallelogram but vice-versa is not true.

Question. 120 Every square is a trapezium.
Solution. True
As a square has all the properties of a trapezium. So, we can say that, every square is a trapezium but vice-versa is not true.

Question. 121 Every rhombus is a trapezium.
Solution. True
Since a rhombus satisfies all the properties of a trapezium. So, we can say that, every rhombus is a trapezium but vice-versa is not true.

Question. 122 A quadrilateral can be drawn if only measures of four sides are given.
Solution. False
As we require at least five measurements to determine a quadrilateral uniquely.

Question. 123 A quadrilateral can have all four angles as obtuse.
Solution. False
If all angles will be obtuse, then their sum will exceed 360°. This is not possible in case of a quadrilateral.

Question. 124 A quadrilateral can be drawn, if all four sides and one diagonal is known.
Solution. True
A quadrilateral can be constructed uniquely, if four sides and one diagonal is known.

Question. 125 A quadrilateral can be drawn, when all the four angles and one side is given.
Solution. False
We cannot draw a unique-quadrilateral, if four angles and one side is known.

Question. 126 A quadrilateral can be drawn, if all four sides and one angle is known.
Solution. True
A quadrilateral can be drawn, if all four sides and one angle is known.

Question. 127 A quadrilateral can be drawn, if three sides and two diagonals are given.
Solution. True
A quadrilateral can be drawn, if three sides and two diagonals are given.

Question. 128 If diagonals of a quadrilateral bisect each other, it must be a parallelogram.
Solution. True
It is the property of a parallelogram.

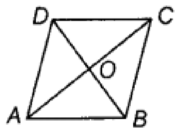
Question. 129 A quadrilateral can be constructed uniquely, if three angles and any two included sides are given.
Solution. True
We can construct a unique quadrilateral with given three angles given and two included sides.

Question. 130 A parallelogram can be constructed uniquely, if both diagonals and the angle between them is given.
Solution. True
We can draw a unique parallelogram, if both diagonals and the angle between them is given.

Question. 131 A rhombus can be constructed uniquely, if both diagonals are given.
Solution. True
A rhombus can be constructed uniquely, if both diagonals are given.

Question. 132 The diagonals of a rhombus are 8 cm and 15 cm. Find its side.
Solution.

Given, $AC = 15\,\text{cm}$, $BD = 8\,\text{cm}$



Since, the diagonals of a rhombus bisect each other at 90°, therefore in the ΔAOB, we have

$$AB^2 = OA^2 + OB^2$$

$$\Rightarrow \quad AB^2 = \left(\frac{15}{2}\right)^2 + \left(\frac{8}{2}\right)^2 = (7.5)^2 + (4)^2 = 56.25 + 16$$

$$\Rightarrow \quad AB^2 = 72.25$$

$$\Rightarrow \quad AB = \sqrt{72.25}$$

$$\Rightarrow \quad AB = 8.5\,\text{cm}$$

Since it is a rhombus, the length of each side is 8.5 cm.

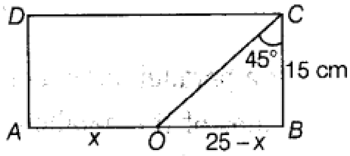
Question. 133 Two adjacent angles of a parallelogram are in the ratio 1 : 3. Find its angles.
Solution. Let the adjacent angles of a parallelogram be x and 8c.
Then, we have x + (3 x) = 180° [adjacent angles of parallelogram are supplementary]
=> 4 x = 180°
=> x = 45°
Thus, the angles are 45°, 135°.
Hence, the angles are 45°, 135, 45°, 135°. [opposite angles in a parallelogram are equal]

Question. 134 Of the four quadrilaterals – square, rectangle, rhombus and trapezium-one is somewhat different from the others because of its design. Find it and give justification.
Solution. In square, rectangle and rhombus, opposite sides are parallel and equal. Also, opposite angles are equal, i.e. they all are parallelograms.
But in trapezium, there is only one pair of parallel sides, i.e. it is not a parallelogram. Therefore, trapezium has different design.

Question. 135 In a rectangle ABCD, AB = 25 cm and BC = 15 cm. In what ratio, does the bisector of $\angle C$ divide AB?
Solution.

Given, $AB = 25\,\text{cm}$ and $BC = 15\,\text{cm}$

Now, in rectangle $ABCD$,



CO is the bisector of $\angle C$ and it divides AB.

$\therefore \quad \angle OCB = \angle OCD = 45°$

$\therefore \qquad \angle OCB = \angle OCD = 45°$

In ΔOCB, we have

$\angle CBO + \angle OCB + \angle COB = 180°$ [angle sum property of triangle]

$90° + 45° + \angle COB = 180°$

$\angle COB = 180° - 90° + 45°$

$\angle COB = 180° - 135° = 45°$

Now, in ΔOCB,

$\angle OCB = \angle COB$

Then, $\quad OB = BC$

$\Rightarrow \quad OB = 15\text{ cm}$

CO divides AB in the ratio $AO : OB$.

Let AO be x, then

$OB = AB - x = 25 - x$

Hence, $\quad AO : OB = x : 25 - x$

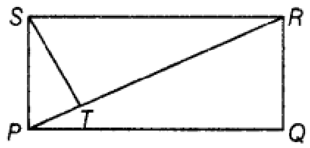
$\Rightarrow \quad 10 : 15 \quad \Rightarrow \quad 2 : 3$

Question. 136 PQRS is a rectangle. The perpendicular ST from S on PR divides $\angle S$ in the ratio 2 : 3. Find $\angle TPQ$.

Solution.

Given, $ST \perp PR$ and ST divides $\angle S$ in the ratio 2 : 3.

So, sum of ratio = 2 + 3 = 5



Now, $\angle TSP = \frac{2}{5} \times 90° = 36°$, $\angle TSR = \frac{3}{5} \times 90° = 54°$

Also, by the angle sum property of a triangle,

$\angle TPS = 180° - (\angle STP + \angle TSP)$

$= 180° - (90° + 36°) = 54°$

We know that, $\angle SPQ = 90°$

$\Rightarrow \angle TPS + \angle TPQ = 90°$

$\Rightarrow \quad 54° + \angle TPQ = 90°$

$\Rightarrow \quad \angle TPQ = 90° - 54° = 36°$

Question. 137 A photo frame is in the shape of a quadrilateral, with one diagonal longer than the other. Is it a rectangle? Why or why not?

Solution. No, it cannot be a rectangle, as in rectangle, both the diagonals are of equal lengths.

Question. 138 The adjacent angles of a parallelogram are (2x – 4)° and (3x – 1)°. Find the measures of all angles of the parallelogram.

Solution. Since, the adjacent angles of a parallelogram are supplementary.

(2 x – 4)° + (3* – 1)° = 180°

$\Rightarrow \quad 5x - 5° = 180°$

$\Rightarrow \quad 5x = 185°$

$\Rightarrow \quad x = \frac{185°}{5} \quad \Rightarrow \quad x = 37°$

Thus, the adjacent angles are

$2x - 4 = 2 \times 37° - 4 = 74 - 4 = 70°$

and $3x - 1 = 3 \times 37° - 1 = 111 - 1 = 110°$

Hence, the angles are 70°, 110°, 70°, 110°

[$\because$ opposite angles in a parallelogram are equal]

Question. 139 The point of intersection of diagonals of a quadrilateral divides one diagonal in the ratio 1: 2. Can it be a parallelogram? Why or why not?
Solution. No, it can never be a parallelogram, as the diagonals of a parallelogram intersect each other in the ratio 1 : 1.

Question. 140 The ratio between exterior angle and interior angle of a regular polygon is 1 : 5. Find the number of sides of the polygon.
Solution.
Let the exterior angle and interior angle be x and $5x$, respectively.
Then, $x + 5x = 180°$
[$\because$ exterior angle and corresponding interior angle are supplementary]

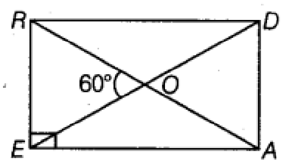
$$\Rightarrow \quad 6x = 180°$$
$$\Rightarrow \quad x = \frac{180°}{6}$$
$$\Rightarrow \quad x = 30°$$
$$\therefore \text{ The number of sides} = \frac{360°}{\text{Exterior angle}} = \frac{360°}{30°} = 12$$

Question. 141 Two sticks each of length 5 cm are crossing each other such that they bisect each other. What shape is formed by joining their end points? Give reason.
Solution. Sticks can be taken as the diagonals of a quadrilateral.
Now, since they are bisecting each other, therefore the shape formed by joining their end points will be a parallelogram.
Hence, it may be a rectangle or a square depending on the angle between the sticks.

