EXERCISE 1.1

1. Every rational number is
(*a*) a natural number (*b*) an integer
(*c*) a real number (*d*) a whole number

Sol. We know that rational and irrational numbers taken together are known as real numbers. Therefore, every real number is either a rational number or an irrational number. Hence, every rational number is a real number.
Therefore, (*c*) is the correct answer.

2. Between two rational numbers
(*a*) there is no rational number
(*b*) there is exactly one rational number
(*c*) there are infinitely many rational numbers
(*d*) there are only rational numbers and no irrational numbers

Sol. Between two rational numbers there are infinitely many rational numbers.
Hence, (*c*) is the correct answer.

3. Decimal representation of a rational number cannot be
(*a*) terminating
(*b*) non-terminating
(*c*) non-terminating repeating
(*d*) non-terminating non-repeating

Sol. The decimal representation of a rational number cannot be non-terminating and non-repeating.
Hence, (*d*) is the correct answer.

4. The product of any two irrational numbers is
(*a*) always an irrational number
(*b*) always a rational number
(*c*) always an integer
(*d*) sometimes rational, sometimes irrational

Sol. The product of any two irrational numbers is sometimes rational and sometimes irrational.
Hence, (*d*) is the correct answer.

5. The decimal expansion of the number $\sqrt{2}$ is

(*a*) a finite decimal

(*b*) 1.41421

(*c*) non-terminating recurring

(*d*) non-terminating non-recurring

Sol. The decimal expansion of the number $\sqrt{2}$ is 1.41421.....

6. Which of the following is irrational?

(*a*) $\sqrt{\frac{4}{9}}$ (*b*) $\frac{\sqrt{12}}{\sqrt{3}}$ (*c*) $\sqrt{7}$ (*d*) $\sqrt{81}$

Sol. (*a*) $\sqrt{\frac{9}{4}} = \frac{3}{2}$, which is a rational number.

(*b*) $\frac{\sqrt{12}}{\sqrt{3}} = \frac{\sqrt{4 \times 3}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3}} = 2$, which is a rational number.

(*c*) $\sqrt{7}$ is an irrational number.

(*d*) $\sqrt{81} = 9$, which is a rational number.

Hence, (*c*) is the correct answer.

7. Which of the following is irrational?

(*a*) 0.14 (*b*) $0.14\overline{16}$

(*c*) $0.\overline{1416}$ (*d*) 0.4014001400014...

Sol. A number is irrational if and only if its decimal representation is non-terminating and non-recurring.

(*a*) 0.14 is a terminating decimal and therefore cannot be an irrational number.

(*b*) $0.14\overline{16}$ is a non-terminating and recurring decimal and therefore cannot be irrational.

(*c*) $0.\overline{1416}$ is a non-terminating and recurring decimal and therefore cannot be irrational.

(*d*) 0.4014001400014.... is a non-terminating and non-recurring decimal and therefore is an irrational number.

Hence, (*d*) is the correct answer.

8. A rational number between $\sqrt{2}$ and $\sqrt{3}$ is

(*a*) $\frac{\sqrt{2}+\sqrt{3}}{2}$ (*b*) $\frac{\sqrt{2}\cdot\sqrt{3}}{2}$

(*c*) 1.5 (*d*) 1.8

Sol. We know that

$\sqrt{2} = 1.4142135....$ and $\sqrt{3} = 1.732050807...$

We see that 1.5 is a rational number which lies between 1.4142135..... and 1.732050807.....

Hence, (*c*) is the correct answer.

9. The value of 1.999... in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is

(*a*) $\frac{19}{10}$ (*b*) $\frac{1999}{1000}$ (*c*) 2 (*d*) $\frac{1}{9}$

Sol. Let $x = 1.999.... = 1.\overline{9}$ . ...(1)

Then, $10x = 19.999.... = 19.\overline{9}$ ...(2)

Subtracting (1) from (2), we get

$$9x = 18 \Rightarrow x = 18 \div 9 = 2$$

$\therefore$ The value of 1.999... in the form $\frac{p}{q}$ is 2 or $\frac{2}{1}$.

Hence, (*c*) is the correct answer.

10. $2\sqrt{3} + \sqrt{3}$ is equal to

(*a*) $2\sqrt{6}$ (*b*) 6 (*c*) $3\sqrt{3}$ (*d*) $4\sqrt{6}$

Sol. Given $2\sqrt{3} + \sqrt{3} = (2+1)\sqrt{3} = 3\sqrt{3}$

Hence, (*c*) is the correct answer.

11. $\sqrt{10} \times \sqrt{15}$ is equal to

(*a*) $6\sqrt{5}$ (*b*) $5\sqrt{6}$ (*c*) $\sqrt{25}$ (*d*) $10\sqrt{5}$

Sol. We have $\sqrt{10} \times \sqrt{15} = \sqrt{10 \times 15} = \sqrt{5 \times 2 \times 5 \times 3} = 5\sqrt{6}$

Hence, (*b*) is the correct answer.

12. The number obtained on rationalising the denominator of $\frac{1}{\sqrt{7}-2}$ is

(*a*) $\frac{\sqrt{7}+2}{3}$ (*b*) $\frac{\sqrt{7}-2}{3}$ (*c*) $\frac{\sqrt{7}+2}{5}$ (*d*) $\frac{\sqrt{7}+2}{45}$

Sol. $$\frac{1}{7-\sqrt{2}} = \frac{1}{7-\sqrt{2}} \times \frac{7+\sqrt{2}}{7+\sqrt{2}} = \frac{7+\sqrt{2}}{(7)^2 - (\sqrt{2})^2} = \frac{7+\sqrt{2}}{49-2} = \frac{7+\sqrt{2}}{47}$$

Hence, (*d*) is the correct answer.

13. $\dfrac{1}{\sqrt{9}-\sqrt{8}}$ is equal to

(*a*) $\frac{1}{2}(3-2\sqrt{2})$ (*b*) $\dfrac{1}{3+2\sqrt{2}}$ (*c*) $3-2\sqrt{2}$ (*d*) $3+2\sqrt{2}$

Sol.

$$\frac{1}{\sqrt{9}-\sqrt{8}} = \frac{1}{\sqrt{3\times 3}-\sqrt{4\times 2}} = \frac{1}{3-2\sqrt{2}}$$

$$= \frac{1}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}} = \frac{3+2\sqrt{2}}{(3)^2-\left(2\sqrt{2}\right)^2}$$

$$= \frac{3+2\sqrt{2}}{9-8} = \frac{3+2\sqrt{2}}{1} = 3+2\sqrt{2}$$

Hence, (*d*) is the correct answer.

14. After rationalising the denominator of $\dfrac{7}{3\sqrt{3}-2\sqrt{2}}$, we get the denominator as

(*a*) 13 (*b*) 19 (*c*) 5 (*d*) 35

Sol.

$$\frac{7}{3\sqrt{3}-2\sqrt{2}} = \frac{7}{3\sqrt{3}-2\sqrt{2}} \times \frac{3\sqrt{3}+2\sqrt{2}}{3\sqrt{3}+2\sqrt{2}}$$

$$= \frac{7(3\sqrt{3}+2\sqrt{2})}{(3\sqrt{3})^2-(2\sqrt{2})^2} = \frac{7(3\sqrt{3}+2\sqrt{2})}{27-8}$$

$$= \frac{7(3\sqrt{3}+2\sqrt{2})}{19}$$

Therefore, we get the denominator as 19.
Hence, (*b*) is the correct answer.

