EXERCISE 7.1

In each of the following, write the correct answer:

1. Which of the following is not a criterion for congruence of triangles?

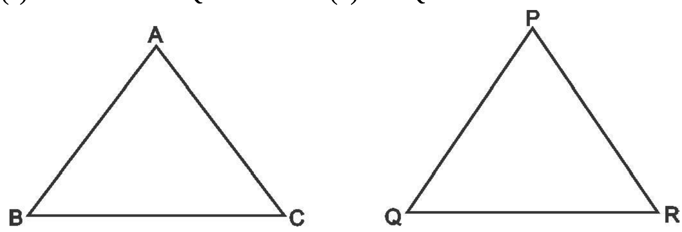
   (a) SAS (b) ASA (c) SSA (d) SSS

Sol. SSA is not a criterion for congruence of triangles.
Hence, (c) is the correct answer.

2. If AB = QR, BC = PR and CA = PQ, then

   (a) $\Delta ABC \cong \Delta PQR$ (b) $\Delta CBA \cong \Delta PRQ$
   (c) $\Delta BAC \cong \Delta RPQ$ (d) $\Delta PQR \cong \Delta BCA$

Sol.



We have AB = QR, BC = PR and CA = PQ.
There is one-one correspondence between the vertices. That is, P correspondence to C, Q to A and R to B which is written as

$$P \leftrightarrow C,\ Q \leftrightarrow A,\ R \leftrightarrow B$$

Under this correspondence, we have

$$\Delta CBA \cong \Delta PRQ$$

Hence, (b) is the correct answer.

3. In ΔABC, AB = AC and $\angle B = 50°$. Then, $\angle C$ is equal to

   (a) 40° (b) 50° (c) 80° (d) 130°

Sol. In ΔABC, we have

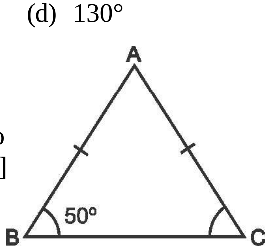
$AB = AC$ [Given]

$\therefore$ $\angle C = \angle B$

[$\because$ Angles opposite to equal sides are equal]

But, $\angle B = 50°$

$\therefore$ $\angle C = 50°$



Hence, (b) is the correct answer.

4. In ΔABC, BC = AB and $\angle B = 80°$. Then, $\angle A$ is equal to

   (a) 80° (b) 40° (c) 50° (d) 100°

Sol. In ΔABC, we have

$$BC = AB \quad \text{[Given]}$$

$\therefore \quad \angle A = \angle C \quad$ [$\because$ Angles opposite to equal sides are equal]

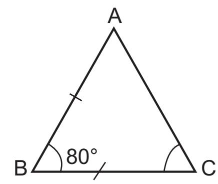
But, $\angle B = 80°$

$\therefore \quad \angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle A + 80° + \angle A = 180°$

$\Rightarrow \quad 2\angle A = 100°$

$\Rightarrow \quad \angle A = 100° \div 2 = 50°$

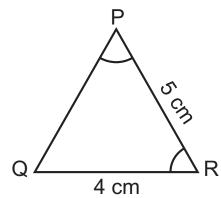


Hence, (*c*) is the correct answer.

5. In ΔPQR, $\angle R = \angle P$ and $QR = 4$ cm and $PR = 5$ cm. Then, the length of PQ is

(*a*) 4 cm (*b*) 5 cm

(*c*) 2 cm (*d*) 2.5 cm



Sol. In ΔPQR, we have $\angle R = \angle P$ [Given]

$\therefore \quad PQ = QR$

[$\because$ Sides opposite to equal angles are equal]

Now, $QR = 4$ cm, therefore, $PQ = 4$ cm.

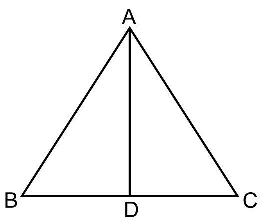
Hence, the length of PQ is 4 cm.

Hence, (*a*) is the correct answer.

6. D is a point on the side BC of a ΔABC such that AD bisects $\angle BAC$. Then,

(*a*) $BD = CD$ (*b*) $BA > BD$

(*c*) $BD > BA$ (*d*) $CD > CA$



Sol. In ΔADC,

Ext. $\angle ADB >$ Int. opp. $\angle DAC$

$\therefore \quad \angle ADB > \angle BAD \quad$ [$\because \angle BAD = \angle DAC$]

$\Rightarrow \quad AB > BD$

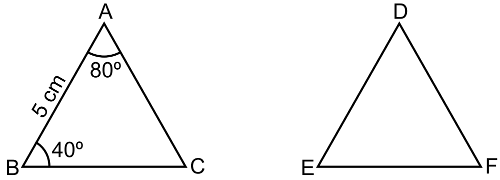
[$\because$ Side opposite to greater angle is longer.]

7. It is given that $\Delta ABC \cong \Delta FDE$ and $AB = 5$ cm, $\angle B = 40°$ and $\angle A = 80°$. Then, which of the following is true?

(*a*) $DF = 5$ cm, $\angle F = 60°$ (b) $DF = 5$ cm, $\angle E = 60°$

(c) $DE = 5$ cm, $\angle E = 60°$ (d) $DE = 5$ cm, $\angle D = 60°$

Sol.



It is given that $\Delta ABC \cong \Delta FDE$ and $AB = 5$cm, $\angle B = 40°$ and $\angle A = 80°$, so $\angle C = 60°$.

The sides of ΔABC fall on corresponding equal sides of ΔFDE. A corresponds to F, B corresponds to D, and C corresponds to E.

So, only $DF = 5cm$, $\angle E = 60°$ is true.

Hence, (*b*) is the correct answer.

8. Two sides of a triangle are of lengths 5 cm and 1.5 cm. The length of the third side of the triangle cannot be

(*a*) 3.6 cm (*b*) 4.1 cm (*c*) 3.8 cm (*d*) 3.4 cm

Sol. Since sum of any two sides of a triangle is always greater than the third side, so third side of the triangle cannot be 3.4 cm because then

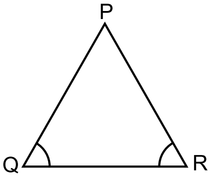
$1.5\text{ cm} + 3.4\text{ cm} = 4.9 < \text{third side } (5\text{ cm})$.