Question. 142 Two sticks each of length 7 cm are crossing each other such that they bisect each other at right angles. What shape is formed by joining their end points? Give reason.
Solution. Sticks can be treated as the diagonals of a quadrilateral.
Now, since the diagonals (sticks) are bisecting each other at right angles, therefore the shape formed by joining their end points will be a rhombus.

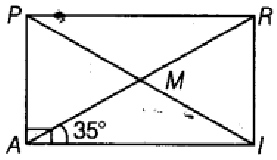
Question. 143 A playground in the town is in the form of a kite. The perimeter is 106 m. If one of its sides is 23 m, what are the lengths of other three sides?
Solution. Let the length of other non-consecutive side be x cm.
Then, we have, perimeter of playground = 23 + 23+ x + x
=> 106 = 2 (23+ x)
=>46 + 2x = 106 2x = 106 – 46
=>2x = 60
=>x = 30 m
Hence, the lengths of other three sides are 23m, 30m and 30m. As a kite has two pairs of equal consecutive sides.

Question. 144 In rectangle READ , find $\angle EAR$, $\angle RAD$ and $\angle ROD$.



Solution.
Given, a rectangle $READ$, in which $\angle ROE = 60°$
$\therefore \quad \angle EOA = 180° - 60° = 120°$ [linear pair]
Now, in ΔEOA, $\angle OEA = \angle OAE = 30°$ [$\because OE = OA$ and equal sides make equal angles]
$\therefore \quad \angle EAR = 30°$, $\angle RAD = 90° - \angle EAR = 60°$ and $\angle ROD = \angle EOA = 120°$

Question. 145 In rectangle PAIR, find $\angle ARI$, ZRMI and $\angle PMA$.



Solution.

Given, $\angle RAI = 35°$

$\therefore$ $\angle PRA = 35°$ [$PR \parallel AI$ and AR is transversal]

$\Rightarrow$ $\angle ARI = 90° - \angle PRA = 90° - 35° = 55°$

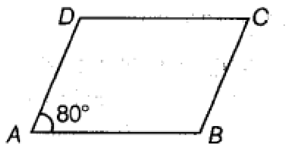
$\because$ $AM = IM$, $\angle MIA = \angle MAI = 35°$

In ΔAMI, $\angle RMI = \angle MAI + \angle MIA = 70°$ [exterior angle]

Also, $\angle RMI = \angle PMA$

$\Rightarrow$ $\angle PMA = 70°$ [vertically opposite angles]

Question. 146 In parallelogram ABCD, find $\angle B$, $\angle C$ and $\angle D$.



Solution.

In a parallelogram, the opposite angles are equal, therefore $\angle C = \angle A = 80°$

Also, adjacent angles are supplementary.

$\therefore$ $\angle A + \angle B = 180°$

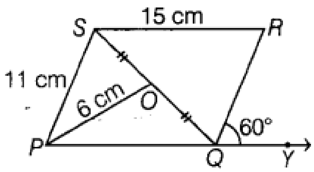
$80° + \angle B = 180°$

$\angle B = 180° - 80° \Rightarrow \angle B = 100°$

Now, $\angle B = \angle D$

$\therefore$ $\angle D = 100°$

Question. 147 In parallelogram PQRS, 0 is the mid-point of SQ. Find $\angle S$, $\angle R$, PQ, QR and diagonal PR.



Solution.

Given, $\angle RQY = 60°$

$\therefore$ $\angle RQP = 120°$ [linear pair]

$\therefore$ $\angle S = 120°$ [$\because$ opposite angles are equal in a parallelogram]

By the angle sum property of a quadrilateral, $\angle P + \angle R + \angle S + \angle Q = 360°$

$\Rightarrow$ $\angle P + \angle R + 120° + 120° = 360°$

$\Rightarrow$ $\angle P + \angle R = 120°$

$\Rightarrow$ $2\angle P = 120°$

$\Rightarrow$ $\angle P = 60°$ [$\because$ opposite angles are equal in parallelogram]

$\Rightarrow$ $\angle P = \angle R = 60°$

Also, $SR = 15\text{ cm}$

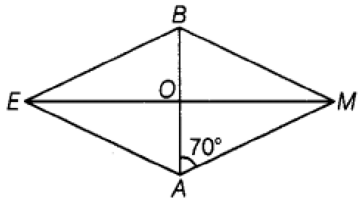
$\therefore$ $PQ = 15\text{ cm}$ [$\because$ opposite sides of a parallelogram are equal]

And $PS = 11\text{ cm}$

$\therefore$ $QR = 11\text{ cm}$ [$\because$ opposite sides of a parallelogram are equal]

and $PR = 2 \times PO = 2 \times 6 = 12$ [$\because$ diagonals of a parallelogram bisect each other]

Question. 148 In rhombus BEAM, find $\angle AME$ and $\angle AEM$.



Solution.

Given, $\angle BAM = 70°$

We know that, in rhombus, diagonals bisect each other at right angles.

$\therefore\ \angle BOM = \angle BOE = \angle AOM = \angle AOE = 90°$

Now, in ΔAOM,

$\angle AOM + \angle AMO + \angle OAM = 180°$ [angle sum property of triangle]

$\Rightarrow\quad 90° + \angle AMO + 70° = 180°$

$\Rightarrow\quad \angle AMO = 180° - 90° - 70°$

$\Rightarrow\quad \angle AMO = 20°$

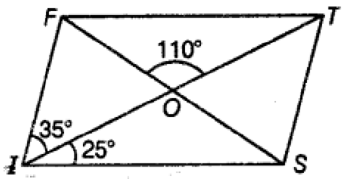
Also, $AM = BM = BE = EA$

In ΔAME, we have,

$AM = EA$

$\therefore\ \angle AME = \angle AEM = 20°$ [$\because$ equal sides make equal angles]

Question. 149 In parallelogram FIST, find $\angle SFT$, $\angle OST$ and $\angle STO$.



Solution.

Given, $\angle FIS = 60°$

Now, $\angle FTS = \angle FIS = 60°$ [$\because$ opposite angles of a parallelogram are equal]

Now, $FT \parallel IS$ and TI is a transversal, therefore $\angle FTO = \angle SIO = 25°$ [alternate angles]

$\therefore\quad \angle STO = \angle FTS - \angle FTO = 60° - 25° = 35°$

Also, $\angle FOT + \angle SOT = 180°$ [linear pair]

$\Rightarrow\quad 110° + \angle SOT = 180°$

$\Rightarrow\quad \angle SOT = 180° - 110° = 70°$

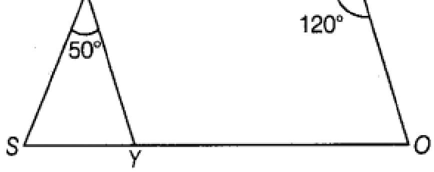
In ΔTOS, $\angle TSO + \angle OTS + \angle TOS = 180°$ [angle sum property of triangle]

$\therefore\quad \angle OST = 180° - (70° + 35°) = 75°$

In ΔFOT, $\angle FOT + \angle FTO + \angle OFT = 180°$

$\Rightarrow\quad \angle SFT = \angle OFT = 180° - (\angle FOT + \angle FTO) = 180° - (110° + 25°) = 45°$

Question. 150 In the given parallelogram YOUR, $\angle RUO$= 120° and 0Y is extended to points, such that $\angle SRY$ = 50°. Find $\angle YSR$.



Solution.

Given, $\angle RUO = 120°$ and $\angle SRY = 50°$

$\angle RYO = \angle RUO = 120°$ [$\because$ opposite angles of a parallelogram]

Now, $\angle SYR = 180° - \angle RYO$ [linear pair]

$= 180° - 120° = 60°$

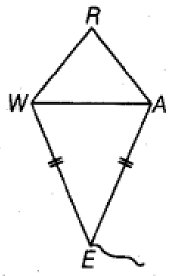
In ΔSRY,

By the angle sum property of a triangle, $\angle SRY + \angle RYS + \angle YSR = 180°$

$\Rightarrow \quad 50° + 60° + \angle YSR = 180°$

$\Rightarrow \quad \angle YSR = 180° - (50° + 60°) = 70°$

Question.151 In kite WEAR, $\angle WEA$ = 70° and $\angle ARW$ = 80°. Find the remaining two angles.



Solution.