15. The value of $\dfrac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}}$ is equal to

(*a*) $\sqrt{2}$ (*b*) 2 (*c*) 4 (*d*) 8

Sol.

$$\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}} = \frac{\sqrt{16\times 2}+\sqrt{16\times 3}}{\sqrt{4\times 2}+\sqrt{4\times 3}}$$

$$= \frac{4\sqrt{2}+4\sqrt{3}}{2\sqrt{2}+2\sqrt{3}} = \frac{4(\sqrt{2}+\sqrt{3})}{2(\sqrt{2}+\sqrt{3})} = \frac{4}{2} = 2$$

Hence, (*b*) is the correct answer.

16. If $\sqrt{2} = 1.4142$, then $\sqrt{\dfrac{\sqrt{2}-1}{\sqrt{2}+1}}$ is equal to

(*a*) 2.4142 (*b*) 5.8282 (*c*) 0.4142 (*d*) 0.1718

Sol. $$\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = \sqrt{\frac{(\sqrt{2}-1)\times(\sqrt{2}-1)}{(\sqrt{2}+1)\times(\sqrt{2}-1)}}$$

$$= \sqrt{\frac{(\sqrt{2}-1)^2}{(\sqrt{2})^2-1^2}} = \frac{\sqrt{2}-1}{\sqrt{2-1}} = \frac{\sqrt{2}-1}{1} = \sqrt{2}-1$$

$$= 1.4142 - 1 = 0.4142$$

Hence, (*c*) is the correct answer.

17. $\sqrt[4]{\sqrt[3]{2^2}}$ equals

(*a*) $2^{\frac{1}{6}}$ (*b*) 2^{-6} (*c*) $2^{1/6}$ (*d*) 2^6

Sol. $$\sqrt[4]{\sqrt[3]{2^2}} = \sqrt[4]{(2^2)^{\frac{1}{3}}} = \left(2^{\frac{2}{3}}\right)^{\frac{1}{4}} = 2^{\frac{2}{3}\times\frac{1}{4}} = 2^{\frac{1}{6}}$$

Hence, (*c*) is the correct answer.

18. The product $\sqrt[3]{2}\cdot\sqrt[4]{2}\cdot\sqrt[12]{32}$ equals

(*a*) $\sqrt{2}$ (*b*) 2 (*c*) $\sqrt[12]{2}$ (*d*) $\sqrt[12]{32}$

Sol. We have,

$$\sqrt[3]{2}.\sqrt[4]{2}.\sqrt[12]{32} = 2^{\frac{1}{3}} \times 2^{\frac{1}{4}} \times (2^5)^{\frac{1}{12}} = 2^{\frac{1}{3}} \times 2^{\frac{1}{4}} \times 2^{\frac{5}{12}}$$

$$= 2^{\frac{1}{3}+\frac{1}{4}+\frac{5}{12}} = 2^{\frac{4+3+5}{12}} = 2^{\frac{12}{12}} = 2^1 = 2$$

Hence, (*b*) is the correct answer.

19. Value of $\sqrt[4]{(81)^{-2}}$ is

(*a*) $\frac{1}{9}$ (*b*) $\frac{1}{3}$ (*c*) 9 (*d*) $\frac{1}{81}$

Sol. $$\sqrt[4]{(81)^{-2}} = \sqrt[4]{\left(\frac{1}{81}\right)^2} = \sqrt[4]{\left\{\left(\frac{1}{9}\right)^2\right\}^2} = \sqrt[4]{\left(\frac{1}{9}\right)^4} = \left(\frac{1}{9}\right)^{4\times\frac{1}{4}} = \frac{1}{9}$$

Hence, (*a*) is the correct answer.

20. Value of $(256)^{0.16} \times (256)^{0.09}$ is

(*a*) 4 (*b*) 16 (*c*) 64 (*d*) 256.25

Sol. $(256)^{0.16} \times (256)^{0.09} = (256)^{0.16+0.09}$

$= (256)^{0.25} = (256)^{\frac{1}{4}} = (4^4)^{\frac{1}{4}} = 4^{4\times\frac{1}{4}} = 4$

Hence, (*a*) is the correct answer.

21. Which of the following is equal to x?

(*a*) $x^{\frac{12}{7}} - x^{\frac{5}{7}}$ (*b*) $\sqrt[12]{(x^4)^{\frac{1}{3}}}$ (*c*) $\left(\sqrt{x^3}\right)^{\frac{2}{3}}$ (*d*) $x^{\frac{12}{7}} \times x^{\frac{7}{12}}$

Sol. (*a*) $x^{\frac{12}{7}} - x^{\frac{5}{7}} \neq x$

(*b*) $\sqrt[12]{(x^4)^{\frac{1}{3}}} = \sqrt[12]{x^{4\times\frac{1}{3}}} = \left(x^{\frac{4}{3}}\right)^{\frac{1}{12}} = x^{\frac{4}{3}\times\frac{1}{12}} = x^{\frac{1}{9}} \neq x$

(*c*) $((x^3)^{\frac{1}{2}})^{\frac{2}{3}} = (x)^{\frac{3}{2}\times\frac{2}{3}} = x^1 = x$

(*d*) $x^{\frac{12}{7}} \times x^{\frac{7}{12}} = x^{\frac{12}{7}+\frac{7}{12}} = x^{\frac{193}{84}} \neq x$

Hence, (*c*) is the correct answer.

EXERCISE 1.2

1. Let x and y be rational and irrational numbers, respectively. Is $x + y$ necessarily an irrational number? Give an example in support of your answer.

Sol. Yes, $x + y$ is necessary an irrational number.

Let $x = 5$ and $y = \sqrt{2}$.

Then, $x + y = 5 + \sqrt{2} = 5 + 1.4142.... = 6.4142....$ which is non-terminating and non-repeating.

Hence, $x + y$ is an irrational number.

2. Let x be rational and y be irrational. Is xy necessarily irrational? Justify your answer by an example.

Sol. Let $x = 0$ (a rational number) and $y = \sqrt{3}$ be an irrational number. Then, $xy = 0\,(\sqrt{3}) = 0$, which is not an irrational number.

Hence, xy is not necessarily an irrational number.

3. State whether the following statements are true or false. Justify your answer.

(*i*) $\frac{\sqrt{2}}{3}$ is a rational number.

(*ii*) There are infinitely many integers between any two integers.

(*iii*) Number of rational numbers between 15 and 18 is finite.

(*iv*) There are numbers which cannot be written in the form $\frac{p}{q}$, $q \neq 0$, p, q both are integers.

(*v*) The square of an irrational number is always rational.

(*vi*) $\frac{\sqrt{12}}{\sqrt{3}}$ is not a rational number as $\sqrt{12}$ and $\sqrt{3}$ are not integers.