Hence, (*d*) is the correct answer.

9. In ΔPQR, if $\angle R > \angle Q$, then

(*a*) $QR > PR$ (*b*) $PQ > PR$

(*c*) $PQ < PR$ (*d*) $QR < PR$



Sol. In ΔPQR, we have $\angle R > \angle Q$

$\therefore$ $PQ > PR$ [$\because$ Side opposite to greater angle is longer]

Hence, (*b*) is the correct answer.

10. In triangles ABC and PQR, $AB = AC$, $\angle C = \angle P$ and $\angle B = \angle Q$. The two triangles are

(*a*) isosceles but not congruent.

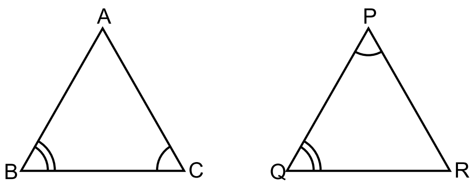
(*b*) isosceles and congruent.

(*c*) congruent but not isosceles.

(*d*) neither congruent nor isosceles.

Sol. $AB = AC$ [Given]

$\therefore$ $\angle B = \angle C$ [$\because$ Angles opposite to equal sides are equal]



Also, $\angle B = \angle Q$ and $\angle C = \angle P$ [Given]

$\therefore$ $\angle Q = \angle P$

$\Rightarrow$ $PR = RQ$ [$\because$ Sides opp. to equal $\angle s$ equal]

Hence, (*a*) is the correct option.

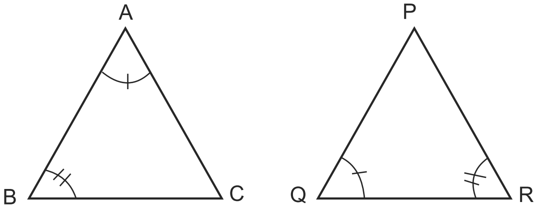
11. In triangles ABC and DEF, $AB = FD$ and $\angle A = \angle D$. The two triangles will be congruent by SAS axiom. Then,

(*a*) $BC = EF$ (*b*) $AC = DE$ (*c*) $AC = EF$ (*d*) $BC = DE$

Sol. (*b*) $AC = DE$.

EXERCISE 7.2

1. In triangles ABC and PQR, $\angle A = \angle Q$ and $\angle B = \angle R$. Which side of ΔPQR should be equal to side AB of ΔABC so that the two triangles are congruent? Give reasons for your answer.



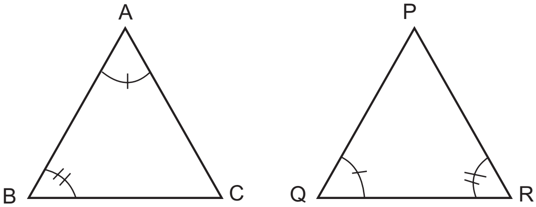
Sol. In triangles ABC and PQR, we have

$\angle A = \angle Q$ [Given]

$\angle B = \angle R$ [Given]

For the triangles to be congruent, we must have AB = QR. They will be congruent by ASA congruence rule.

2. In triangles ABC and PQR, $\angle A = \angle Q$ and $\angle B = \angle R$. Which side of ΔPQR should be equal to side BC of ΔABC so that the two triangles are congruent? Give reasons for your answer.



Sol. In triangles ABC and PQR, we have

$\angle A = \angle Q$ and $\angle B = \angle R$ [Given]

For the triangles to be congruent, we must have

$BC = RP$

They will be congruent by AAS congruence rule.

3. "If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles must be congruent." Is the statement true? Why?

Sol. This statement is not true. Angles must be the included angles.

4. "If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles must be congruent." Is the statement true? Why?

Sol. This statement is true. Sides must be corresponding sides.

5. Is it possible to construct a triangle with lengths of sides 4 cm, 3 cm and 7 cm? Give reasons for your answer.

Sol. We know that the sum of any two sides of a triangle is always greater than the third side.

Here, the sum of two sides whose lengths are

4 cm and 3 cm = 4 cm + 3 cm = 7 cm,

which is equal to the length of third side, i.e., 7 cm.

Hence, it is not possible to construct a triangle with lengths of sides 4 cm, 3 cm and 7 cm.

6. It is given that $\Delta ABC \cong \Delta RPQ$. Is it true to say that BC = QR? Why?

Sol. It is false that BC = QR because BC = PQ as $\Delta ABC \cong \Delta RPQ$.

7. It is given that $\Delta PQR \cong \Delta EDF$, then is it true to say that PR = EF? Give reasons for your answer.

Sol. Yes, PR = EF because they are the corresponding sides of Δ PQR and Δ EDF.

8. In ΔPQR, $\angle P = 70°$ and $\angle R = 30°$. Which side of this triangle is the longest? Give reasons for your answer.

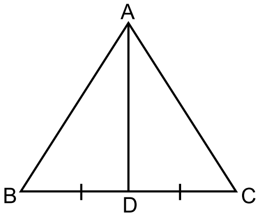
Sol. In Δ PQR, we have

$$\angle Q = 180° - (\angle P + \angle R)$$
$$= 180° - (70° + 30°) = 180° - 100° = 80°$$

Now, in Δ PQR, $\angle Q$ is the larger (greater) and side opposite to greater angle is longer.

Hence, PR is the longest side.

9. AD is a median of the triangle ABC. Is it true that AB + BC + CA > 2AD? Give reason for your answer.



Sol. In ΔABD, we have

$AB + BD > AD$ ...(1)

[$\because$ Sum of the lengths of any two sides of a triangle must be greater than the third side]

Now, in ΔADC, we have

$AC + CD > AD$ ...(2)

[$\because$ Sum of the lengths of any two sides of a triangle must be greater than the third side]

Adding (1) and (2), we get

$AB + BD + CD + AC > 2AD$

$\Rightarrow \quad AB + BC + CA > 2AD \quad$ [$\because$ BD = CD as AD is median of ΔABC]

10. M is a point on side BC of a triangle ABC such that AM is the bisector of $\angle BAC$. Is it true to say that perimeter of the triangle is greater than 2AM? Give reason for your answer.