Given, in a kite $WEAR$, $\angle WEA = 70°$, $\angle ARW = 80°$

Now, by the interior angle sum property of a quadrilateral,

$\angle RWE + \angle WEA + \angle EAR + \angle ARW = 360°$

$\Rightarrow \quad \angle RWE + 70° + \angle EAR + 80° = 360°$

$\Rightarrow \quad \angle RWE + \angle EAR = 360° - 150°$

$\Rightarrow \quad \angle RWE + \angle EAR = 210°$ ...(i)

Now, $\angle RWA = \angle RAW$ [$\because RW = RA$] ...(ii)

and $\angle AWE = \angle WAE$ [$\because WE = AE$] ...(iii)

On adding Eqs. (ii) and (iii), we get $\angle RWA + \angle AWE = \angle RAW + \angle WAE$

$\Rightarrow \quad \angle RWE = \angle RAE$

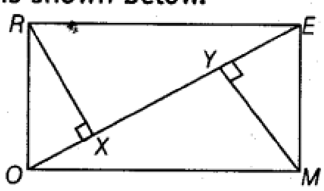
From Eq. (i),

$2\angle RWE = 210°$

$\angle RWE = 105° \quad \Rightarrow \quad \angle RWE = \angle RAE = 105°$

Question.152

A rectangle *MORE* is shown below.



Answer the following questions by giving appropriate reason.

(i) Is $RE = OM$? (ii) Is $\angle MYO = \angle RXE$?

(iii) Is $\angle MOY = \angle REX$? (iv) Is $\Delta MYO \cong \Delta RXE$?

(v) Is $MY = RX$?

Solution.

(i) Yes, $RE = OM$

Given, $MORE$ is a rectangle. Therefore, opposite sides are equal.

(ii) Yes, $\angle MYO = \angle RXE$

Here, MY and RX are perpendicular to OE.

Since, $\angle RXO = 90° \Rightarrow \angle RXE = 90°$ and $\angle MYE = 90° \Rightarrow \angle MYO = 90°$

(iii) Yes, $\angle MOY = \angle REX$

$\because$ $RE \parallel OM$ and EO is a transversal.

$\therefore$ $\angle MOE = \angle OER$ [$\because$ alternate interior angles]

$\Rightarrow \angle MOY = \angle REX$

(iv) Yes, $\Delta MYO \cong \Delta RXE$

In ΔMYO and ΔRXE

$MO = RE$ [proved in (i)]

$\angle MOY = \angle REX$ [proved in (iii)]

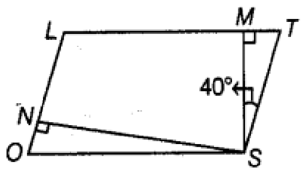
$\angle MYO = \angle RXE$ [proved in (ii)]

$\therefore$ $\Delta MYO \cong \Delta RXE$ [by AAS]

(v) Yes, $MY = RX$

Since, these are corresponding parts of congruent triangles.

Question.153 In parallelogram LOST, SNLOL and $SM \perp LT$. Find $\angle STM$, $\angle SON$ and $\angle NSM$.



Solution.

Given, $\angle MST = 40°$

In ΔMST,

By the angle sum property of a triangle, $\angle TMS + \angle MST + \angle STM = 180°$

$\Rightarrow$ $\angle STM = 180° - (90° + 40°)$ [$\because SM \perp LT$, $\angle TMS = 90°$]

$= 50°$

$\therefore$ $\angle SON = \angle STM = 50°$ [$\because$ opposite angles of a parallelogram are equal]

Now, in the ΔONS,

$\angle ONS + \angle OSN + \angle SON = 180°$ [angle sum property of triangle]

$\angle OSN = 180° - (90° + 50°)$

$= 180° - 140° = 40°$

Moreover, $\angle SON + \angle TSO = 180°$

[$\because$ adjacent angles of a parallelogram are supplementary]

$\Rightarrow$ $\angle SON + \angle TSM + \angle NSM + \angle OSN = 180°$

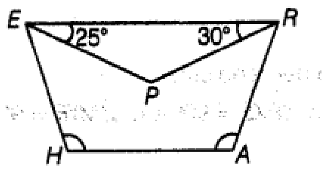
$\Rightarrow$ $50° + 40° + \angle NSM + 40° = 180°$

$\Rightarrow$ $90° + 40° + \angle NSM = 180°$

$\Rightarrow$ $130° + \angle NSM = 180°$

$\Rightarrow$ $\angle NSM = 180° - 130° = 50°$

Question. 154 In trapezium HARE, EP and RP are bisectors of $\angle E$ and $\angle R$, respectively. Find $\angle HAR$ and $\angle EHA$.



Solution.

As EP and PR are angle bisectors of $\angle REH$, and $\angle ARE$ respectively. [given]

Since, $HARE$ is a trapezium,

Therefore, $\angle E + \angle H = 180°$ and $\angle R + \angle A = 180°$

$\Rightarrow \angle PER + \angle PEH + \angle H = 180°$ and $\angle ERP + \angle PRA + \angle RAH = 180°$

$\Rightarrow$ $25° + 25° + \angle H = 180°$ and $30° + 30° + \angle A = 180°$

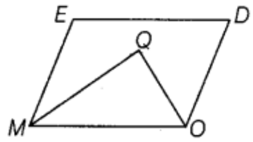
$\Rightarrow$ $50° + \angle H = 180°$ and $60° + \angle A = 180°$

$\Rightarrow$ $\angle H = 130°$ and $\angle A = 120°$, i.e. $\angle EHA = 130°$ and $\angle HAR = 120°$

Question. 155 In parallelogram MODE, the bisectors of $\angle M$ and $\angle O$ meet at Q. Find the measure of $\angle MQO$.

Solution.

Let $MODE$ be a parallelogram and Q be the point of intersection of the bisector of $\angle M$ and $\angle O$.



Since, $MODE$ is a parallelogram.

$\therefore$ $\angle EMO + \angle DOM = 180°$ [$\because$ adjacent angles are supplementary]

$\Rightarrow$ $\frac{1}{2}\angle EMO + \frac{1}{2}\angle DOM = 90°$ [dividing both sides by 2]

$\Rightarrow$ $\angle QMO + \angle QOM = 90°$ ...(i)

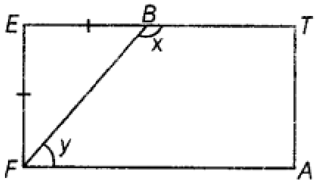
Now, in ΔMOQ,

$\angle QOM + \angle QMO + \angle MQO = 180°$ [angle sum property of triangle]

$\Rightarrow$ $90° + \angle MQO = 180°$ [from Eq. (i)]

$\therefore$ $\angle MQO = 180° - 90° = 90°$

Question. 156 A playground is in the form of a rectangle ATEF. Two players are standing at the points F and B, where EF =EB. Find the values of x and y.



Solution.

Given, a rectangle $ATEF$ in which $EF = EB$. Then, ΔFEB is an isosceles triangle. Therefore, by the angle sum property of a triangle, we have

$\angle EFB + \angle EBF + \angle FEB = 180°$ [angle sum property of triangle]

$\Rightarrow$ $\angle EFB + \angle EBF + 90° = 180°$ [$\because$ in a rectangle, each angle is of 90°]

$\Rightarrow$ $2\angle EFB = 90°$ [$\because \angle EFB = \angle EBF$]

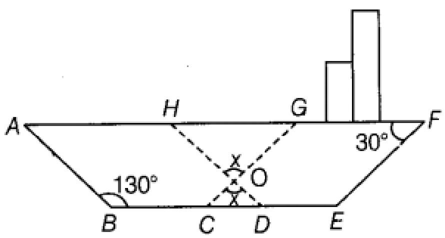
$\angle EFB = 45°$ and $\angle EBF = 45°$

Now, $\angle x = 180° - 45° = 135°$ [linear pair]

and $\angle EFB + \angle y = 90°$ [$\because$ in a rectangle, each angle is of 90°]

$\Rightarrow$ $\angle y = 90° - 45° = 45°$

Question. 157 In the following figure of a ship, ABDH and CEFG are two parallelograms. Find the value of x.



Solution.

We have, two parallelograms $ABDH$ and $CEFG$.

Now, in $ABDH$,

$\therefore \quad \angle ABD = \angle AHD = 130°$ [$\because$ opposite angles of a parallelogram are equal]

and $\angle GHD = 180° - \angle AHD = 180° - 130°$ [linear pair]

$\Rightarrow \quad 50° = \angle GHO$

Also, $\angle EFG + \angle FGC = 180°$ [$\because$ adjacent angles of a parallelogram are supplementary]

$\Rightarrow \quad 30° + \angle FGC = 180° \Rightarrow \angle FGC = 180° - 30° = 150°$

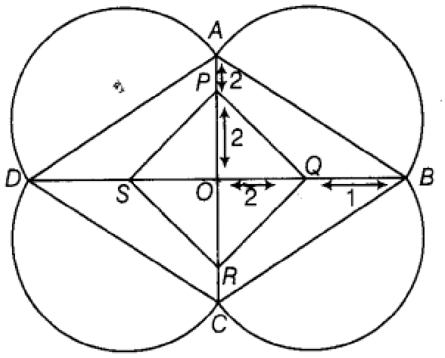
and $\angle HGC + \angle FGC = 180°$ [linear pair]

$\therefore \quad \angle HGC = 180° - \angle FGC = 180° - 150° = 30° = \angle HGO$

In ΔHGO, by using angle sum property, $\angle OHG + \angle HGO + \angle HOG = 180°$

$\Rightarrow \quad 50° + 30° + x = 180° \Rightarrow x = 180° - 80° = 100°$

Question. 158 A rangoli has been drawn on the floor of a house. ABCD and PQRS both are in the shape of a rhombus. Find the radius of semi-circle drawn on each side of rhombus ABCD.