(*vii*) $\frac{\sqrt{15}}{\sqrt{3}}$ is written in the form $\frac{p}{q}$, where $q \neq 0$ and so it is a rational number.

Sol. (*i*) The given statement is false. $\frac{\sqrt{2}}{3}$ is of the form $\frac{p}{q}$ but $p = \sqrt{2}$ is not an integer.

(*ii*) The given statement is false. Consider two integers 3 and 4. There is no integer between 3 and 4.

(*iii*) The given statement is false. There lies infinitely many rational numbers between any two rational numbers. Hence, number of rational numbers between 15 and 18 are infinite.

(*iv*) The given statement is true. For example, $\frac{\sqrt{3}}{\sqrt{5}}$ is of the form $\frac{p}{q}$ but $p = \sqrt{3}$ and $q = \sqrt{5}$ are not integers.

(*v*) The given statement is false. Consider an irrational number $\sqrt[4]{2}$.

Then, its square $\left(\sqrt[4]{2}\right)^2 = \sqrt{2}$, which is not a rational number.

(*vi*) The given statement is false. $\sqrt{\frac{12}{3}} = \sqrt{4} = 2$, which is a rational number.

(*vii*) The given statement is false. $\frac{\sqrt{15}}{\sqrt{3}} = \sqrt{\frac{15}{3}} = \sqrt{5} = \frac{\sqrt{5}}{1}$, where $p = \sqrt{5}$ is an irrational number.

4. Classify the following numbers as rational or irrational with justification.

(*i*) $\sqrt{196}$ (*ii*) $3\sqrt{18}$ (*iii*) $\sqrt{\frac{9}{27}}$ (*iv*) $\frac{\sqrt{28}}{\sqrt{343}}$

(*v*) $-\sqrt{0.4}$ (*vi*) $\frac{\sqrt{12}}{\sqrt{75}}$ (*vii*) 0.5918 (*viii*) $(1+\sqrt{5})-(4+\sqrt{5})$ (*ix*) 10.124124...

(*x*) 1.010010001....

Sol. (*i*) $\sqrt{196}=14$, which is a rational number.

(*ii*) $3\sqrt{18}=3\sqrt{9\times 2}=3\times 3\sqrt{2},=9\sqrt{2}$, which is the product of a rational and an irrational number.

Hence, $3\sqrt{18}$ is an irrational number.

(*iii*) $\sqrt{\frac{9}{27}}=\frac{1}{\sqrt{3}}$, which is the quotient of a rational and an irrational number and therefore an irrational number.

(*iv*) $\frac{\sqrt{28}}{\sqrt{343}}=\sqrt{\frac{4}{49}}=\frac{2}{7}$, which is a rational number.

(*v*) $-\sqrt{0.4}=-\frac{2}{\sqrt{10}}$, which is the quotient of a rational and an irrational number and so it is an irrational number.

(*vi*) $\frac{\sqrt{12}}{\sqrt{75}}=\sqrt{\frac{4}{25}}=\frac{2}{5}$, which is a rational number.

(*vii*) 0.5918 is a terminating decimal expansion.
Hence, it is a rational number.

(*viii*) $\left(1+\sqrt{5}\right)-\left(4+\sqrt{5}\right)=-3$, which is a rational number.

(*ix*) 10.124124...... is a decimal expansion which is non-terminating recurring. Hence, it is a rational number.

(*x*) 1.010010001..... is a decimal expansion which is non-terminating non-recurring.

Hence, it is an irrational number.

EXERCISE 1.3

1. Find which of the variables x, y, z and u represent rational numbers and which irrational numbers:

(*i*) $x^2 = 5$ (*ii*) $y^2 = 9$ (*iii*) $z^2 = .04$ (*iv*) $u^2 = \frac{17}{4}$

Sol. (*i*) $x^2 = 5 \Rightarrow x = \sqrt{5}$, which is an irrational number.

(*ii*) $y^2 = 9 \Rightarrow y = \sqrt{9} = 3$, which is a rational number.

(*iii*) $z^2 = .04 \Rightarrow z = \sqrt{.04} = 0.2$, which is a terminating decimal. Hence, it is rational number.

(*iv*) $u^2 = \frac{17}{4} \Rightarrow u = \sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{2}$, which is of the form $\frac{p}{q}$, where $p = \sqrt{17}$ is not an integer.

Hence, u is an irrational number.

2. Find three rational numbers between

(*i*) – 1 and – 2 (*ii*) 0.1 and 0.11

(*iii*) $\frac{5}{7}$ and $\frac{6}{7}$ (*iv*) $\frac{1}{4}$ and $\frac{1}{5}$

Sol. (*i*) – 1.1, – 1.2, – 1.3 (terminating decimals) are three rational numbers lying between – 1 and – 2.

(*ii*) 0.101, 0.102, 0.103 (terminating decimals) are three rational numbers which lie between 0.1 and 0.11.

(*iii*) $\frac{5}{7} = \frac{5}{7} \times \frac{10}{10} = \frac{50}{70}$ and $\frac{6}{7} = \frac{6}{7} \times \frac{10}{10} = \frac{60}{70}$

$\Rightarrow \frac{51}{70}, \frac{52}{70}, \frac{53}{70}$ are three rational numbers lying between $\frac{50}{70}$ and $\frac{60}{70}$ and therefore lie between $\frac{5}{7}$ and $\frac{6}{7}$.

(*iv*) $\frac{1}{4} = \frac{1}{4} \times \frac{20}{20} = \frac{20}{80}$ and $\frac{1}{5} = \frac{1}{5} \times \frac{16}{16} = \frac{16}{80}$

Now, $\sqrt{2} \times \sqrt{3}\ \frac{18}{80} \left(= \frac{9}{40}\right), \frac{19}{80}$ are three rational numbers lying between $\frac{1}{4}$ and $\frac{1}{5}$.

3. Insert a rational number and an irrational number between the following:

(*i*) 2 and 3 (*ii*) 0 and 0.1 (*iii*) $\frac{1}{3}$ and $\frac{1}{2}$

(*iv*) $\frac{-2}{5}$ and $\frac{1}{2}$ (*v*) 0.15 and 0.16 (*vi*) $\sqrt{2}$ and $\sqrt{3}$

(*vii*) 2.357 and 3.121 (*viii*) 0.0001 and 0.001

(*ix*) 3.623623 and 0.484848

(*x*) 6.375289 and 6.375738

Sol. (*i*) A rational number between 2 and 3 is $\frac{2+3}{2} = \frac{5}{2} = 2.5$.

Also, 2.1 (terminating decimal) is a rational between 2 and 3.

Again, 2.010010001... (a non-terminating and non-recurring decimal) is an irrational number between 2 and 3.

(*ii*) 0.04 is a terminating decimal and also it lies between 0 and 0.1.

Hence, 0.04 is a rational number which lies between 0 and 0.1.

Again, 0.003000300003 ..... is a non-terminating and non-recurring decimal which lies between 0 and 0.1.

Hence, 0.003000300003 .... is an irrational number between 0 and 0.1.