Sol. We have to prove that

$$AB + BC + AC > 2\,AM.$$

As sum of any two sides of a triangle is greater than the third side, so in Δ ABM, we have

$$AB + BM > AM \quad ...(1)$$

and in ΔACM, $\quad AC + CM > AM \quad$ ...(2)

Adding (1) and (2), we get

$$AB + BM + AC + CM > 2AM$$

or $AB + (BM + CM) + AC > 2AM$

$\Rightarrow \quad AB + BC + AC > 2\,AM$

Hence, it is true to say that perimeter of the triangle is greater than 2AM.

11. Is it possible to construct a triangle with lengths of sides as 9 cm, 7 cm and 17 cm? Give reason for your answer.

Sol. No, it is not possible to construct a triangle whose sides are 9 cm, 7cm and 17cm because

$$9\,\text{cm} + 7\text{cm} = 16\,\text{cm} < 17\text{cm}$$

where as sum of any two sides of a triangle is always greater than the third side.

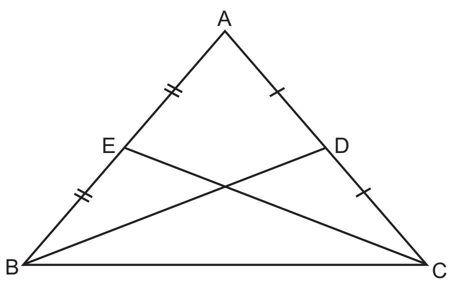
12. Is it possible to construct a triangle with lengths of its sides as 8 cm, 7 cm and 4 cm? Give reason for your answer.

Sol. Yes, it is possible to construct a triangle with lengths of sides as 8 cm, 7 cm and 4 cm as sum of any two sides of a triangle is greater than the third side.

EXERCISE 7.3

1. ABC is an isosceles triangle with AB = AC and BD and CE are its two medians. Show that BD = CE.

Sol. Given : ΔABC with AB = AC and AD = CD, AE = BE.



To prove : BD = CE

Proof : In ΔABC, we have

AB = AC [Given]

$\Rightarrow \quad \frac{1}{2}AB = \frac{1}{2}AC$

$\Rightarrow \quad AE = AD$

[∵ D is the mid-point of AC and E is the mid-point of AB]

Now, in ΔABD and ΔACE, we have

AB = AC [Given]

∠A = ∠A [Common angle]

AE = AD [Proved above]

So, by SAS criterion of congruence, we have

Δ ABD ≅ ΔACE

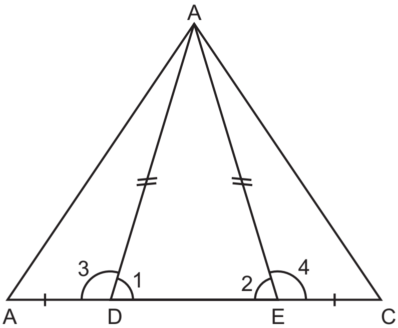
⇒ BD = CE [CPCT]

Hence, proved.

2. In the given figure, D and E are points on side BC of a ΔABC such that BD = CE and AD = AE. Show that ΔABD ≅ ACE.

Sol. Given : Δ ABC in which BD = CE and AD = AE.

To prove: Δ ABD ≅ ΔACE



Proof : In ΔADE, we have

$AD = AE$ [Given]

$\Rightarrow \angle 2 = 1$

[$\because$ Angles opposite to equal sides of a triangle are equal]

Now, $\angle 1 + \angle 3 = 180°$ ...(1)

[Linear pair axiom]

$\angle 2 + \angle 4 = 180°$ ...(2)

[Linear pair axiom]

From equations (1) and (2), we get

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

$\Rightarrow \angle 3 = \angle 4$ [$\because \angle 1 = \angle 2$]

Now, in ΔABD and ΔACE, we have

$AD = AE$ [Given]

$\angle 3 = \angle 4$ [Proved above]

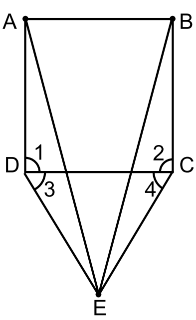
$BD = CE$ [Given]

So, by SAS criterion of congruence, we have

$\Delta ABD \cong \Delta ACE$

Hence, proved.

3. CDE is an equilateral triangle formed on a side CD of a square ABCD (See fig.). Show that $\Delta ADE \cong \Delta BCE$.



Sol. Given : An equilateral triangle CDE formed on side CD of square ABCD.

To prove : $\Delta ADE \cong \Delta BCE$

Proof : In square ABCD, we have

$\angle 1 = \angle 2$ … (1) [$\because$ Each = 90°]

Now, in ΔDCE, we have

$\angle 3 = \angle 4$ …(2) [$\because$ Each = 60°]

Adding (1) of (2), we get

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

$\Rightarrow \angle ADE = \angle BCE$

Now, in ΔADE and ΔBCE, we have

$DE = CE$ [Sides of an equilateral triangle are equal]

$\angle ADE = \angle BCE$ [Proved above]

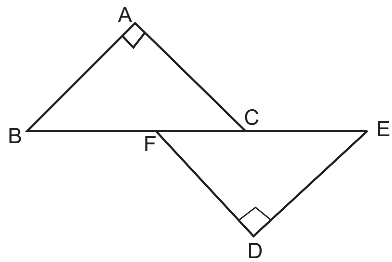
$AD = BC$ [Sides of a square are equal in length]

So, by SAS criterion of congruence, we have

$\Delta ADE \cong \Delta BCE$

Hence, proved

4. In the given figure, $BA \perp AC$, $DE \perp DF$ such that $BA = DE$ and $BF = EC$. Show that $\Delta ABC \cong \Delta DEF$.



Sol. We have BF = EC

$\therefore$ $BF + FC = CE + FC \Rightarrow BC = EF$

In ΔABC, $\angle A = 90°$ and in ΔDEF, $\angle D = 90°$.