Solution.

In rhombus $ABCD$,

$AO = OP + PA = 2 + 2 = 4$ units and $OB = OQ + QB = 2 + 1 = 3$ units

We know that, diagonals of rhombus bisect each other at 90°.

Now,

In ΔOAB, $(AB)^2 = (OA)^2 + (OB)^2$ [by Pythagoras theorem]

$\Rightarrow \quad (AB)^2 = (4)^2 + (3)^2 = 25$

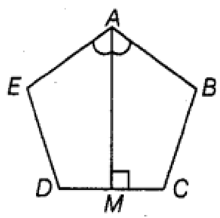
$\Rightarrow \quad AB = \sqrt{25} \quad \Rightarrow \quad AB = 5$ units

Since, AB is diameter of semi-circle.

$\therefore \quad \text{Radius} = \dfrac{\text{Diameter}}{2} = \dfrac{AB}{2} = \dfrac{5}{2} = 2.5$ units

Hence, radius of the semi-circle is 2.5 units.

Question. 159 ABCDE is a regular pentagon. The bisector of angle A meets the sides CD at M. Find $\angle AMC$



Solution.

Given, a pentagon $ABCDE$. The line segment AM is the bisector of the $\angle A$.

Now, since the measure of each interior angle of a regular pentagon is 108°.

$\therefore \quad \angle BAM = \frac{1}{2} \times 108° = 54°$

By the angle sum property of a quadrilateral, we have (in quadrilateral $ABCM$)

$$\angle BAM + \angle ABC + \angle BCM + \angle AMC = 360°$$

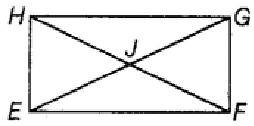
$\Rightarrow \quad 54° + 108° + 108° + \angle AMC = 360°$

$\Rightarrow \quad \angle AMC = 360° - 270° \Rightarrow \angle AMC = 90°$

Question. 160 Quadrilateral EFGH is a rectangle in which J is the point of intersection of the diagonals. Find the value of x, if JF = 8x + 4 and EG = 24 x − 8.

Solution.

Given, $EFGH$ is a rectangle in which diagonals are intersecting at the point J.



We know that, the diagonals of a rectangle bisect each other and are equal.

Then, $\quad EG = 2 \times JF$

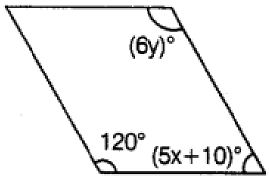
$\Rightarrow \quad 24x - 8 = 2(8x + 4)$

$\Rightarrow \quad 24x - 8 = 16x + 8$

$\Rightarrow \quad 24x - 16x = 8 + 8$

$\Rightarrow \quad 8x = 16 \Rightarrow x = 2$

Question. 161 Find the values of x and y in the following parallelogram.



Solution.

In a parallelogram, adjacent angles are supplementary.

$\therefore \quad 120° + (5x + 10)° = 180°$

$\Rightarrow \quad 5x + 10° + 120° = 180°$

$\Rightarrow \quad 5x = 180° - 130°$

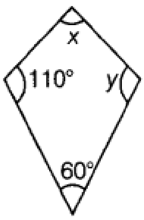
$\Rightarrow \quad 5x = 50°$

$\Rightarrow \quad x = 10°$

Also, opposite angles are equal in a parallelogram.

Therefore, $6y = 120° \Rightarrow y = 20°$

Question. 162 Find the values of x and y in the following kite.



Solution.

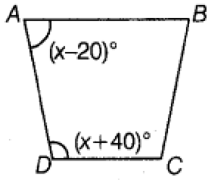
The given figure is a kite.
In a kite, one pair of opposite angles are equal.

$\therefore \quad y = 110°$

Now, by the angle sum property of a quadrilateral, we have

$$110° + 60° + 110° + x = 360°$$
$$\Rightarrow \quad x = 360° - 280° \Rightarrow x = 80°$$

Question. 163 Find the value of x in the trapezium ABCD given below.



Solution.

Given, a trapezium $ABCD$ in which $\angle A = (x - 20)°$, $\angle D = (x + 40)°$

Since, in a trapezium, the angles on either side of the base are supplementary, therefore

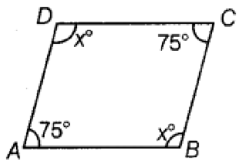
$$(x - 20) + (x + 40) = 180°$$
$$\Rightarrow \quad x - 20 + x + 40 = 180°$$
$$\Rightarrow \quad 2x + 20° = 180°$$
$$\Rightarrow \quad 2x = (180° - 20°) = 160° \Rightarrow x = 80°$$

Question. 164 Two angles of a quadrilateral are each of measure 75° and the other two angles are equal. What is the measure of these two angles? Name the possible figures so formed.

Solution.

Let $ABCD$ be a quadrilateral,

where $\angle A = \angle C = 75°$ and $\angle B = \angle D = x$ [say]



Then, by the angle sum property of a quadrilateral, we have

$$\angle A + \angle B + \angle C + \angle D = 360°$$
$$\Rightarrow \quad 75° + x + 75° + x = 360°$$
$$\Rightarrow \quad 2x = 360° - 150°$$
$$\Rightarrow \quad 2x = 210° \Rightarrow x = 105°$$

Thus, other two angles are of 105° each.

Since, opposite angles are equal, therefore the quadrilateral is a parallelogram.

Question. 165 In a quadrilateral PQRS, $\angle P$ = 50°, $\angle Q$ = 50°, $\angle R$ = 60°. Find $\angle S$. Is this quadrilateral convex or concave?

Solution.

Given a quadrilateral $PQRS$, where

$$\angle P = 50°, \angle Q = 50° \text{ and } \angle R = 60°$$

Now, by the angle sum property of a quadrilateral, we have

$$\angle P + \angle Q + \angle R + \angle S = 360°$$
$$\Rightarrow \quad 50° + 50° + 60° + \angle S = 360°$$
$$\Rightarrow \quad \angle S = 360° - 160°$$
$$\Rightarrow \quad \angle S = 200°$$

Since, one interior angle of the given quadrilateral is obtuse, therefore the quadrilateral is concave.

Question. 166 Both the pairs of opposite angles of a quadrilateral are equal and

supplementary. Find the measure of each angle.
Solution.

Let $ABCD$ be a quadrilateral, such that

$\angle A = \angle C$, $\angle B = \angle D$ and also $\angle A + \angle C = 180°$, $\angle B + \angle D = 180°$

Now, $\angle A + \angle A = 180°$ $[\because \angle C = \angle A]$

$\Rightarrow$ $2\angle A = 180°$

$\Rightarrow$ $\angle A = 90°$

Similarly, $\angle B = 90°$

Hence, each angle is a right angle.

Question. 167 Find the measure of each angle of a regular octagon.
Solution.

Number of sides(n) in octagon = 8

Now, the sum of interior angles of a regular octagon $= (n-2) \times 180°$

$= (8-2) \times 180°$

$= 6 \times 180° = 1080°$

Since, the octagon is regular, measure of each angle $= \dfrac{1080°}{8} = 135°$

Question. 168 Find the measure of an exterior angle of a regular pentagon and an exterior angle of a regular decagon. What is the ratio between these two angles?
Solution.

We know that, number of sides in pentagon is 5 and in decagon is 10.

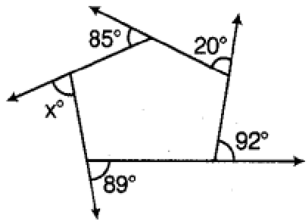
Now, exterior angle of a regular pentagon $= \dfrac{360°}{5} = 72°$

Exterior angle of a regular decagon $= \dfrac{360°}{10} = 36°$

$\therefore$ Required ratio $= \dfrac{72}{36} = 2:1$

So, the ratio between these two angles is 2:1.

Question. 169 In the figure, find the value of x.



Solution.

We observe that, the given figure is a pentagon.

Now, we know that, sum of all the exterior angles of a pentagon is 360°.

$\therefore$ $92° + 20° + 85° + x° + 89° = 360°$

$\Rightarrow$ $286° + x° = 360°$

$\Rightarrow$ $x° = 360° - 286° = 74°$

Question. 170 Three angles of a quadrilateral are equal. Fourth angle is of measure 120°. What is the measure of equal angles?
Solution.

Let the measures of equal angles be $x°$ each.