(*iii*) $\frac{1}{3} = \frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$ and $\frac{1}{2} = \frac{1}{2} \times \frac{6}{6} = \frac{6}{12}$

Now, $\frac{5}{12}$ is a rational number between $\frac{4}{12}$ and $\frac{6}{12}$. So, $\frac{5}{12}$ is a rational number lying between $\frac{1}{3}$ and $\frac{1}{2}$.

Again, $\frac{1}{3} = 0.33333...$ and $\frac{1}{2} = 0.5$.

Now, 0.414114111.... is a non-terminating and non-recurring decimal.

Hence, 0.414114111..... is an irrational number lying between $\frac{1}{3}$ and $\frac{1}{2}$.

(*iv*) $\frac{-2}{5} = -0.4$ and $\frac{1}{2} = 0.5$

Now, 0 is a rational number between – 0.4 and 0.5 *i.e.,* 0 is a rational number lying between $\frac{-2}{5}$ and $\frac{1}{2}$.

Again, 0.131131113..... is a non-terminating and non-recurring decimal which lies between – 0.4 and 0.5.

Hence, 0.131131113..... is an irrational number lying between $\frac{-2}{5}$ and $\frac{1}{2}$.

(*v*) 0.151 is a rational number between 0.15 and 0.16. Similarly, 0.153 , 0.157, etc. are rational numbers lying between 0.15 and 0.16.

Again, 0.151151115.... (a non-terminating and non-recurring decimal) is an irrational number between 0.15 and 0.16.

(*vi*) $\sqrt{2} = 1.4142135....$ and $\sqrt{3} = 1.732050807...$

Now, 1.5 (a terminating decimal) which lies between 1.4142135.... and 1.732050807.....

Hence, 1.5 is a rational number between $\sqrt{2}$ and $\sqrt{3}$.

Again, 1.575575557..... (a non-terminating and non-recurring decimal) is an irrational number lying between $\sqrt{2}$ and $\sqrt{3}$.

(*vii*) 3 is a rational number between 2.357 and 3.121.

Again, 3.101101110... (a non-terminating and non-recurring decimal) is an irrational number lying between 2.357 and 3.121.

(*viii*) 0.00011 is a rational number between 0.0001 and 0.001.

Again, 0.0001131331333....... (a non-terminating and non-recurring decimal) is an irrational number between 0.0001 and 0.001.

(*ix*) 1 is a rational number between 0.484848 and 3.623623.

Again, 1.909009000..... (a non-terminating and non-recurring decimal) is an irrational number lying between 0.484848 and 3.623623.

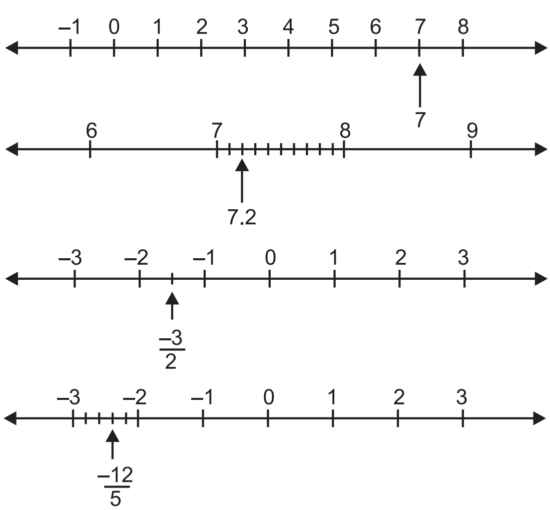
(*x*) 6.3753 (a terminating decimal) is a rational number between 6.375289 and 6.375738.

Again, 6.375414114111....(a non-terminating and non-recurring decimal) is an irrational number lying between 6.375289 and 6.375738.

4. Represent the following numbers on the number line :

$$7, 7.2, \frac{-3}{2}, \frac{-12}{5}$$

Sol.



5. Locate $\sqrt{5}$, $\sqrt{10}$ and $\sqrt{17}$ on the number line.

Sol. Presentation of $\sqrt{5}$ on number line:

We write 5 as the sum of the squares of two natural numbers:

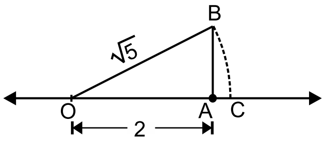
$$5 = 1 + 4 = 1^2 + 2^2$$

On the number line, take OA = 2 units.

Draw BA = 1 unit, perpendicular to OA. Join OB.

By Pythagoras theorem, OB= $\sqrt{5}$

Using a compass with centre O and radius OB, draw an arc which intersects the number line at the point C. Then, C crossponds to $\sqrt{5}$.



Presentation of $\sqrt{10}$ on the number line:

We write 10 as the sum of the squares of two natural numbers:

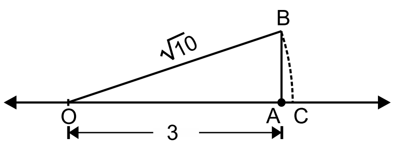
$$10 = 1 + 9 = 1^2 + 3^2$$

On the number line, take OA = 3 units.

Draw BA = 1 unit, perpendicular to OA. Join OB.

By Pythagoras theorem, OB= $\sqrt{10}$

Using a compass with centre O and radius OB, draw an arc which intersects the number line at the point C. Then, C corresponds to $\sqrt{10}$.



Presentation of $\sqrt{17}$ on the number line:

We write 17 as the sum of the squares of two natural numbers:

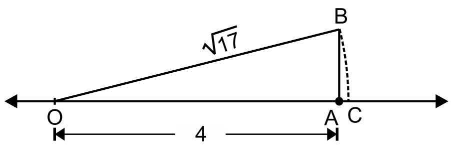
$$17 = 1 + 16 = 1^2 + 4^2$$

On the number line, take OA = 4 units.

Draw BA = 1 unit, perpendicular to OA. Join OB.

By Pythagoras theorem, OB= $\sqrt{17}$

Using a compass with centre O and radius OB, draw an arc which intersects the number line at the point C. Then, C corresponds to $\sqrt{17}$.

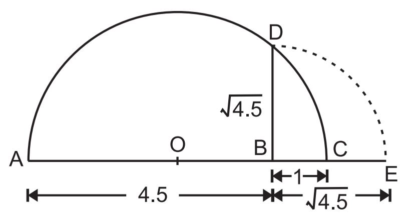


6. Represent geometrically the following numbers on the number line :

(*i*) $\sqrt{4.5}$ (*ii*) $\sqrt{5.6}$ (*iii*) $\sqrt{8.1}$ (*iv*) $\sqrt{2.3}$

Sol. (*i*) $\sqrt{4.5}$

Presentation of $\sqrt{4.5}$ on number line:



Mark the distance 4.5 units from a fixed point A on a given line to obtain a point B such that AB = 4.5 units. From B, mark a distance of 1 unit and mark the new point as C.

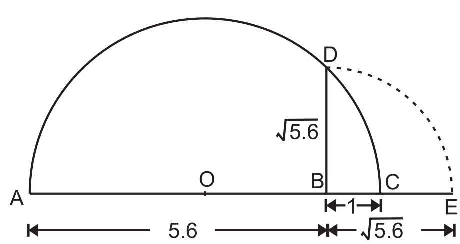
Find the mid-point of AC and mark that point as O. Draw a semicircle with centre O and radius OC. Draw a line perpendicular to AC passing through B and intersecting the semicircle at D. Then, BD = $\sqrt{4.5}$.