$\therefore$ ΔABC and ΔDEF are right triangles.

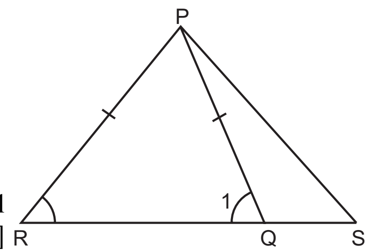
Now, in right triangles ABC and DEF, we have

$BA = DE$ [Given]

and $BC = EF$ [Proved above]

$\therefore$ $\Delta ABC \cong \Delta DEF$ [By RHS congruence rule]

5. O is a point on the side SR of a ΔPSR such that PQ = PR. Prove that PS > PQ.



Sol. Given : PQ = PR

To prove: PS > PQ

Proof: In Δ PRQ, we have

$PR = PQ$ [Given]

$\Rightarrow$ $\angle 1 = \angle R$

[$\because$ Angles opposite to equal sides of a triangle are equal]

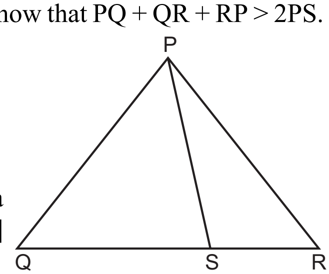
But, $\angle 1 > \angle S$ [$\because$ Exterior angle of a triangle is greater than each of the remote interior angles]

$\Rightarrow$ $\angle R > \angle S$ [$\because$ $\angle 1 = \angle R$]

$\Rightarrow$ $PS > PR$ [$\because$ In a triangle, side opposite to the large is longer]

Hence, proved.

6. S is any point on side QR of a ΔPQR. Show that PQ + QR + RP > 2PS.



Sol. Given: A point S on side QR of ΔPQR.

To prove : $PQ + QR + RP > 2PS$

Proof : In ΔPQS, we have

$PQ + QS > PS$ ...(1)

[$\because$ Sum of the lengths of any two sides of a triangle must be greater than the third side]

Now, in Δ PSR, we have

$RS + RP > PS$ ...2 [$\because$ Sum of the lengths of any two sides of a triangle must be greater than the third side]

Adding (1) and (2), we get

$$PQ + QS + RS + RP > 2\,PS$$

$$\Rightarrow \quad PQ + QR + RP > 2PS$$

Hence, proved

7. D is any point on side AC of a ΔABC with $AB = AC$. Show that $CD < BD$.

Sol. In ΔABC, we have

$AB = AC$ [Given]

$\therefore \quad \angle ABC = \angle ACB$

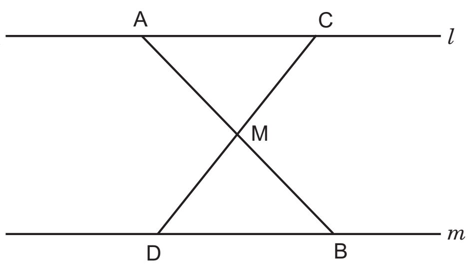
[$\because$ Angles opp. to equal sides of a triangle are equal]

Now, $\angle DBC < \angle ABC$

$\therefore \quad \angle DBC < \angle ACB$ or $\angle DBC < \angle DCB$

Hence, $CD < BD$. [$\because$ Side opposite to greater angle is longer]

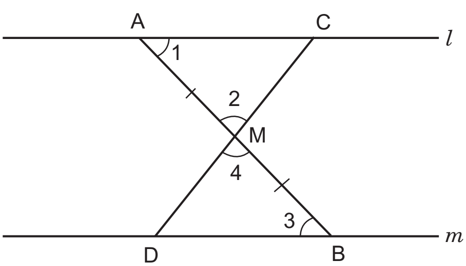
8. In the given figure, $l \parallel m$ and M is the mid-point of the line segment AB. Prove that M is also the mid-point of any line segment CD having its end-points on l and m respectively.



Sol. In ΔAMC and ΔBMD, we have

$\angle 1 = \angle 3$ [Alt. $\angle s$ because $l \parallel m$]

$\angle 2 = \angle 4$ [Vert. opp. $\angle s$]



$AM = BM$ [Given]

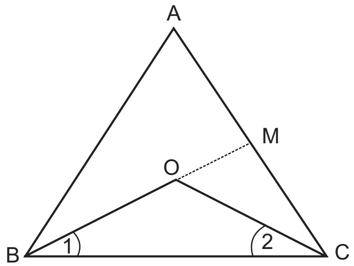
$\therefore \quad \Delta AMC \cong \Delta BMD$ [By AAS congruence rule]

$\therefore \quad CM = DM$ [CPCT]

Hence, M is also the mid-point of CD

9. Bisectors of the angles B and C of an isosceles triangle with $AB = AC$ intersect each other at O. BO is produced to a point M. Prove that $\angle MOC = \angle ABC$.

Sol. Bisectors of the angles B and C of an isosceles triangle ABC with $AB = AC$ intersect each other at O. BO is produced to a point M.



In ΔABC, we have

$$AB = AC$$

$\therefore \quad \angle ABC = \angle ACB$

[$\because$ Angles opposite to equal sides of a triangle are equal]

$$\Rightarrow \quad \frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB,$$

i.e., $\angle 1 = \angle 2$ [$\because$ BO and CO are bisectors of $\angle B$ and $\angle C$]

In ΔOBC, Ext. $\angle MOC = \angle 1 + \angle 2$

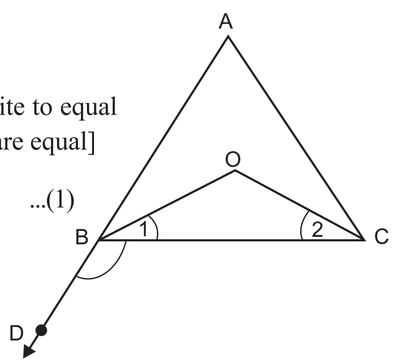
[$\because$ Exterior angle of a triangle is equal to the sum of interior opposite angles]

$\Rightarrow \quad$ Ext. $\angle MOC = 2\angle 1$ [$\because \angle 1 = \angle 2$]

Hence, $\angle MOC = \angle ABC$.

