EXERCISE 5.1

1. The three steps from solids to points are:
   (a) Solids-surfaces-lines-points
   (b) Solids-lines-surfaces-points
   (c) Lines-points-surfaces-solids
   (d) Lines-surfaces-points-solids

Sol. The three steps from solids to points are Solids-surfaces-lines-points.
Hence, (a) is the correct answer.

2. The number of dimensions, a solid has:
   (a) 1 (b) 2 (c) 3 (d) 0

Sol. A solid has shape, size, position and can be moved from one place to another. So, a solid has three dimensions. For example: cuboid, cube, cylinder, cone etc.
Hence, (c) is the correct answer.

3. The number of dimensions, a surface has:
   (a) 1 (b) 2 (c) 3 (d) 0

Sol. A surface has 2 dimensions.
Hence, (b) is the correct answer.

4. The number of dimension, a point has:
   (a) 0 (b) 1 (c) 2 (d) 3

Sol. According to Euclid, a point is that which has no part, i.e., no length, no breadth and no height. So, it has no dimension.
Hence, (a) is the correct answer.

5. Euclid divided his famous treatise “The Elements” into:
   (a) 13 chapters (b) 12 chapters
   (c) 11 chapters (d) 9 chapters

Sol. Euclid divided his famous treatise ‘The Elements’ into 13 chapters.
Hence, (a) is the correct answer.

6. The total number of propositions in the Elements are:
   (a) 465 (b) 460 (c) 13 (d) 55

Sol. The total number of propositions in the Elements are 465.
Hence, (a) is the correct answer.

7. Boundaries of solids are:

(*a*) surfaces (*b*) curves (*c*) lines (*d*) points.

Sol. Boundaries of solids are surfaces.
Hence, (*a*) is the correct answer.

8. Boundaries of surfaces are:

(*a*) surfaces (*b*) curves (*c*) lines (*d*) points

Sol. Boundaries of surfaces are curves.
Hence, (*b*) is the correct answer.

9. In Indus valley Civilisation (about 3000 B.C.), the bricks used for construction work were having dimensions in the ratio

(*a*) 1 : 3 : 4 (*b*) 4 : 2: 1 (*c*) 4 : 4 : 1 (*d*) 4 : 3 : 2

Sol. Bricks used for construction work were having dimensions in the ratio are 4 : 2 : 1.
Hence, (*b*) is the correct answer.

10. A pyramid is a solid figure, the base of which is

(*a*) only a triangle (*b*) only a square

(*c*) only a rectangle (*d*) any polygon

Sol. A pyramid is a solid figure, the base of which is any polygon.
Hence, (*d*) is the correct answer.

11. The side faces of a pyramid are:

(*a*) Triangles (*b*) Squares (*c*) Polygons (*d*) Trapeziums

Sol. The sides faces of a pyramid are triangles.
Hence, (*a*) is the correct answer.

12. It is known that if $x + y = 10$ then $x + y + z = 10 + z$. The Euclid's axiom that illustrates this statement is:

(*a*) First Axiom (*b*) Second Axiom

(*c*) Third Axiom (*d*) Fourth Axiom

Sol. If $x + y = 10$ then $x + y + z = 10 + z$. The Euclid's axiom that illustrates this statement is axiom 2 which states that; If equals are added to equals, the wholes are equal.
Hence, (*b*) is the correct answer.

13. In ancient India, the shapes of altars used for house hold rituals were:

(*a*) Squares and circles (*b*) Triangles and rectangles

(*c*) Trapeziums and pyramids (*d*) Rectangles and squares

Sol. In Ancient India, the shapes of altars used for house hold rituals were squares and circles.
Hence, (*a*) is the correct answer.

14. The number of interwoven isosceles triangles in Sriyantra (in the Atharva Veda) is:

(*a*) Seven (*b*) Eight (*c*) Nine (*d*) Eleven

Sol. The number of interwoven isosceles triangles in Sriyantra (in the Atharva Veda) is nine.

Hence, (*c*) is the correct answer.