Now, draw an arc with centre B and radius BD, which intersects the number line in E.

Thus, E represents $\sqrt{4.5}$.

(*ii*) $\sqrt{5.6}$

Presentation of $\sqrt{5.6}$ on number line:



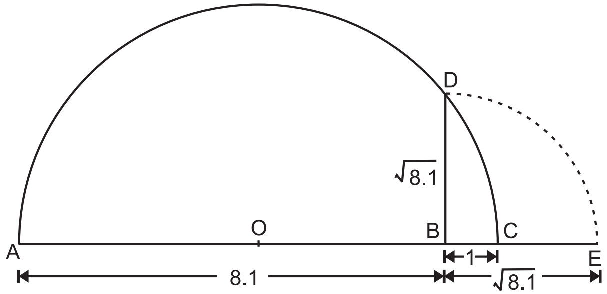
Mark the distance 5.6 units from a fixed point A on a given line to obtain a point B such that AB = 5.6 units. From B, mark a distance of 1 unit and mark the new point as C. Find the mid-point of AC and mark that point as O. Draw a semicircle with centre O and radius OC. Draw a line perpendicular to AC passing through B and intersecting the semicircle at D. Then, BD = $\sqrt{5.6}$.

Now, draw an arc with centre B and radius BD, which intersects the number line in E.

Thus, E represents $\sqrt{5.6}$.

(*iii*) $\sqrt{8.1}$

Presentation of $\sqrt{8.1}$ on number line:

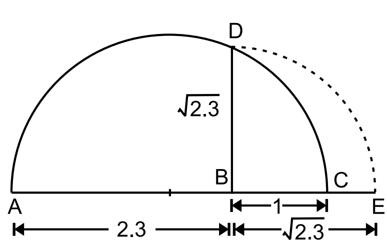


Mark the distance 8.1 units from a fixed point A on a given line to obtain a point B such that AB = 8.1 units. From B, mark a distance of 1 unit and mark the new point as C. Find the mid-point of AC and mark that point as O. Draw a semicircle with centre O and radius OC. Draw a line perpendicular to AC passing through B and intersecting the semicircle at D. Then, BD = $\sqrt{8.1}$.

Now, draw an arc with centre B and radius BD, which intersects the number line in E. Thus, E represents $\sqrt{8.1}$.

(*iv*) $\sqrt{2.3}$

Presentation of $\sqrt{2.3}$ on number line:



Mark the distance 2.3 units from a fixed point A on a given line to obtain a point B such that AB = 2.3 units. From B, mark a distance of 1 unit and mark the new point as C. Find the mid-point of AC and mark that point as O. Draw a semicircle with centre O and radius OC. Draw a line perpendicular to AC passing through B and intersecting the semicircle at D. Then, BD = $\sqrt{2.3}$.

Now, draw an arc with centre B and radius BD, which intersects the number line in E. Thus, E represents $\sqrt{2.3}$.

7. Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(*i*) 0.2 (*ii*) 0.888... (*iii*) $5.\overline{2}$ (*iv*) $0.\overline{001}$

(*v*) 0.2555.... (*vi*) $0.1\overline{34}$ (*vii*) .00323232...

(*viii*) 0.404040....

Sol.(*i*) $0.2 = \frac{2}{10} = \frac{1}{5}$.

(*ii*) Let $x = 0.888... = 0.\overline{8}$. ...(1)

$\therefore$ $10x = 8.\overline{8}$ ...(2)

Subtracting (1) from (2), we get

$9x = 8$

Hence, $x = \frac{8}{9}$

(*iii*) Let $x = 5.\overline{2} = 5.2222...$ ...(1)

Multiplying both sides by 10, we get

$10x = 52.222... = 52.\overline{2}$ ...(2)

If we subtract $5.\overline{2}$ from $52.\overline{2}$, the repeating portion of the decimal cancels out.

$\therefore$ Subtracting (1) from (2), we get

$$10x - x = 47 \Rightarrow 9x = 47 \Rightarrow x = \frac{47}{9}$$

Hence, $5.\overline{2} = \frac{47}{9}$.

(*iv*) Let $x = 0.\overline{001} = 0.001001.$ ...(1)

$\therefore$ $1000x = 1.001001...$ ...(2)

Subtracting (1) from (2), we get

$$999x = 1$$

Hence, $x = \frac{1}{999}$.

(*v*) Let $x = 0.2555... = 0.2\overline{5}$. ...(1)

$\therefore$ $10x = 2.\overline{5}...$ ...(2)

and $100x = 25.\overline{5}$ ...(3)

Subtracting (2) from (3), we get

$$90x = 23$$

$$\therefore \quad x = \frac{23}{90}$$

(*vi*) Let $x = 0.1\overline{34} = 0.1343434...$ ...(1)

Multiplying both sides by 100, we get

$100x = 13.43434... = 13.4\overline{34}$ ...(2)

If we subtract $0.1\overline{34}$ from $13.4\overline{34}$, the repeating portion of the decimal cancels out.

$\therefore$ Subtracting (1) from (2), we get

$$100x - x = 13.3 \Rightarrow 99x = \frac{133}{10} \Rightarrow x = \frac{133}{990}$$

Hence, $0.1\overline{34} = \frac{133}{990}$.

(*vii*) Let $x = 0.00323232.... = 0.00\overline{32}$. ...(1)

$\therefore$ $100x = 0.\overline{32}$ ...(2)

and $10{,}000x = 32.\overline{32}$ ...(3)

Subtracting (2) from (3), we get

$$9900x = 32$$

$$\therefore \quad x = \frac{32}{9900} = \frac{8}{2475}$$

(*viii*) Let $x = 0.404040.... = 0.\overline{40}$. ...(1)

$\therefore$ $100x = 40.\overline{40}$ ...(2)

Subtracting (1) from (2), we get

$$99x = 40$$

$$\therefore \quad x = \frac{40}{99}$$

8. Show that $0.142857142857...... = \frac{1}{7}$.

Sol. Let $x = 0.142857142857...$ ...(1)

$\therefore$ $1000000x = 142857.\overline{142857}$ ...(2)

Subtracting (1) from (2), we get

$$999999x = 142857 \quad ...(3)$$

$$\Rightarrow \quad x = \frac{142875}{999999} = \frac{1}{7}$$

Hence, $0.142857142857.... = \frac{1}{7}$.