10. Bisectors of the angles B and C of an isosceles triangle ABC with AB = AC intersect each other at O. Show that external angle adjacent to $\angle ABC$ is equal to $\angle BOC$.

Sol. In Δ ABC, we have



$$AB = AC$$

$\therefore \quad \angle B = \angle C$

[$\because$ Angles opposite to equal sides of a triangle are equal]

$\therefore \quad \frac{1}{2}\angle B = \frac{1}{2}\angle C \quad$ ...(1)

In Δ OBC, we have

$$\angle 1 = \frac{1}{2}\angle B$$

and $\quad \angle 2 = \frac{1}{2}\angle C$

$\therefore \quad \angle 1 = \angle 2$ [By (1)]

$\angle DBC + \angle 1 + \angle OBA = 180°$ [$\because$ ABD is a straight line]

$\Rightarrow \quad \angle DBC + 2\angle 1 = 180°$ [$\because \angle 1 = \angle OBA$] ...(1)

In ΔOBC,

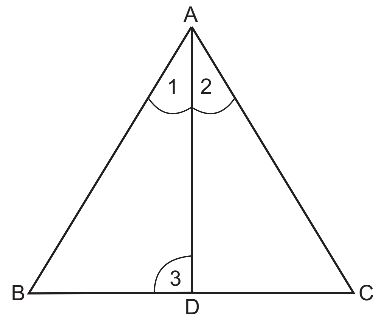
$\angle 1 + \angle 2 + \angle BOC = 180°$

$\Rightarrow \quad 2\angle 1 + \angle BOC = 180°$ [$\because \angle 1 = \angle 2$]...(2)

From (1) and (2), we get

$$\angle DBC + 2\angle 1 = 2\angle 1 + \angle BOC$$

$$\Rightarrow \quad \angle DBC = \angle BOC$$



11. In the given figure, AD is the bisector of $\angle BAC$. Prove that $AB > BD$.

Sol. Since exterior angle of a triangle is greater than either of the interior opposite angles, therefore, in ΔACD,

Ext. $\angle 3 > \angle 2 \Rightarrow \angle 3 > \angle 1$

[$\because$ AD is the bisector of $\angle BAC$, so $\angle 1 = \angle 2$]

Now, in ΔABD, we have

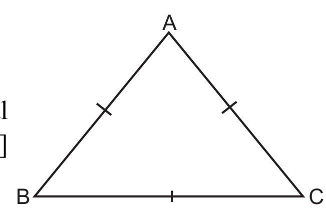
$$\angle 3 > \angle 1$$

Hence, $AB > BD$. [$\because$ In a triangle, side opposite to greater angle is longer]

EXERCISE 7.4

1. Find all the angles of an equilateral triangle.

Sol. In Δ ABC, we have



$$AB = AC$$

$$\Rightarrow \quad \angle C = \angle B \quad ...(1)$$

[$\because$ Angles opposite to equal sides of a triangle are equal]

$$BC = AC$$

$$\Rightarrow \quad \angle A = \angle B \quad ...(2)$$

[$\because$ Angles opposite to equal sides of a triangle are equal]

Now, $\angle A + \angle B + \angle C = 180°$ [$\because$ Angle sum property of a triangle]

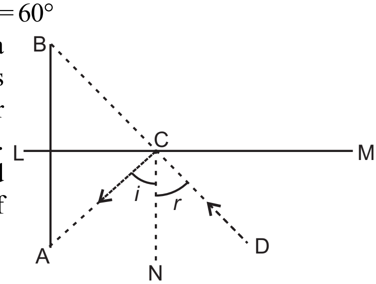
$\Rightarrow$ $\angle A + \angle A + \angle A = 180°$ [From (1) and (2)]

$\Rightarrow$ $3\angle A = 180°$

$$\Rightarrow \quad \angle A = \frac{180^\circ}{3} = 60^\circ$$

$\therefore$ $\angle A = \angle B = \angle C = 60°$

2. The image of an object placed at a point A before a plane mirror LM is seen at the point B by an observer at D as shown in the given figure. Prove that the image is as far behind the mirror as the object is in front of the mirror.



[**Hint:** CN is normal to the mirror. Also, angle of incidence = angle of reflection].

Sol. Let AB intersect LM at O. We have to prove that AO = BO.

Now, $\angle i = \angle r$ ...(1)

[$\because$ Angle of incidence = Angle of reflection]

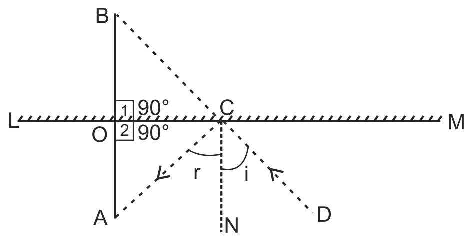
$\angle B = \angle i$ [Corres. $\angle s$] ...(2)

and $\angle A = \angle r$ [Alternate int. $\angle s$]...(3)

From (1), (2) and (3), we get

$\angle B = \angle A$

$\Rightarrow$ $\angle BCO = \angle ACO$



In ΔBOC and ΔAOC, we have

$\angle 1 = \angle 2$ [Each = 90°]

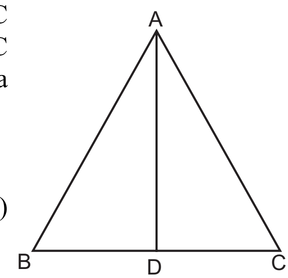
$OC = OC$ [Common side]

and $\angle BCO = \angle ACO$ [Proved above]

$\therefore$ $\Delta BOC \cong \Delta AOC$ [ASA congruence rule]

Hence, $AO = BO$ [CPCT]

3. ABC is an isosceles triangle with AB = AC and D is a point on BC such that AD $\perp$ BC (See fig.). To prove that $\angle BAD = \angle CAD$, a student proceeded as follows:



In ΔABD and ΔACD,

$AB = AC$ (Given)

$\angle B = \angle C$ (because $AB = AC$)

and $\angle ADB = \angle ADC$

Therefore,

$\Delta ABD = \Delta ACD$ (AAS)

So, $\angle BAD = \angle CAD$ (CPCT)

What is the defect in the above arguments?