15. Greek's emphasised on:

(*a*) Inductive reasoning (*b*) Deductive reasoning

(*c*) Both (*a*) and (*b*) (*d*) Practical use of geometry

Sol. The Greeks were interested in establishing the truth of the statements they discovered using deductive reasoning. A Greek mathematician, Thales is credited with giving the first known proof.

Hence, (*b*) is the correct answer.

16. In Ancient India, Altars with combination of shapes like rectangles, triangles and trapeziums were used for:

(*a*) Public worship (*b*) Households rituals

(*c*) Both (*a*) and (*b*) (*d*) None of (*a*), (*b*), (*c*)

Sol. In Ancient India, Altars with combination of shapes like rectangles, triangles and trapeziums were used for public worship.

Hence, (*a*) is the correct answer.

17. Euclid belongs to the country:

(*a*) Babylonia (*b*) Egypt

(*c*) Greece (*d*) India

Sol. Euclid belongs to the country Greece. Euclid around 300 B.C. collected all known work in the field of mathematics and arranged it in his famous treatise called Elements.

Hence, (*c*) is the correct answer.

18. Thales belongs to the country:

(*a*) Babylonia (*b*) Egypt (*c*) Greece (*d*) Rome

Sol. Thales belongs to the country Greece. The Greeks were interested in establishing the truth of the statements they discovered using deductive reasoning. Thales, a Greek mathematician, is credited with giving the first known proof.

Hence, (*c*) is the correct answer.

19. Pythagoras was a student of:

(*a*) Thales (*b*) Euclid

(*c*) Both (*a*) and (*b*) (*d*) Archimedes

Sol. Pythagoras (572 BC) was a student of Thales. Pythagoras and his group discovered many geometric properties and developed the theory of geometry to a great extent. This process continued till 300 BC. At that time Euclid, a teacher of mathematics at Alexandria in Egypt, collected all the known work and arranged it in his famous treatise.

Hence, (*a*) is the correct answer.

20. Which of the following needs proof?

(*a*) Theorem (*b*) Axiom (*c*) Definition (*d*) Postulate

Sol. Theorem

Hence, (*a*) is the correct answer.

21. Euclid stated that all right angles are equal to each other in the form of

(*a*) an axiom (*b*) a definition (*c*) a postulate (*d*) a proof

Sol. a postulate

Hence, (*c*) is the correct answer.

22. "Lines are parallel if they do not intersect" is stated in the form of

(*a*) an axiom (*b*) a definition

(*c*) a postulate (*d*) a proof

Sol. "Lines are parallel if they do not intersect" is the form of a definition.

Hence, (*b*) is the correct answer.

EXERCISE 5.2

Write whether the following statements are true or false. Justify your answer.

1. Euclidean geometry is valid only for curved surfaces.

Sol. The given statement is false because Euclidean geometry is valid only for the figures in the plane.

2. The boundaries of the solids are curves.

Sol. The given statement is false because boundaries of solids are surfaces.

3. The edges of a surface are curves.

Sol. The given statement is false because the edges of surfaces are line.

4. The things which are double of the same thing are equal to one another.

Sol. True

Since, it is one of the Euclid's axioms. Some of Euclid's axioms: (1) Things which are equal to the same thing are equal to one another. (2) If equals are added to equals, the wholes are equal. (3) If equals are subtracted from equals, the remainders are equal. (4) Things which coincide with one another are equal to one another. (5) The whole is greater than the part. (6) Things which are double of the same things are equal to one another. (7) Things which are halves of the same things are equal to one another.

5. If a quantity B is a part of another quantity A, then A can be written as the sum of B and some third quantity C.

Sol. The given statement is true because it is one of Euclid's axiom.

6. The statements that are proved are called axioms.

Sol. The given statement is false because the statement that are proved are called theorems.

7. "For every line *l* and for every point P not lying on a given line *l*, there exists a unique line *m* passing through P and parallel to *l* " is known as Playfair's axiom.

Sol. The given statement is true, because it is an equivalent version of Euclid's fifth postulate.

8. Two distinct intersecting lines cannot be parallel to the same line.

Sol. The given statement is true because it is an equivalent version of Euclid's fifth postulate.