9. Simplify:

(*i*) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$ (*ii*) $\frac{\sqrt{24}}{8} + \frac{\sqrt{54}}{9}$

(*iii*) $\sqrt[4]{12} \times \sqrt[6]{7}$ (*iv*) $4\sqrt{28} \div 3\sqrt{7}$

(*v*) $3\sqrt{3} + 2\sqrt{27} + \frac{7}{\sqrt{3}}$ (*vi*) $(\sqrt{3} - \sqrt{2})^2$

(*vii*) $\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$

(*viii*) $\frac{3}{\sqrt{8}} + \frac{1}{\sqrt{2}}$ (*ix*) $\frac{2\sqrt{3}}{3} - \frac{\sqrt{3}}{6}$

Sol. (*i*) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5} = \sqrt{9 \times 5} - 3\sqrt{4 \times 5} + 4\sqrt{5}$

$$= 3\sqrt{5} - 3 \times 2\sqrt{5} + 4\sqrt{5} = (3 - 6 + 4)\sqrt{5} = \sqrt{5}$$

(*ii*) $\frac{\sqrt{24}}{8} + \frac{\sqrt{54}}{9} = \frac{\sqrt{4 \times 6}}{8} + \frac{\sqrt{9 \times 6}}{9} = \frac{2\sqrt{6}}{8} + \frac{3\sqrt{6}}{9} = \frac{\sqrt{6}}{4} + \frac{\sqrt{6}}{3}$

$$= \sqrt{6}\left(\frac{1}{4} + \frac{1}{3}\right) = \sqrt{6}\left(\frac{3+4}{12}\right) = \frac{7\sqrt{6}}{12}$$

(*iii*) $4\sqrt{12} \times 7\sqrt{6} = 4\sqrt{2 \times 2 \times 3} \times 7\sqrt{2 \times 3}$

$$= 8\sqrt{3} \times 7\sqrt{2} \times \sqrt{3}$$

$$= 24 \times 7\sqrt{2} = 168\sqrt{2}$$

(*iv*) $4\sqrt{28} \div 3\sqrt{7} = 4\sqrt{2 \times 2 \times 7} \times \frac{1}{3\sqrt{7}}$

$$= \frac{8\sqrt{7}}{3\sqrt{3}} = \frac{8}{3}$$

(*v*) $3\sqrt{3} + 2\sqrt{27} + \frac{1}{\sqrt{3}}$

$$= 3\sqrt{3} + 2 \times 3\sqrt{3} + \frac{7}{\sqrt{3}} = 3\sqrt{3} + 6\sqrt{3} + \frac{7}{\sqrt{3}}$$

$$= 3\sqrt{3} + 6\sqrt{3} + \frac{7\sqrt{3}}{3}$$

$$= \sqrt{3}\left(3+6+\frac{7}{3}\right) = \sqrt{3}\left(9+\frac{7}{3}\right) = \sqrt{3}\times\frac{34}{3}$$

$$= \frac{34}{3}\sqrt{3}$$

(*vi*) $\left(\sqrt{3}-\sqrt{2}\right)^2 = (\sqrt{3})^2 + (\sqrt{2})^2 - 2(\sqrt{3})\ (\sqrt{2})$

$$= 3+2-2\sqrt{3\times 2} = 5-2\sqrt{6}$$

(*vii*) $\sqrt[4]{81}-8\sqrt[3]{216}+15\sqrt[5]{32}+\sqrt{225}$

$$= \sqrt[4]{3^4} - 8\sqrt[3]{6^3} + 15\sqrt[5]{2^5} + \sqrt{(15)^2}$$
$$= 3-(8\times 6)+(15\times 2)+15$$
$$= 3-48+30+15=0$$

(*viii*) $\frac{3}{\sqrt{8}}+\frac{1}{\sqrt{2}} = \frac{3}{\sqrt{4\times 2}}+\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{3}{2}+1\right)$

$$= \frac{5}{2\sqrt{2}} = \frac{5}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{4}$$

(*ix*) $\frac{2\sqrt{3}}{3}-\frac{\sqrt{3}}{6} = \sqrt{3}\left(\frac{2}{3}-\frac{1}{6}\right) = \sqrt{3}\left(\frac{4-1}{6}\right) = \sqrt{3}\times\frac{3}{6} = \frac{\sqrt{3}}{2}$

10. Rationalise the denominator of the following:

(*i*) $\frac{2}{3\sqrt{3}}$ (*ii*) $\frac{\sqrt{40}}{\sqrt{3}}$ (*iii*) $\frac{3+\sqrt{2}}{4\sqrt{2}}$

(*iv*) $\frac{16}{\sqrt{41}-5}$ (*v*) $\frac{2+\sqrt{3}}{2-\sqrt{3}}$ (*vi*) $\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

(*vii*) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ (*viii*) $\frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ (*ix*) $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

Sol. (*i*) $\frac{2}{3\sqrt{3}} = \frac{2}{3\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{9}$

(*ii*) $\frac{\sqrt{40}}{\sqrt{3}} = \frac{\sqrt{40}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{30}}{3}$

(*iii*) $\frac{3+\sqrt{2}}{4\sqrt{2}} = \frac{3+\sqrt{2}}{4\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}(3+\sqrt{2})}{4\times 2} = \frac{3\sqrt{2}+2}{8}$

(*iv*) $$\frac{16}{\sqrt{41}-5} = \frac{16}{\sqrt{41}-5}\times\frac{\sqrt{41}+5}{\sqrt{41}+5}$$

$$= \frac{16(\sqrt{41}+5)}{41-25} = \frac{16(\sqrt{41}+5)}{16}$$

$$= \sqrt{41}+5$$

(*v*) $$\frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{(2+\sqrt{3})^2}{(2)^2-(\sqrt{3})^2} = \frac{4+3+4\sqrt{3}}{4-3}$$

$$= \frac{7+4\sqrt{3}}{1} = 7+4\sqrt{3}$$

(*vi*) $$\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} = \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}\times\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

$$= \frac{\sqrt{6}(\sqrt{2}-\sqrt{3})}{(\sqrt{2})^2-(\sqrt{3})^2} = \frac{\sqrt{6}(\sqrt{2}-\sqrt{3})}{2-3}$$

$$= \sqrt{6}(\sqrt{3}-\sqrt{2}) = \sqrt{6\times 3}-\sqrt{6\times 2}$$

$$= \sqrt{18}-\sqrt{12} = \sqrt{9\times 2}-\sqrt{4\times 3} = 3\sqrt{2}-2\sqrt{3}$$

(*vii*) $$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{3+2+2\sqrt{3}\times\sqrt{2}}{3-2}$$

$$= \frac{5+2\sqrt{6}}{1} = 5+2\sqrt{6}$$

(*viii*) $$\frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{15+3\sqrt{15}+\sqrt{15}+3}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$= \frac{18+4\sqrt{15}}{5-3} = \frac{2(9+2\sqrt{15})}{2} = 9+2\sqrt{15}$$

$$(ix)\quad \frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = \frac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}}$$

$$= \frac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} \times \frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}}$$

$$= \frac{48-12\sqrt{6}+20\sqrt{6}-30}{(4\sqrt{3})^2-(3\sqrt{2})^2}$$

$$= \frac{18+8\sqrt{5}}{48-18} = \frac{18+8\sqrt{6}}{30} = \frac{9+4\sqrt{6}}{15}$$

11. Find the values of a and b in each of the following:

(i) $\dfrac{5+2\sqrt{3}}{7+4\sqrt{3}} = a-6\sqrt{3}$ $\qquad$ (ii) $\dfrac{3-\sqrt{5}}{3+2\sqrt{5}} = a\sqrt{5}-\dfrac{19}{11}$