[**Hint**: Recall how $\angle B = \angle C$ is proved when $AB = AC$]

Sol. In Δ ADB and ΔADC, we have

$\angle ADB = \angle ADC$

[$\therefore$ Each equal to 90°]

$AB = AC$ [Given]

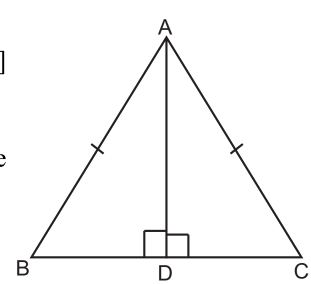
$AD = AD$ [Common side]

So, by RHS criterion of congruence, we have

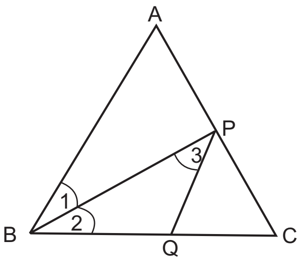
$\Delta ADB \cong \Delta ADC$

$\therefore$ $\angle BAD = \angle CAD$ [CPCT]

Hence, proved.



4. P is a point on the bisector of $\angle ABC$. If the line through P, parallel to BA meet BC at Q, prove that BPQ is an isosceles triangle.



Sol. We have to prove that BPQ is an isosceles triangle.

$\angle 1 = \angle 2$ ...(1)

[$\because$ BP is the bisector of $\angle ABC$]

Now, PQ is parallel to BA and BP cuts them

$\therefore$ $\angle 1 = \angle 3$ [Alt. $\angle$s] ...(2)

From (1) and (2), we get

$\angle 2 = \angle 3$

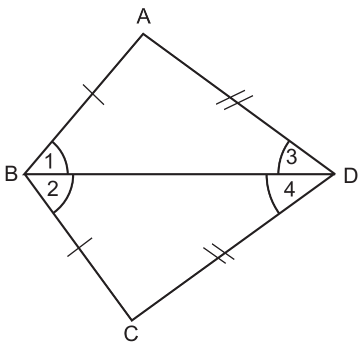
In ΔBPQ, we have

$\angle 2 = \angle 3$ [Proved above]

$\therefore$ $PQ = BQ$ [$\because$ Sides opp. to equal angles are equal]

Hence, BPQ is an isosceles triangle.

5. ABCD is a quadrilateral in which AB = BC and AD = CD. Show that BD bisects both the angles ABC and ADC.



Sol. In ΔABD and ΔCBD, we have

$AB = BC$ [Given]

$AD = CD$ [Given]

$BD = BD$ [Common side]

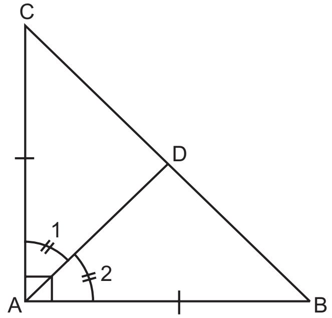
$\therefore$ $\Delta ABD \cong \Delta CBD$ [By SSS congruence rule]

$\Rightarrow$ $\angle 1 = \angle 2$ [CPCT]

and $\angle 3 = \angle 4$

Hence, BD bisects both the angles ABC and ADC.

6. ABC is a right triangle with AB = AC. Bisector of $\angle A$ meets BC at D. Prove that BC = 2AD.



Sol. Given : A right angled triangle with AB = AC and bisector of $\angle A$ meets BC at D.

To prove: BC = 2 AD

Proof : In right Δ ABC,

$AB = AC$ [Given]

$\Rightarrow$ BC is hypotenuse

[$\because$ Hypotenuse is the longest side.]

$\therefore$ $\angle BAC = 90°$

Now, in ΔCAD and ΔBAD, we have

$AC = AB$ [Given]

$\angle 1 = \angle 2$ [$\because$ AD is the bisector of $\angle A$]

$AD = AD$ [Common side]

So, by SAS criterion of congruence, we have

$\Delta CAD \cong \Delta BAD$

$\therefore$ $CD = BD$ [CPCT]

$\Rightarrow$ $AD = BD = CD$ ...(1)

[$\because$ Mid-point of hypotenuse of a rt. Δ is equidistant from the three vertices of a Δ]

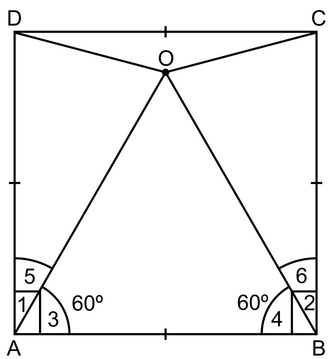
Now, $BC = BD + CD$

$\Rightarrow$ $BC = AD + AD$ [Using (1)]

$\Rightarrow$ $BC = 2AD$

Hence, proved.

7. O is a point in the interior of a square ABCD such that OAB is an equilateral triangle. Show that ΔOCD is an isosceles triangle.



Sol. Given: A square ABCD and $OA = OB = AB$.

To prove: ΔOCD is an isosceles triangle.

Proof: In square ABCD,

$\angle 1 = \angle 2$ ...(1)

[$\because$ Each equal to 90°]

Now, in ΔOAB, we have

$\angle 3 = \angle 4$ ...(2) [$\because$ Each equal to 60°]

Subtracting (2) from (1), we get

$\angle 1 - \angle 3 = \angle 2 - \angle 4$

$\Rightarrow$ $\angle 5 = \angle 6$

Now, in ΔDAO and ΔCBO,

$AD = BC$ [Given]

$\angle 5 = \angle 6$ [Proved above]

$OA = OB$ [Given]

So, by SAS criterion of congruence, we have

$\Delta DAO \cong \Delta CBO$

$\therefore$ $OD = OC$

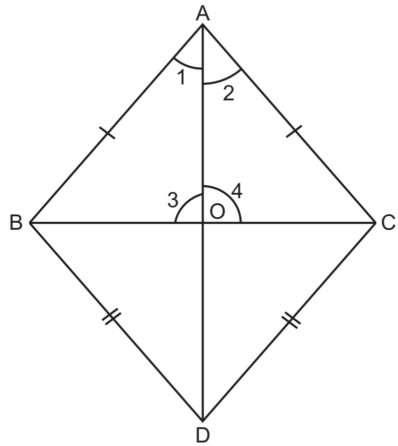
$\Rightarrow$ ΔOCD is an isosceles triangle.