9. Attempts to prove Euclid's fifth postulate using the other postulates and axioms led to the discovery of several other geometries.

Sol. The given statement is true because these geometries are different from Euclidean geometry called non-Euclidean geometries.

EXERCISE 5.3

Solve each of the following question using appropriate Euclid's axiom:

1. Two salesmen make equal sales during the month of August. In September, each salesman doubles his sale of the month of August. Compare their sales in September.

Sol. Let the sales of two salesman in the month of August be x and y. As, they make equal sale during the month of August, so $x = y$. In September, each salesman doubles his sale of the month of August, so $2x = 2y$.

Now, by Euclid's axiom, things which are double of the same things are equal to one another.

Hence, we can say that in the month of September also, two salesman make equal sales.

2. It is known that $x + y = 10$ and that $x = z$. Show that $z + y = 10$.

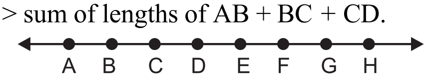
Sol. It is known that $x + y = 10$ and $x = z$ ...(1)

$\therefore$ $x + y = z + y$ [$\because$ By Euclid's axiom 2, If equals are added to equals, the wholes are equal]

$\Rightarrow$ $10 = y + z$ [Using (1), $x + y = 10$)]

Hence, $z + y = 10$.

3. Look at the figure, and show that length AH > sum of lengths of AB + BC + CD.



Sol. We see that AB, BC and CD are parts of line.

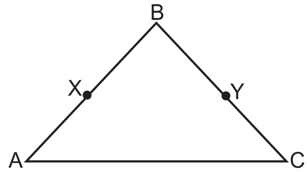
Now, AB + BC + CD = AD ...(1)

By Euclid's axiom 5, the whole is greater than the part, so

AH > AD

i.e., length AH > Sum of lengths of AB + BC + CD [Using (1)]

4. In the given figure, we have AB = BC, BX = BY. Show that AX = CY.



Sol. We have AB = BC ...(1) [Given]

and BX = BY ...(2) [Given]

Subtracting (2) from (1), we get

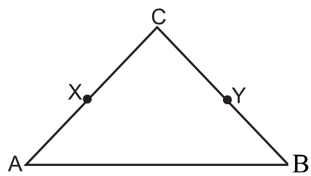
$$AB - BX = BC - BY$$

Now, by Euclid's axiom 3, we have

If equals are subtracted from equals, the remainders are equal.

Hence, AX = CY.

5. In the given figure, we have X and Y are the mid-points of AC and BC and AX = CY. Show that AC = BC.



Sol. We have AX = CY [Given]

Now, by Euclid's axiom 6, we have things which are double of the same thing are equal to one another,

so 2AX = 2 CY

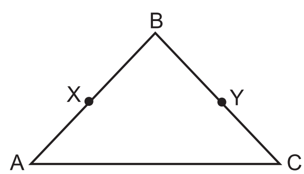
Hence, AC = BC. [$\because$ X and Y are the mid-points of AC and BC]

6. In the given figure, we have

$$BX = \frac{1}{2}AB$$

$$BY = \frac{1}{2}BC \text{ and } AB = BC.$$

Show that BX = BY.



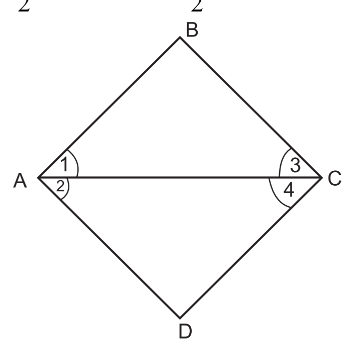
Sol. We have AB = BC [Given]

Now, by Euclid's axiom 7, we have Things which are halves of the same thing are equal to one another.

$$\therefore \frac{1}{2}AB = \frac{1}{2}BC$$

Hence, BX = BY. [$\because BX = \frac{1}{2}AB$ and $BY = \frac{1}{2}BC$ (Given)]