(iii) $\dfrac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = 2-b\sqrt{6}$ $\qquad$ (iv) $\dfrac{7+\sqrt{5}}{7-\sqrt{5}} - \dfrac{7-\sqrt{5}}{7+\sqrt{5}} = a+\dfrac{7}{11}\sqrt{5}\,b$

Sol. (i) $\text{LHS} = \dfrac{5+2\sqrt{3}}{7+4\sqrt{3}} = \dfrac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \dfrac{7-4\sqrt{3}}{7-4\sqrt{3}}$

$$= \frac{(5+2\sqrt{3})(7-4\sqrt{3})}{(7)^2-(4\sqrt{3})^2}$$

$$= \frac{35-20\sqrt{3}+14\sqrt{3}-24}{49-48}$$

$$= \frac{11-6\sqrt{3}}{1} = 11-6\sqrt{3}$$

Now, $11-6\sqrt{3} = a-6\sqrt{3}$

$\Rightarrow$ $a = 11$

(ii) $\text{LHS} = \dfrac{3-\sqrt{5}}{3+2\sqrt{5}} = \dfrac{3-\sqrt{5}}{3+2\sqrt{5}} \times \dfrac{3-2\sqrt{5}}{3-2\sqrt{5}}$

$$= \frac{(3-\sqrt{5})(3-2\sqrt{5})}{(3)^2-(2\sqrt{5})^2}$$

$$= \frac{9 - 6\sqrt{5} - 3\sqrt{5} + 10}{9 - 20} = \frac{19 - 9\sqrt{5}}{-11}$$

Now, $\frac{19 - 9\sqrt{5}}{-11} = a\sqrt{5} - \frac{19}{11}$

$\Rightarrow \frac{-19}{11} + \frac{9}{11}\sqrt{5} = a\sqrt{5} - \frac{19}{11}$

$\Rightarrow \frac{9}{11}\sqrt{5} - \frac{19}{11} = a\sqrt{5} - \frac{19}{11}$

Hence, $a = \frac{9}{11}$.

(*iii*) LHS $= \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$

$$= \frac{(\sqrt{2}+\sqrt{3})(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2})^2 - (2\sqrt{3})^2}$$

$$= \frac{6 + 2\sqrt{6} + 3\sqrt{6} + 6}{18 - 12}$$

$$= \frac{12 + 5\sqrt{6}}{6} = 2 + \frac{5\sqrt{6}}{6}$$

Now, $2 - b\sqrt{6} = 2 + \frac{5}{6}\sqrt{6} \Rightarrow b = -\frac{5}{6}$

(*iv*) $\frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} = a + \frac{7}{11}\sqrt{5}\, b$

$$\frac{7+\sqrt{5}}{7-\sqrt{5}} \times \frac{7+\sqrt{5}}{7+\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} \times \frac{7-\sqrt{5}}{7-\sqrt{5}} = a + \frac{7}{11}\sqrt{5}\, b$$

$$\frac{(7+\sqrt{5})^2}{(7)^2 - (\sqrt{5})^2} - \frac{(7-\sqrt{5})^2}{(7)^2 - (\sqrt{5})^2} = a + \frac{7}{11}\sqrt{5}\, b$$

$$\frac{49 + 5 + 14\sqrt{5}}{49 - 5} - \frac{49 + 5 - 14\sqrt{5}}{49 - 5} = a + \frac{7}{11}\sqrt{5}\, b$$

$$= \frac{54 + 14\sqrt{5}}{44} - \frac{54 - 14\sqrt{5}}{44} = a + \frac{7}{11}\sqrt{5}\, b$$

$$= \frac{54+14\sqrt{5}-54+14\sqrt{5}}{44} \quad = a + \frac{7}{11}\sqrt{5}\,b = \frac{28\sqrt{5}}{44}$$

$$\Rightarrow \quad \frac{7\sqrt{5}}{11} = a + \frac{7}{11}\sqrt{5}\,b$$

$$\Rightarrow \quad 0 + \frac{7\sqrt{5}}{11} = a + \frac{7}{11}\sqrt{5}\,b$$

Thus, $a = 0$ and $b = 1$.

12. If $a = 2+\sqrt{3}$, then find the value of $a - \frac{1}{a}$.

Sol. We have $a = 2+\sqrt{3}$

$$\therefore \quad \frac{1}{a} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$$

$$= \frac{2-\sqrt{3}}{4-3} = \frac{2-\sqrt{3}}{1} = 2-\sqrt{3}$$

$$\therefore \quad a - \frac{1}{a} = 2+\sqrt{3}-2+\sqrt{3} = 2\sqrt{3}$$

13. Rationalise the denominator in each of the following and hence evaluate by taking $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$ and $\sqrt{5} = 2.236$, upto three places of decimal.

(*i*) $\frac{4}{\sqrt{3}}$ (*ii*) $\frac{6}{\sqrt{6}}$ (*iii*) $\frac{\sqrt{10}-\sqrt{5}}{2}$

(*iv*) $\frac{\sqrt{2}}{2+\sqrt{2}}$ (*v*) $\frac{1}{\sqrt{3}+\sqrt{2}}$

Sol. (*i*) $\frac{4}{\sqrt{3}} = \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3} = \frac{4 \times 1.732}{3} = \frac{6.928}{3} = 2.309$

(*ii*) $\frac{6}{\sqrt{6}} = \frac{6}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{6\sqrt{6}}{6} = \sqrt{6} = \sqrt{2 \times 3} = \sqrt{2} \times \sqrt{3}$

$= 1.414 \times 1.732 = 2.44909 = 2.449$ (approx.)

(*iii*) $$\frac{\sqrt{10}-\sqrt{5}}{2} = \frac{\sqrt{2}\times\sqrt{5}-\sqrt{5}}{2} = \frac{\sqrt{5}(\sqrt{2}-1)}{2} = \frac{2.236(1.414-1)}{2}$$

$$= 1.118\times 0.414 = 0.463$$

(*iv*) $$\frac{\sqrt{2}}{2+\sqrt{2}} = \frac{\sqrt{2}}{2+\sqrt{2}}\times\frac{2-\sqrt{2}}{2-\sqrt{2}} = \frac{\sqrt{2}(2-\sqrt{2})}{(2)^2-(\sqrt{2})^2} = \frac{\sqrt{2}(2-\sqrt{2})}{4-2}$$

$$= \frac{\sqrt{2}\left(2-\sqrt{2}\right)}{2} = \frac{2\sqrt{2}-2}{2}$$

$$= \sqrt{2}-1 = 1.414-1 = 0.414$$

(*v*) $$\frac{1}{\sqrt{3}+\sqrt{2}} = \frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{\sqrt{3}-\sqrt{2}}{3-2}$$

$$= \frac{\sqrt{3}-\sqrt{2}}{1} = \sqrt{3}-\sqrt{2}$$

$$= 1.732-1.414 = 0.318$$

EXERCISE 1.4

1. Express $0.6 + 0.\overline{7} + 0.4\overline{7}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Sol. We have $0.6 = \frac{6}{10}$ ...(1)

Let $x = 0.\overline{7} = 0.777...$ ...(2)

Subtracting (1) from (2), we get

$$9x = 7 \Rightarrow x = \frac{7}{9} \quad \text{or} \quad 0.\overline{7} = \frac{7}{9}$$

Now, let $y = 0.4\overline{7} = 0.4777...$

$\therefore \quad 10y = 4.\overline{7}$ ...(3)

and $\quad 100y = 47.\overline{7}$ ...(4)

Subtracting (3) from (4), we get

$$90y = 43 \Rightarrow y = \frac{43}{90}$$

$$\therefore \quad 0.4\overline{7} = \frac{43}{90}$$

Now, $0.6 + 0.\overline{7} + 0.4\overline{7} = \frac{6}{10} + \frac{7}{9} + \frac{43}{90} = \frac{54 + 70 + 43}{90} = \frac{167}{90}$

So, $\frac{167}{90}$ is of the form $\frac{p}{q}$ and $q \neq 0$.