Hence, proved.

8. ABC and DBC are two triangles on the same base BC such that A and D lie on the opposite sides of BC, $AB = AC$ and $DB = DC$. Show that AD is the perpendicular bisector of BC.

Sol. Given : ΔABC and ΔDBC on the same base BC. Also, $AB = AC$ and $DB = DC$.

To prove : AD is the perpendicular bisector of BC i.e., $OB = OC$



Proof : In ΔBAD and ΔCAD, we have

$AB = AC$ [Given]

$BD = CD$ [Given]

$AD = AD$ [Common side]

So, by SSS criterion of congruence, we have

$\Delta BAD \cong \Delta CAD$

$\therefore \quad \angle 1 = \angle 2$ [CPCT]

Now, in ΔBAO and ΔCAO, we have

$AB = AC$ [Given]

$\angle 1 = \angle 2$ [Proved above]

$AO = AO$ [Common side]

So, by SAS criterion of congruence, we have

$\Delta BAO \cong \Delta CAO$

$\therefore \quad BO = CO$ [CPCT]

and $\quad \angle 3 = \angle 4$ [CPCT]

But, $\angle 3 + \angle 4 = 180°$ [Linear pair axiom]

$\Rightarrow \quad \angle 3 + \angle 3 = 180°$

$\Rightarrow \quad 2\angle 3 = 180°$

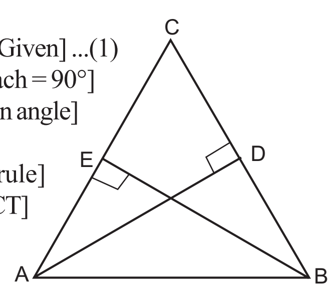
$\Rightarrow \quad \angle 3 = \dfrac{180°}{2} = 90°$

$\therefore$ AD is perpendicular bisector of BC [$\because$ $BO = CO$ and $\angle 3 = 90°$]

Hence, proved.

9. ABC is an isosceles triangle in which $AC = BC$. AD and BE are respectively two altitudes to the sides BC and AC. Prove that $AE = BD$.

Sol. In ΔADC and ΔBEC, we have



$AC = BC$ [Given] ...(1)

$\angle ADC = \angle BEC$ [Each $= 90°$]

$\angle ACD = \angle BCE$ [Common angle]

$\therefore \quad \Delta ADC \cong \Delta BEC$

[By AAS congruence rule]

$\therefore \quad CE = CD$ ...(2) [CPCT]

Subtracting (2) from (1), we get

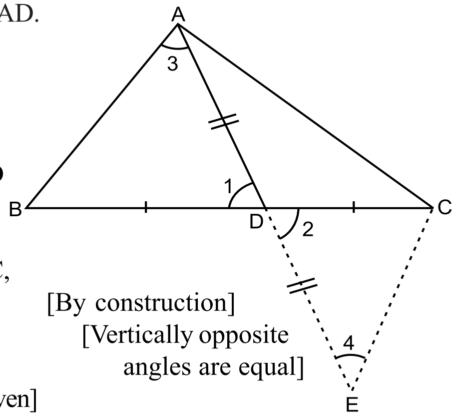
$AC - CE = BC - CD$

$\Rightarrow \qquad AE = BD$

Hence, proved.

10. Prove that sum of any two sides of a triangle is greater than twice the median with respect to the third side.

Sol. Given : ΔABC with median AD.



To prove :

$AB + AC > 2\,AD$

$AB + BC > 2\,AD$

$BC + AC > 2\,AD$

Construction: Produce AD to E such that DE = AD and join EC.

Proof : In ΔADB and ΔEDC,

$AD = ED$ [By construction]

$\angle 1 = \angle 2$ [Vertically opposite angles are equal]

$DB = DC$ [Given]

So, by SAS criterion of congruence, we have

$\Delta ADB \cong \Delta EDC$

$\therefore \qquad AB = EC$ [CPCT]

and, $\qquad \angle 3 = \angle 4$ [CPCT]

Now, in ΔAEC, we have

$AC + CE > AE$ [$\because$ Sum of the lengths of any two sides of a triangle must be greater than the third side]

$\Rightarrow \qquad AC + CE > AD + DE$

$\Rightarrow \qquad AC + CE > AD + AD$ [$\because$ AD = DE]

$\Rightarrow \qquad AC + CE > 2AD$

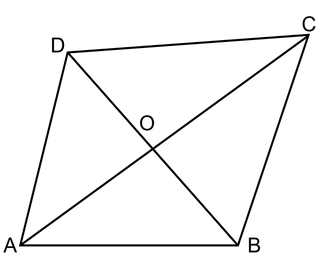
$\Rightarrow \qquad AC + AB > 2AD$ [$\because$ AB = EC]

Hence, proved.

Similarly, $AB + BC > 2AD$ and $BC + AC > 2AD$.

11. Show that in a quadrilateral ABCD, $AB + BC + CD + DA < 2(BD + AC)$.

Sol. Given : A quadrilateral ABCD.