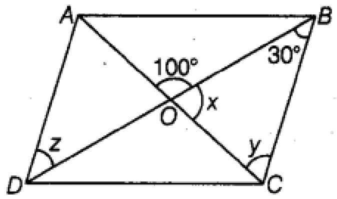
Then, by the angle sum property of a quadrilateral, we have

$$x°+x°+x°+120° = 360°$$
$$\Rightarrow \quad 3x° + 120° = 360°$$
$$\Rightarrow \quad 3x° = 240°$$
$$\Rightarrow \quad x° = 80°$$

Question. 171 In a quadrilateral HOPE, PS and ES are bisectors of $\angle P$ and $\angle E$ respectively. Give reason.

Solution. Data insufficient.

Question. 172 ABCD is a parallelogram. Find the values of x, y and z.



Solution.

Given, a parallelogram $ABCD$.

In the ΔOBC, we have

$$y + 30° = 100° \qquad \text{[exterior angle property of triangle]}$$
$$\Rightarrow \quad y = 70°$$

By the angle sum property of a triangle,

we have, $x + y + 30 = 180°$

$\Rightarrow \quad x + 70° + 30° = 180° \quad \Rightarrow \quad x = 180° - 100° = 80°$

Now, since $AD \parallel BC$ and BD is transversal, therefore

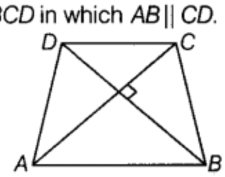
$$\angle ADO = \angle OBC \qquad \text{[alternate interior angles]}$$
$$\Rightarrow \quad z = 30°$$

Question. 173 Diagonals of a quadrilateral are perpendicular to each other. Is such a quadrilateral always a rhombus? Give a figure to justify your answer.

Solution.

False, it is not necessary that a quadrilateral having perpendicular diagonals is a rhombus.

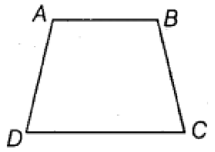
e.g. Consider a trapezium $ABCD$ in which $AB \parallel CD$.



Question. 174 ABCD is a trapezium such that AB || CD, $\angle A$: $\angle D$ = 2:1, $\angle B$: $\angle C$ = 7:5. Find the angles of the trapezium.

Solution.

Let $ABCD$ be a trapezium, where $AB \parallel CD$.



Let the angles A and D be of measures $2x$ and x, respectively.

Then, $2x + x = 180°$

[$\because$ in trapezium, the angles on either side of the base are supplementary]

$\Rightarrow \quad 3x = 180° \quad \Rightarrow \quad x = 60°$

$\therefore \quad \angle A = 2 \times 60° = 120°, \angle D = 60°$

Again, let the angles B and C be $7x$ and $5x$ respectively. Then, $7x + 5x = 180°$

$\Rightarrow \quad 12x = 180° \quad \Rightarrow \quad x = 15°$

Thus, $\quad \angle B = 7 \times 15 = 105°$ and $\angle C = 5 \times 15 = 75°$

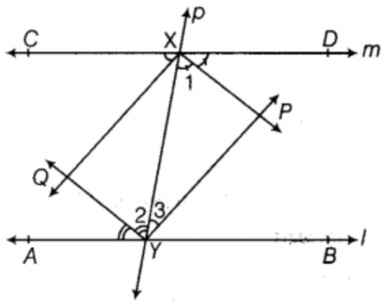
Question. 175 A line / is parallel to Line m and a-transversal p intersects them at X, Y respectively. Bisectors of interior angles at X and Y intersect at P and Q. Is PXQY a rectangle? Give reason.

Solution.

Given, $l \parallel m$

$\therefore \quad \angle DXY = \angle XYA$ [alternate interior angles]

$\Rightarrow \quad \frac{\angle DXY}{2} = \frac{\angle XYA}{2}$ [dividing both the sides by 2]



Now, $\quad \angle 1 = \angle 2$ [$\because XP$ and YQ are bisectors]

$\because$ Alternate angles are equal, i.e. $\angle 1 = \angle 2$

$\therefore \quad XP \parallel QY$ ...(i)

Similarly, $\quad XQ \parallel PY$ ...(ii)

From Eqs. (i) and (ii), we get

$PXQY$ is a parallelogram. ...(iii)

$\Rightarrow \quad \angle DXY + \angle XYB = 180°$

[$\because$ interior angles on the same side of transversal are supplementary]

$$\frac{\angle DXY}{2} + \frac{\angle XYB}{2} = \frac{180°}{2}$$ [dividing both the sides by 2]

$\angle 1 + \angle 3 = 90°$ ...(iv)

In ΔXYP,

$\angle 1 + \angle 3 + \angle P = 180°$

$90° + \angle P = 180°$ [from Eq. (iv)]

$\angle P = 90°$ ...(v)

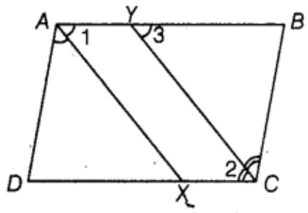
From Eqs. (iii) and (v), $PXQY$ is a rectangle.

Question. 176 ABCD is a parallelogram. The bisector of angle A intersects CD at X and bisector of angle C intersects AB at Y. Is AXCY a parallelogram? Give reason.

Solution.

Given, $ABCD$ is a parallelogram.

So, $\angle A = \angle C$ [$\because$ opposite angles of a parallelogram are equal]



$\therefore \quad \frac{\angle A}{2} = \frac{\angle C}{2}$ [dividing both the sides by 2]

$\angle 1 = \angle 2$ [alternate angles]

But $\angle 2 = \angle 3$ [$\because AB \parallel CD$ and CY is the transversal]

$\therefore \quad \angle 1 = \angle 3$

But they are pair of corresponding angles.

$\therefore \quad AX \parallel YC$ ...(i)

$AY \parallel XC$ [$\because AB \parallel DC$] ...(ii)

From Eqs. (i) and (ii), we get

$AXCY$ is a parallelogram.

Question. 177 A diagonal of a parallelogram bisects an angle. Will it also bisect the other angle? Give reason.

Solution. Consider a parallelogram ABCD.

Given, $\angle 1 = \angle 2$

Since, $ABCD$ is a parallelogram.

$AB \parallel CD$ and AC is the transversal.

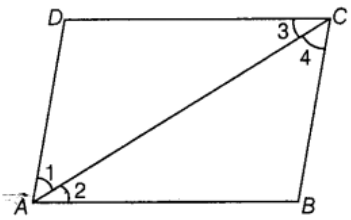
$\therefore \quad \angle 1 = \angle 4$ [alternate angles] ...(i)

Similarly,

$\angle 2 = \angle 3$ [alternate angles]...(ii)

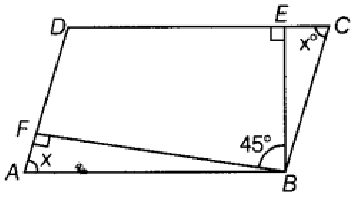
But given, $\angle 1 = \angle 2$

$\therefore \quad \angle 3 = \angle 4$ [from Eqs. (i) and (ii)]



Question. 178 The angle between the two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 45°. Find the angles of the parallelogram.

Solution. Let ABCD be a parallelogram, where BE and BF are the perpendiculars through the vertex B to the sides DC and AD, respectively.



Let $\angle A = \angle C = x$, $\angle B = \angle D = y$ [$\because$ opposite angles are equal in parallelogram]

Now, $\angle A + \angle B = 180°$ [$\because$ adjacent sides of a parallelogram are supplementary]

$\Rightarrow \quad x + \angle ABF + \angle FBE + \angle EBC = 180°$

$\Rightarrow \quad x + 90° - x + 45° + 90° - x = 180°$

[$\because$ In ΔABF, $\angle ABF = 90° - x$ and in ΔBEC, $\angle EBC = 90° - x$]

$\Rightarrow \quad -x = 180° - 225°$

$\Rightarrow \quad x = 45°$

$\therefore \quad \angle A = \angle C = 45°$

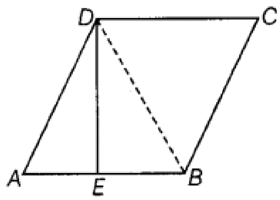
$\angle B = 45° + 45° + 45° = 135°$

$\Rightarrow \quad \angle D = 135°$

Hence, the angles are 45°, 135°, 45°, 135°.

Question. 179 ABCD is a rhombus such that the perpendicular bisector of AB passes through D. Find the angles of the rhombus.[Hint Join BD. Then, AABD is equilateral.]

Solution. Let ABCD be a rhombus in which DE is perpendicular bisector of AB.