7. In the given figure, we have $\angle 1 = \angle 2$, $\angle 2 = \angle 3$. Show that $\angle 1 = \angle 3$.



Sol. We have

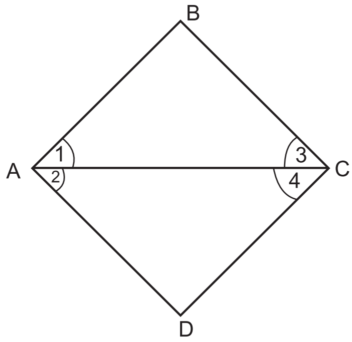
$$\angle 1 = \angle 2 \quad \text{[Given]}$$

$$\angle 2 = \angle 3 \quad \text{[Given]}$$

Now, by Euclid's axiom 1, things which are equal to the same thing are equal to one other.

Hence, $\angle 1 = \angle 3$.

8. In the given figure, we have $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$. Show that $\angle A = \angle C$.



Sol. We have $\angle 1 = \angle 3$ ...(1)[Given]

and $\angle 2 = \angle 4$ ...(2)[Given]

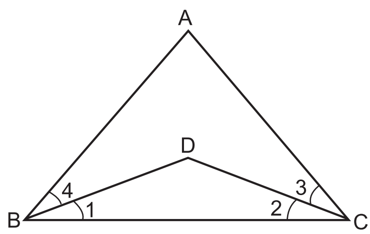
Now, by Euclid's axiom 2, we have if equal are added to equals, the whole are equal.

Adding (1) and (2), we get

$$\angle 1 + \angle 2 = \angle 3 + \angle 4$$

Hence, $\angle A = \angle C$.

9. In the given figure, we have $\angle ABC = \angle ACB$, $\angle 4 = \angle 3$. Show that $\angle 1 = \angle 2$.



Sol. We have

$\Rightarrow$ $\angle ABC = \angle ACB$ ...(1) [Given]

and $\angle 4 = \angle 3$ ...(2) [Given]

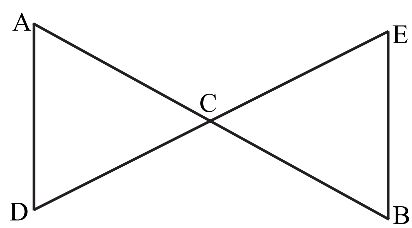
Now, subtracting (2) from (1), we get

Now, by Euclid's axiom 3, if equals are subtracted from equals, the remainders are equal.

$$\angle ABC - \angle 4 = \angle ACB - \angle 3$$

Hence, $\angle 1 = \angle 2$.

10. In the given figure, we have AC = DC, CB= CE. Show that AB = DE.



Sol. We have $AC = DC$ ...(1) [Given]

and $CB = CE$ ...(2) [Given]

Now, by axiom 2, if equals are added to equals, the wholes are equal.

Adding (1) and (2), we get

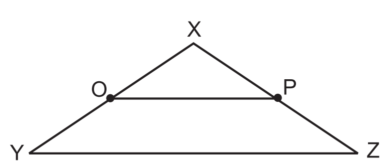
$$AC + CB = DC + CE$$

Hence, AB = DE.

11. In the given figure, if

$OX = \frac{1}{2}XY$, $PX = \frac{1}{2}XZ$ and

OX = PX, show that XY = XZ.



Sol. We have $OX = PX$ [Given]

Now

$\because \quad OX = \frac{1}{2}XY$ (Given) and $PX = \frac{1}{2}XZ$ [Given]

$\therefore \quad \frac{1}{2}XY = \frac{1}{2}XZ$

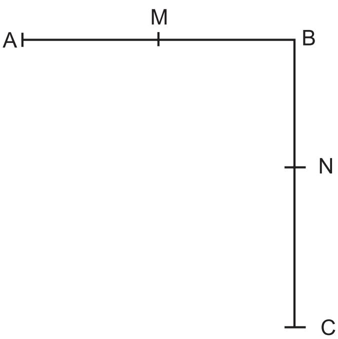
[$\because$ Things which are halves of the same thing are equal to one another (Euclid's axiom 7)]

$\therefore \quad XY = XZ$ [$\because$ Things which are double of the same thing are equal to one another (Euclid's axiom 6)]

12. In the given figure,

(*i*) AB = BC, M is the mid-point of AB and N is the mid-point of BC. Show that AM = NC.