2. Simplify: $\frac{7\sqrt{3}}{\sqrt{10} + \sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6} + \sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15} + 3\sqrt{2}}$.

Sol. $\frac{7\sqrt{3}}{\sqrt{10} + \sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6} + \sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15} + 3\sqrt{2}}$

$$= \frac{7\sqrt{3}}{\sqrt{10} + \sqrt{3}} \times \frac{\sqrt{10} - \sqrt{3}}{\sqrt{10} - \sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6} + \sqrt{5}} \times \frac{\sqrt{6} - \sqrt{5}}{\sqrt{6} - \sqrt{5}}$$

$$- \frac{3\sqrt{2}}{\sqrt{15} + 3\sqrt{2}} \times \frac{\sqrt{15} - 3\sqrt{2}}{\sqrt{15} - 3\sqrt{2}}$$

$$= \frac{7\sqrt{3}(\sqrt{10} - \sqrt{3})}{10 - 3} - \frac{2\sqrt{5}(\sqrt{6} - \sqrt{5})}{6 - 5} - \frac{3\sqrt{2}(\sqrt{15} - 3\sqrt{2})}{15 - 18}$$

$$= \sqrt{3}(\sqrt{10} - \sqrt{3}) - 2\sqrt{5}(\sqrt{6} - \sqrt{5}) + \sqrt{2}(\sqrt{15} - 3\sqrt{2})$$

$$= \sqrt{30} - 3 - 2\sqrt{30} + 10 + \sqrt{30} - 6$$

$$= 2\sqrt{30} - 9 - 2\sqrt{30} + 10 = 1$$

3. *If* $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, then find the value of

$\frac{4}{3\sqrt{3} - 2\sqrt{2}} + \frac{3}{3\sqrt{3} + 2\sqrt{2}}$.

Sol. We have $\frac{4}{3\sqrt{3} - 2\sqrt{2}} + \frac{3}{3\sqrt{3} + 2\sqrt{2}}$.

$$= \frac{4(3\sqrt{3} + 2\sqrt{2}) + 3(3\sqrt{3} - 2\sqrt{2})}{(3\sqrt{3} - 2\sqrt{2})\,(3\sqrt{3} + 2\sqrt{2})}$$

$$= \frac{12\sqrt{3} + 8\sqrt{2} + 9\sqrt{3} - 6\sqrt{2}}{(3\sqrt{3})^2 - (2\sqrt{2})^2} = \frac{21\sqrt{3} + 2\sqrt{2}}{27 - 8}$$

$$= \frac{21\sqrt{3} + 2\sqrt{2}}{19} = \frac{21(1.732) + 2(1.414)}{19}$$

$$= \frac{36.372 + 2.828}{19} = 2.063$$

4. If $a = \dfrac{3+\sqrt{5}}{2}$, then find the value of $a^2 + \dfrac{1}{a^2}$.

Sol. We have $a = \dfrac{3+\sqrt{5}}{2}$

$$\Rightarrow \quad a^2 = \frac{(3+\sqrt{5})^2}{4}$$

$$= \frac{9+5+6\sqrt{5}}{4} = \frac{14+6\sqrt{5}}{4} = \frac{7+3\sqrt{5}}{2}$$

Now, $$\frac{1}{a^2} = \frac{2}{7+3\sqrt{5}} = \frac{2}{7+3\sqrt{5}} \times \frac{7-3\sqrt{5}}{7-3\sqrt{5}}$$

$$= \frac{2(7-3\sqrt{5})}{(7)^2-(3\sqrt{5})^2}$$

$$= \frac{2(7-3\sqrt{5})}{49-45} = \frac{2(7-3\sqrt{5})}{4} = \frac{7-3\sqrt{5}}{2}$$

$$\therefore \quad a^2 + \frac{1}{a^2} = \frac{7+3\sqrt{5}}{2} + \frac{7-3\sqrt{5}}{2}$$

$$= \frac{7+3\sqrt{5}+7-3\sqrt{5}}{2} = \frac{14}{2} = 7$$

5. If $x = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, and $y = \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, then find the value of $x^2 + y^2$.

Sol. $$x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{\left(\sqrt{3}+\sqrt{2}\right)^2}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{3+2+2\sqrt{3\times 2}}{3-2}$$

$$\Rightarrow \quad x = \frac{5+2\sqrt{6}}{1} = 5+2\sqrt{6}$$

Similarly, $y = 5-2\sqrt{6}$

Now, $x + y = 5+2\sqrt{6} + 5-2\sqrt{6} = 10$

and, $$xy = \frac{\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}-\sqrt{2}\right)} \times \frac{\left(\sqrt{3}-\sqrt{2}\right)}{\left(\sqrt{3}+\sqrt{2}\right)} = 1$$

$$\therefore \quad x^2 + y^2 = (10)^2-(1)^2 = 100-2 = 98$$

6. Simplify: $(256)^{-\left(4^{-\frac{3}{2}}\right)}$.

Sol. $(256)^{-\left(4^{-\frac{3}{2}}\right)} = \left(2^8\right)^{-\left(4^{-\frac{3}{2}}\right)} = \left(2^8\right)^{-\left(2^{2\times-\frac{3}{2}}\right)} = \left(2^8\right)^{-\left(2^{-3}\right)}$

$$=\left(2^8\right)^{-\left(\frac{1}{8}\right)} = 2^{8\times\left(-\frac{1}{8}\right)} = 2^{-1} = \frac{1}{2}$$

7. Find the value of $\frac{4}{(216)^{-\frac{2}{3}}}+\frac{1}{(256)^{-\frac{3}{4}}}+\frac{2}{(243)^{-\frac{1}{5}}}$

Sol. We have,

$$\frac{4}{(216)^{-\frac{2}{3}}}+\frac{1}{(256)^{-\frac{3}{4}}}+\frac{2}{(243)^{-\frac{1}{5}}} = 4(216)^{\frac{2}{3}}+(256)^{\frac{3}{4}}+2(243)^{\frac{1}{5}}$$

$$= 4(6^3)^{\frac{2}{3}}+(4^4)^{\frac{3}{4}}+2(3^5)^{\frac{1}{5}}$$

$$= 4\times 6^{3\times\frac{2}{3}}+4^{4\times\frac{3}{4}}+2\times 3^{5\times\frac{1}{5}}$$

$$= 4\times 6^2+4^3+2\times 3$$

$$= 144+64+6=214$$