Join BD. Then, in ΔAED and ΔBED, we have

$AE = EB$

$ED = ED$ [common side]

$\angle AED = \angle DEB = 90°$

Then, by SAS rule, $\Delta AED \cong BED$

$\therefore \quad AD = DB = AB$ [$\because$ $ABCD$ is a rhombus. So, $AD = AB$]

Thus, ΔADB is an equilateral triangle.

$\therefore \quad \angle DAB = \angle DBA = \angle ADB = 60°$

$\Rightarrow \quad \angle DCB = 60°$ [opposite angles of a rhombus are equal]

Now, $\angle DAB + \angle ABC = 180°$ [adjacent angles of a rhombus are supplementary]

$\Rightarrow 60° + \angle ABD + \angle DBC = 180°$

$\Rightarrow \quad 60° + 60° + \angle DBC = 180°$

$\Rightarrow \quad \angle DBC = 60°$

$\therefore \quad \angle ABC = \angle ABD + \angle DBC = 60° + 60° = 120°$

$\therefore \quad \angle ADC = 120°$ [opposite angles of a rhombus are equal]

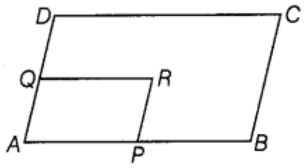
Hence, the angles of the rhombus arc 60°, 120°, 60°, 120°.

Question. 180 ABCD is a parallelogram. Point P and Q are taken on the sides AB and AD, respectively and 4he parallelogram PRQA is formed. If $\angle C$= 45°, find $\angle R$.

Solution.

Let $ABCD$ be a parallelogram,

where $\angle C = 45°$



Since, $ABCD$ is a parallelogram,

$\angle A = \angle C$ [opposite angles of parallelogram are equal]

Again, since $PRQA$ is a parallelogram,

$\angle A = \angle R$ [opposite angles of parallelogram are equal]

$\Rightarrow$ $\angle R = 45°$ [$\because \angle A = \angle C = 45°$]

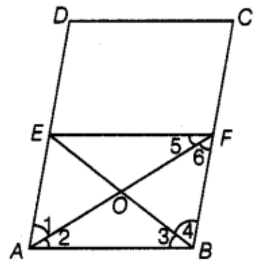
Question. 181 In parallelogram ABCD, the angle bisector of $\angle A$ bisects BC. Will angle bisector of B also bisect AD? Give reason.

Solution. Given, ABCD is a parallelogram, bisector of $\angle A$, bisects BC at F, i.e. $\angle 1 = \angle 2$,CF = FB

Draw FE || BA.

$ABFE$ is a parallelogram by construction [$\because FE \parallel BA$]

$\Rightarrow$ $\angle 1 = \angle 6$ [alternate angle]



But $\angle 1 = \angle 2$ [given]

$\therefore$ $\angle 2 = \angle 6$

$AB = FB$ [opposite sides to equal angles are equal] ...(i)

$\therefore$ $ABFE$ is a rhombus.

Now, in ΔABO and ΔBOF, $AB = BF$ [from Eq. (i)]

$BO = BO$ [common]

$AO = FO$ [diagonals of rhombus bisect each other]

$\therefore$ $\Delta ABO \cong \Delta BOF$ [by SSS]

$\angle 3 = \angle 4$ [by CPCT]

Now, $BF = \frac{1}{2} BC$ [given]

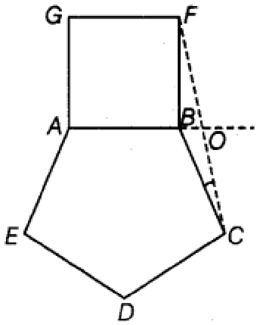
$\Rightarrow$ $BF = \frac{1}{2} AD$ [$BC = AD$]

$\Rightarrow$ $AE = \frac{1}{2} AD$ [$BF = AE$]

$\therefore$ E is the mid-point of AD.

Question. 182 A regular pentagon ABCDE and a square ABFG are formed on opposite sides of AB. Find $\angle BCF$?

Solution.



Given, $ABCDE$ is a regular pentagon.

Then, measure of each interior angle of the regular pentagon

$$= \frac{\text{Sum of interior angles}}{\text{Number of sides}} = \frac{(x-2)\times 180^\circ}{5}$$

$$= \frac{(5-2)\times 180^\circ}{5} = \frac{540^\circ}{5} = 108^\circ$$

$\therefore \qquad \angle CBA = 108^\circ$

Join CF.

Now, $\angle FBC = 360^\circ - (90^\circ + 108^\circ) = 360^\circ - 198^\circ = 162^\circ$

In ΔFBC, by the angle sum property, we have

$$\angle FBC + \angle BCF + \angle BFC = 180^\circ$$

$\Rightarrow \qquad \angle BCF + \angle BFC = 180^\circ - 162^\circ$

$\Rightarrow \qquad \angle BCF + \angle BFC = 18^\circ$

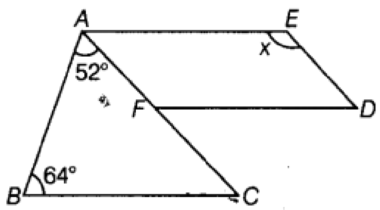
Since, ΔFBC is an isosceles triangle and $BF = BC$.

$\therefore \qquad \angle BCF = \angle BFC = 9^\circ$

Question. 183 Find maximum number of acute angles which a convex quadrilateral, a pentagon and a hexagon can have. Observe the pattern and generalise the result for any polygon.

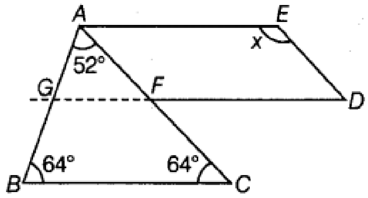
Solution. If an angle is acute, then the corresponding exterior angle is greater than 90°. Now, suppose a convex polygon has four or more acute angles. Since, the polygon is convex, all the exterior angles are positive, so the sum of the exterior angle is at least the sum of the interior angles. Now, supplementary of the four acute angles, which is greater than 4 x 90° = 360° However, this is impossible. Since, the sum of exterior angle of a polygon must equal to 360° and cannot be greater than it. It follows that the maximum number of acute angle in convex polygon is 3.

Question. 184 In the following figure, FD || BC || AE and AC || ED. Find the value of x.



Solution.

Produce DF such that it intersects AB at G.



In ΔABC,

$\angle A + \angle B + \angle C = 180°$ [angle sum property of triangle]

$\Rightarrow \quad 52° + 64° + \angle C = 180°$

$\angle C = 180° - (52° + 64°) = 180° - 116° = 64°$

Now, we see that, $DG \parallel BC$ and $DG \parallel AE$

$\therefore \quad \angle ACB = \angle AFG$ [$\because$ $FG \parallel BC$ and FC is a transversal, so corresponding angles]

$\Rightarrow \quad 64° = \angle AFG$

Also, GFD is a straight line.

$\therefore \quad \angle GFA + \angle AFD = 180°$ [linear pair]

$\Rightarrow \quad 64° + \angle AFD = 180°$

$\Rightarrow \quad \angle AFD = 180° - 64° = 116°$

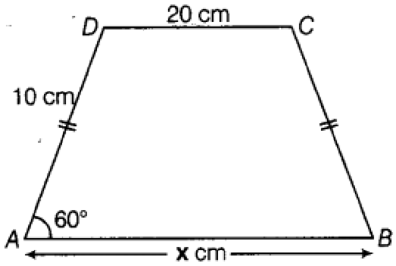
Also, $FD \parallel AE$ and $AF \parallel ED$

So, $AEDF$ is a parallelogram.

$\therefore \quad \angle AFD = \angle AED$ [$\because$ opposite angles in a parallelogram are equal]

$\Rightarrow \quad \angle AED = x = 116°$

Question. 185 In the following figure, AB || DC and AD = BC. Find the value of x.

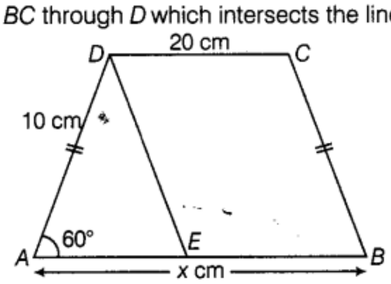


Solution.

Given, an isosceles trapezium, where $AB \parallel DC$, $AD = BC$ and $\angle A = 60°$.

Then, $\angle B = 60°$.

Draw a line parallel to BC through D which intersects the line AB at E (say).



Then, $DEBC$ is a parallelogram, where

$BE = CD = 20$ cm and $DE = BC = 10$ cm

Now, $\quad \angle DEB + \angle CBE = 180°$

[adjacent angles are supplementary in parallelogram]

$\Rightarrow \quad \angle DEB = 180° - 60° = 120°$

$\therefore$ In ΔADE, $\angle ADE = 60°$ [exterior angle]

Also, $\quad \angle DEA = 60°$ [$\because$ $AD = DE = 10$ cm and $\angle DAE = 60°$]

Then, ΔADE is an equilateral triangle.