(*ii*) BM = BN, M is the mid-point of AB and N is the mid-point of BC. Show that AB = BC.



Sol. (*i*) We have $AB = BC$ ...(1) [Given]

Now, A, M, B are three points on a line, and M lies between A and B, then

$AM + MB = AB$ ...(2)

Similarly, $BN + NC = BC$ ...(3)

So, we get $AM + MB = BN + NC$

From (1), (2), (3) and Euclid's axiom 1]

Since M is the mid-point of AB and N is the mid-point of BC, therefore

$$2\,AM = 2\,NC$$

Using axiom 6, things which are double of the same thing are equal to one another.

Hence, $AM = NC$.

(*ii*) We have $BM = BN$ ...(1)

As M is the mid-point of AB, so

$BM = AM$ ...(2)

and N is the mid-point of BC,

$BN = NC$ ...(3)

From (1), (2) and (3) and Euclid's axiom 1, we get

$AM = NC$ ... (4)

Adding (4) and (1), we get

$$AM + BM = NC + BN$$

Hence, $AB = BC$

[By axiom 2, if equals are added to equals, the wholes are equal]

EXERCISE 5.4

1. Read the following statement:

An equilateral triangle is a polygon made up of three line segments out of which two line segments are equal to the third one and all its angles are 60^{o} each.

Define the terms used in this definitions which you feel necessary. Are there any undefined terms in this? Can you justify that all sides and all angles are equal in an equilateral triangle?

Sol. The terms need to be defined are:

Polygon: A simple closed figure made up of three or more line segments.

Line segment: Part of a line with two end points.

Line: Undefined term.

Point: Undefined term.

Angle: A figure formed by two rays with a common initial points.

Acute angle: Angle whose measure is between $0°$ and $90°$.

Undefined terms used are : line, point

Two line segments are equal to third line segment

Therefore, all three sides of an equilateral triangle are equal

All its angles are 60^{o} each. Therefore, all angles are equal (by Euclid's first axiom , things which are equal to same thing are equal to one another.)

Hence, we can say that all sides and all angles are equal in an equilateral triangle.

2. Study the following statement:

"Two intersecting lines cannot be perpendicular to the same line".

Check whether it is an equivalent version to the Euclid's fifth postulate.

[Hint: Identify the two intersecting lines l and m and the line n in the above statement]

Sol. Two intersecting lines cannot be both perpendicular to the same line because if two lines l and m are perpendicular to the same line n, then l and m must be parallel.

The given statement is not an equivalent version of Euclid's fifth postulate.

3. Read the following statements which are taken as axioms:

(*i*) If a transversal intersects two parallel lines, then corresponding angles are not necessarily equal.

(*ii*) If a transversal intersect two parallel lines, then alternate interior angles are equal.

Is this system of axioms consistent? Justify your answer.

Sol. No, this system of axioms is not consistent because if a transversal intersects two parallel lines and if corresponding angles are not equal, then alternate interior angles can not be equal.

4. Read the following two statements which are taken as axioms:

(*i*) If two lines intersect each other, then vertically opposite angles are not equal.

(*ii*) If a ray stands on a line, then the sum of two adjacent angles so formed is equal to 180^{o}.

Is this system of axioms consistent ? Justify your answer.

Sol. The given system of axioms is not consistent because if a ray stands on a line and the sum of two adjacent angles so formed is equal to 180^{o}, then for two lines which intersect each other, the vertically opposite angles becomes equal.

5. Read the following axioms:

(*i*) Things which are equal to the same thing are equal to one another.

(*ii*) If equals are added to equals, the wholes are equal.

(*iii*) Things which are double of the same thing are equal to one another.

Check whether the given system of axioms is consistent or inconsistent.

Sol. The given system of axioms is consistent because (*i*), (*ii*) and (*iii*) are Euclid's axiom.