$\therefore \quad AE = 10$ cm

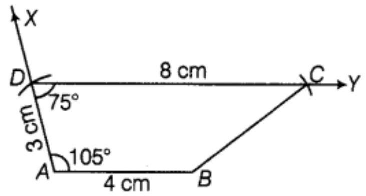
$\Rightarrow \quad AB = AE + EB = 10 + 20 = 30$ cm

Hence, $x = 30$ cm

Question. 186 Construct a trapezium ABCD in which AB || DC, $\angle A$ = 105°, AD = 3 cm, AB = 4 cm and CD = 8 cm.
Solution.

We know that.

$$\angle A + \angle D = 180 \quad [\because \text{ sum of adjacent angle of a trapezium is } 180°]$$
$$105° + \angle D = 180°$$
$$\angle D = 75°$$



Steps of Construction

***Step* I** Draw $AB = 4$ cm.

***Step* II** Draw $\overline{AX}$ such that $\angle BAX = 105°$.

***Step* III** Mark a point D on AX such that $AD = 3$cm.

***Step* IV** Draw $\overline{DY}$ such that $\angle ADY = 75°$.

***Step* V** Mark a point C such that $CD = 8$cm.

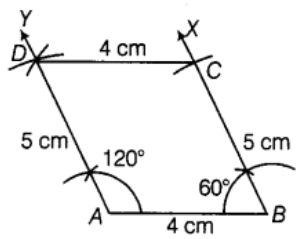
***Step* VI** Join BC.

Hence, $ABCD$ is the required trapezium.

Question. 187 Construct a parallelogram ABCD in which AB =4 cm, BC = 5cm and $\angle B$ = 60°.
Solution.

We know that, the opposite sides of a parallelogram are equal.

So,
$$AB = DC = 4 \text{ cm}$$
$$BC = AD = 5 \text{ cm}$$
$$\angle B = 60°$$
$$\angle A + \angle B = 180° \quad \text{[sum of cointerior angles]}$$
$$\angle A = 120°$$



Steps of Construction

***Step* I** Draw $AB = 4$cm.

***Step* II** Draw ray BX such that $\angle ABX = 60°$.

***Step* III** Mark a point C such that $BC = 5$ cm.

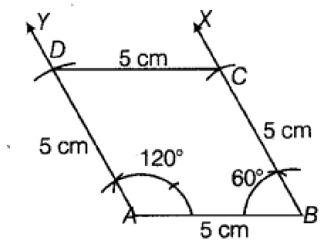
***Step* IV** Draw a ray AY such that $\angle YAB = 120°$.

***Step* V** Mark a point D such that $AD = 5$ cm.

***Step* VI** Join C and D.

Hence, $ABCD$ is required parallelogram.

Question. 188 Construct a rhombus whose side is 5 cm and one angle is of 60°
Solution.



$\angle B = 60°$ [suppose]

$\angle A + \angle B = 180°$ [sum of cointerior angles]

$\angle A + 60° = 180°$

$\angle A = 120°$

$AB = BC = CD = DA = 5\,\text{cm}$

Steps of Construction

Step I Draw $AB = 5\text{cm}$.

Step II Draw a ray AY such that $\angle BAY = 120°$.

Step III Mark a point D such that $AD = 5\text{cm}$.

Step IV Draw a ray BX such that $\angle ABX = 60°$

Step V Mark a point C such that $BC = 5\text{cm}$.

Step VI Joint C and D.

$\therefore$ $ABCD$ is the required rhombus.

Question. 189 Construct a rectangle whose one side is 3 cm and a diagonal is equal to 5 cm.

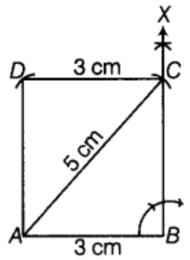
Solution.

We know that, diagonals of a rectangle and opposite sides are equal.

All the angles of rectangle are right angle.

So, $AC = 5\,\text{cm}$

$AB = 3\,\text{cm}$



Steps of Construction

Step I Draw $AB = 3\text{cm}$.

Step II Draw a ray BX such that $\angle ABX = 90°$

Step III Draw an arc such that $AC = 5\text{cm}$.

Step IV With B as centre, draw an arc of radius 5 cm. With C as centre, draw an another arc of radius 3 cm which intersect first arc at a point, suppose D.

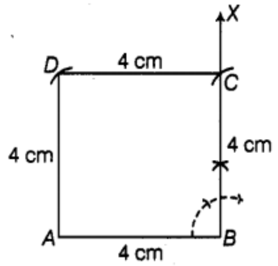
Step V Join CD and AD.

Thus, $ABCD$ is the required rectangle.

Question. 190 Construct a square of side 4 cm.

Solution.

We know that, all sides of a square are equal and each side is perpendicular to adjacent side.
So, $AB = BC = CD = DA = 4$ cm



Steps of Construction

Step I Draw $\overline{AB} = 4$cm.

Step II At B, draw $\overline{BX}$ such that $\angle ABX = 90°$.

Step III From $\overline{BX}$, cut-off $BC = 4$cm.

Step IV With centre C and radius = 4 cm, drawn an arc.

Step V With centre A and radius = 4 cm, draw another arc to intersect the previous arc at D.

Step VI Join DA and CD.

Thus, $ABCD$ is the required square.

Question. 191 Construct a rhombus CLUE in which CL = 7.5 cm and LE = 6 cm.

Solution.

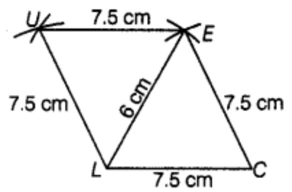
We know that, all sides of a rhombus are equal and opposite sides are parallel to each other.

Steps of Construction

Step I Draw a line segment $CL = 7.5$ cm.

Step II With C as a centre, draw an arc $CE = 7.5$ cm.

Step III With L as a centre, draw an another arc $LU = 7.5$ cm.



Step IV Now, with centre L, draw an arc $LE = 6$cm, which cut-off previous arc CE.

Step V With E as a centre, draw an arc $UE = 7.5$cm, which cut-off previous arc LU.

Step VI Now join UL, CE and EU.

Thus, we have required rhombus $CLUE$.

Question. 192 Construct a quadrilateral BEAR in which BE = 6 cm, EA = 7 cm, RB = RE = 5 cm and BA = 9 cm. Measure its fourth side.

Solution.

Steps of Construction

Step I Draw a line segment $BE = 6$ cm.

Step II With B as center, draw an arc $BR = 5$ cm and with E as a centre, draw an arc $EA = 7$ cm.

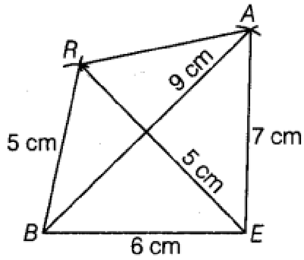
Step III Now, draw an another arc $BA = 9$ cm with B as a centre, which cut-off arc AE.

Step IV Draw an another arc $ER = 5$ cm with E as a centre, which cut-off arc BR.

Step V Now join BR, EA and AR.

Thus, we have required quadrilateral $BEAR$.

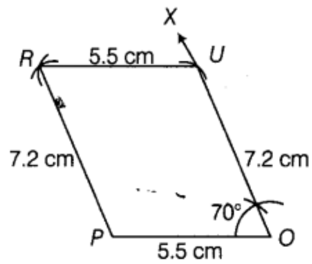
Also, $AR = 5$ cm



Question. 193 Construct a parallelogram POUR in which PO = 5.5 cm, OU = 7.2 cm and $\angle O$ = 70°.

Solution.

Since,



Since, opposite sides of a parallelogram are equal.

$\therefore PO = RU = 5.5$ cm, $OU = RP = 7.2$ cm

Steps of Construction

Step I Draw $PO = 5.5$ cm.

Step II Construct $\angle POX = 70°$.

Step III With O as centre and radius $OU = 7.2$ cm, draw an arc.

Step IV With U as centre and radius $UR = 5.5$ cm, draw an arc.

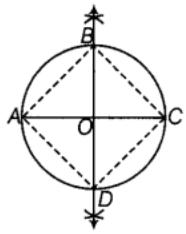
Step V With P as centre and radius $PR = 7.2$ cm, draw an arc to cut the arc drawn in Step IV.

Step VI Join PR and UR.

Hence, $POUR$ is the required parallelogram.

Question. 194 Draw a circle of radius 3 cm and draw its diameter and label it as AC. Construct its perpendicular bisector and let it intersect the circle at B and D. What type of quadrilateral is ABCD? Justify your answer.

Solution.



Steps of Construction

Step I Taking centre $OC = 3$ cm, draw a circle.

Step II Join A to C and draw a perpendicular bisector of AC that cuts the circumference of circle at B and D.

Step III Join B and D.

Step IV Thus, $ABCD$ is a cyclic quadrilateral.

Justification

In cyclic quadrilateral,

$\angle B = \angle D = 90°$ [angle in a semi-circle]

$\angle A = \angle C = 90°$

$\angle B + \angle D = 180°$

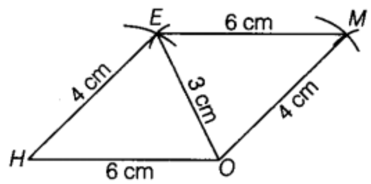
and $\angle A + \angle C = 180°$

[opposite angles are supplementary]

Since, opposite angles are supplementary, thus quadrilateral is a cyclic quadrilateral.

Question. 195 Construct a parallelogram HOME with HO = 6 cm, HE = 4 cm and OE = 3 cm.

Solution.



Steps of Construction

Step I Draw $HO = 6$cm.

Step II With H as centre and radius $HE = 4$cm, draw an arc.

Step III With O as centre and radius $OE = 3$cm, draw an arc, intersecting the arc drawn in step II at E.

Step IV With E as centre and radius $EM = 6$cm, draw an arc opposite to the side HE.

Step V With O as centre and radius $OM = 4$cm, draw an arc, intersecting the arc drawn in step IV at M.

Step VI Join HE, OE, EM and OM.

Hence, $HOME$ is the required parallelogram.

Question. 196 Is it possible to construct a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 5.4 cm, DA = 5.9 cm and diagonal AC = 8 cm? If not, why?

Solution. No,

Given measures are AS = 3 cm, SC = 4 cm,CD = 5.4 cm,

DA = 59cmand AC = 8cm

Here, we observe that AS + SC = 3 + 4 = 7 cm and AC = 8 cm

i.e. the sum of two sides of a triangle is less than the third side, which is absurd.

Hence, we cannot construct such a quadrilateral.

Question. 197 Is it possible to construct a quadrilateral ROAM in which RO = 4 cm, OA = 5 cm, $\angle O$ = 120°,$\angle R$ = 105° and $\angle A$ = 135°? If not, why?

Solution.

No,

Given measures are

$$OA = 5\text{cm}, \angle O = 120°, \angle R = 105° \text{ and } \angle A = 135°$$

Here, we see that, $\angle O + \angle R + \angle A = 120° + 105° + 135° = 360°$

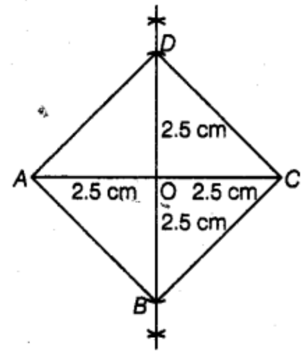
i.e. the sum of three angles of a quadrilateral is 360°.

This is impossible, as the total sum of angles is 360° in a quadrilateral.

Hence, this quadrilateral cannot be constructed.

Question. 198 Construct a square in which each diagonal is 5 cm long.

Solution.



Steps of Construction

Step I Draw $AC = 5\text{cm}$.

Step II With A as centre, draw arc of length slightly greater than $\frac{1}{2}AC$ above and below the line segment AC.

Step III With C as centre, draw an arc of same length as in step II above and below the line segment AC which intersect the arcs drawn in step II.

Step IV Join both the intersection points obtained in step III by a line segment which intersects AC at O (say).

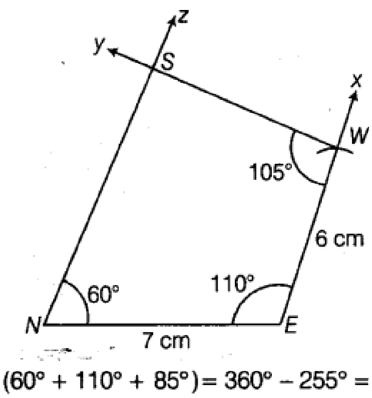
Step V With O as centre cut-off $OB = OD = 2.5\,\text{cm}$ along the bisector line.

Step VI Join AD, CD, AB and CB.

This is the required square $ABCD$.

Question. 199 Construct a quadrilateral NEWS in which NE = 7 cm, EW = 6 cm, $\angle N$ = 60°, $\angle E$= 110° and $\angle S$ = 85°

Solution.



Fourth angle $= 360° - (60° + 110° + 85°) = 360° - 255° = 105°$

Steps of Construction

Step I Draw $NE = 7\,\text{cm}$.

Step II Make $\angle NEX = 110°$.

Step III With E as centre and radius 6 cm, draw an arc, cutting EX at W.

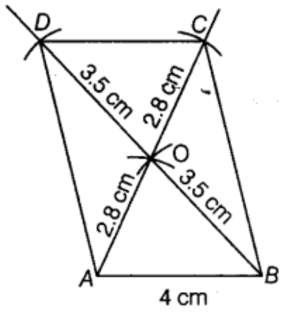
Step IV Make $\angle EWY = 105°$

Step V Make $\angle ENZ = 60°$, so that NZ and WY intersect each other at point S.

Thus, $NEWS$ is the required quadrilateral.

Question. 200 Construct a parallelogram when one of its side is 4 cm and its two diagonals are 5.6 cm and 7 cm. Measure the other side.

Solution.



Steps of Construction

Step I Draw $AB = 4$ cm.

Step II With A as centre and radius 2.8 cm, draw an arc.

Step III With B as centre and radius 3.5 cm, draw another arc cutting the previous arc at O.

Step IV Join OA and OB.

Step V Produce AO to C such that $OC = AO$ and produce BO to D such that $OD = BD$.

Step VI Join AD, BC and CD.

Thus, $ABCD$ is the required parallelogram.

and other side = 5 cm.

Question. 201 Find the measure of each angle of a regular polygon of 20 sides?

Solution.

We know that, the sum of interior angles of an n polygon $= (n - 2) \times 180°$

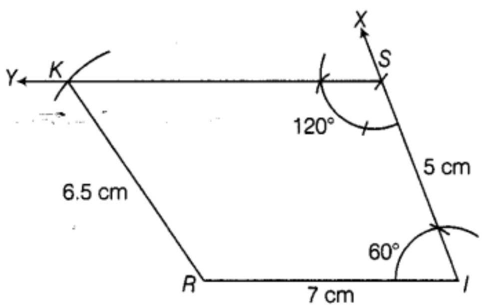
Here, $n = 20$, then

Sum $= (20 - 2) \times 180° = 18 \times 180° = 3240°$

$\therefore$ The measure of each interior angle $= \dfrac{3240}{20} = 162°$

Question. 202 Construct a trapezium RISK in which RI || KS, RI = 7 cm, IS = 5 cm, RK = 6.5 cm and $\angle I$ = 60°.

Solution.



$\angle I + \angle S = 180°$

$60° + \angle S = 180°$ [cointerior angles]

$\angle S = 120°$

Steps of Construction

Step I Draw an arc $RI = 7$ cm.

Step II Make $\angle RIX = 60°$.

Step III With I as centre and radius 5 cm, draw an arc cutting IX at S.

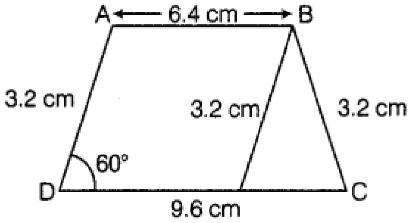
Step IV Make $\angle ISY = 120°$

Step V With R as centre and radius 6.5 cm, draw an arc cutting SY at K.

Step VI Join KR.

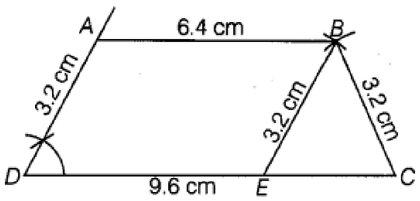
Thus, $RISK$ is the required trapezium.

Question. 203 Construct a trapezium ABCD, where AB|| CD, AD = BC = 3.2 cm, AB = 6.4 cm and CD = 9.6 cm. Measure $\angle B$ and $\angle A$



[Hint Difference of two parallel sides gives an equilateral triangle.]

Solution.



Steps of Construction

***Step* I** Draw a line segment $DC = 9.6$ cm.

***Step* II** With D as center, draw an angle measure 60°. Now, cut-off it with an arc 3.2 cm called point A.

***Step* III** Now, draw a parallel AB to CD.

***Step* IV** Talking C as center, cut an arc B measure 3.2 cm on previous parallel line.

***Step* V** Draw a line segment $BE = 3.2$ cm from arc B.

***Step* VI** Join B to E and C.

Thus, we have required trapezium $ABCD$ in which $\angle A = 120°$ and $\angle B = 60°$.