To prove :

$AB + BC + CD + DA + < 2\,(BD + AC)$

Proof : In ΔAOB, we have

$\Rightarrow OA + OB > AB \qquad \ldots(1)$

[$\because$ Sum of the lengths of any two sides of a triangle must be greater than the third side]

In ΔBOC, we have

$OB + OC > BC$ ...(2) [Same reason]

In ΔCOD, we have

$OC + OD > CD$ ...(3) [Same reason]

In ΔDOA, we have

$OD + OA > DA$ ...(4) [Same reason]

Adding (1), (2), (3) and (4), we get

$OA + OB + OB + OC + OC + OD + OD + OA > AB + BC + CD + DA$

$\Rightarrow \quad 2(OA + OB + OC + OD) > AB + BC + CD + DA$

$\Rightarrow \quad 2\{(OA + OC) + (OC + OD)\} > AB + BC + CD + DA$

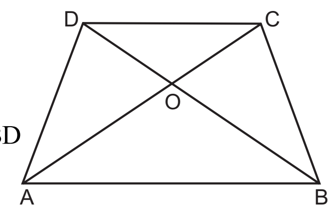
$\Rightarrow \quad 2(AC + BD) > AB + BC + CD + DA$

$\Rightarrow \quad AB + BC + CD + DA < 2(BD + AC)$

Hence, proved.

12. Show that in a quadrilateral ABCD.

$AB + BC + CD + DA > AC + BD$



Sol. Given : A quadrilateral ABCD.

To prove : $AB + BC + CD + DA > AC + BD$

Proof : In ΔABC, we have

$AB + BC > AC$ ...(1)

[$\because$ Sum of the lengths of any two sides of a triangle must be greater than the third side]

In ΔBCD, we have

$BC + CD > BD$ ...(2) [Same reason]

In ΔCDA, we have

$CD + DA > AC$ ...(3) [Same reason]

In ΔDAB, we have

$AD + AB > BD$ ...(4) [Same reason]

Adding (1), (2), (3) and (4), we get

$AB + BC + BC + CD + CD + DA + AD + AB > AC + BD + AC + BD$

$\Rightarrow \quad 2AB + 2BC + 2CD + 2DA > 2AC + 2BD$

$\Rightarrow \quad 2(AB + BC + CD + DA) > 2(AC + BD)$

$\Rightarrow \quad AB + BC + CD + DA > AC + BD$

Hence, proved.

13. In a triangle ABC, D is the mid-point of side AC such that $BD = \frac{1}{2} AC$. Show that $\angle ABC$ is a right angle.

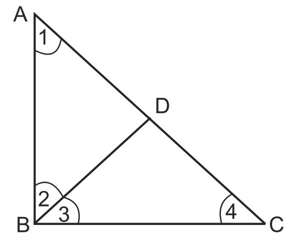
Sol. We have to prove that $\angle ABC = 90°$.

As D is the mid-point of AC,

so, $\quad AD = DC$

Also, $\quad BD = \frac{1}{2} AC = AD \quad$ [$\because$ D is the mid-point of AC]

$\therefore \quad BD = AD = DC$



In ΔABD, we have

$$BD = AD$$

$\therefore \quad \angle 1 = \angle 2$ ...(1)

[$\because$ Angles opposite to equal sides are equal]

In ΔBDC, we have

$$BD = DC,$$

$\therefore \quad \angle 3 = \angle 4$ ...(2)

[$\because$ Angles opposite to equal sides are equal]

In ΔABC, we have

$$\angle 1 + \angle ABC + \angle 4 = 180^\circ$$

$\Rightarrow \quad \angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$ [$\because \angle ABC = \angle 3 + \angle 2$]

$\Rightarrow \quad 2(\angle 2 + \angle 3) = 180^\circ$

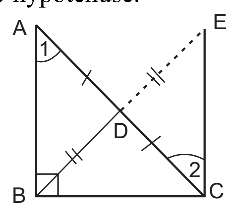
$\Rightarrow \quad \angle 2 + \angle 3 = 90^\circ$ [By (1) and (2)]

$\Rightarrow \quad \angle ABC = 90^\circ$

Hence, proved.

14. In a right triangle, prove that the line segment joining the mid-point of the hypotenuse to the opposite vertex is half the hypotenuse.

Sol. ABC is a right triangle, right angled at B and D is the mid-point of AC. We have to prove that $BD = \frac{1}{2} AC$.



Now, produce BD to E such that BD = DE. Join EC.

In ΔADB and ΔCDE, we have

$AD = CD$ [$\because$ D is the mid-point of AC]

$\angle ADB = \angle CDE$ [Vertically opposite $\angle$s]

$BD = DE$ [By construction]

$\therefore \quad \Delta ADB \cong \Delta CDE$ [By SAS criterion of congruence]

$\therefore \quad AB = EC$ [CPCT]

and $\quad \angle 1 = \angle 2$ [CPCT]

But, $\angle 1$ and $\angle 2$ are alternate angles.

$\therefore \quad EC \parallel BA$

Now, EC is parallel to BA and BC is the transversal

$\therefore \quad \angle ABC + \angle BCE = 180^\circ$

$\Rightarrow \quad 90^\circ + \angle BCE = 180^\circ$

$\Rightarrow \quad \angle BCE = 180^\circ - 90^\circ = 90^\circ$

In ΔABC and ΔEBC, we have

$BC = BC$ [Common side]

$AB = EC$ [Proved above]

$\angle CBA = \angle BCE$ [$\because$ Each $= 90^\circ$]

$\therefore \quad \Delta ABC \cong \Delta EBC$ [By SAS criterion of congruence]

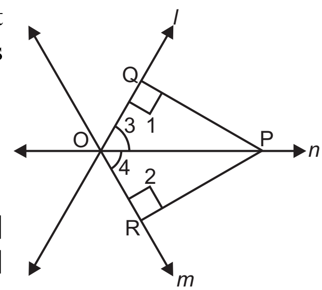
$\therefore \quad AC = EB$ [CPCT]

$\Rightarrow \quad \frac{1}{2}AC = \frac{1}{2}EB \Rightarrow \frac{1}{2}AC = BD$

Hence, $BD = \frac{1}{2}AC$.

15. Two lines l and m intersect at the point O and P is a point on a line n passing through the point O such that P is equidistant from l and m. Prove that n is the bisector of the angle formed by l and m.

Sol. Given : Lines l, m and n intersect at point O. P is a point on line n such that P is equidistant from l and n.



To prove : n is the bisector of $\angle QOR$.

Proof : In ΔOQP and ΔORP, we have

$\angle 1 = \angle 2$ [$\because$ Each equal to 90°]

$OP = OP$ [Common side]

$PQ = QR$ [Given]

So, by RHS criterion of congruence, we have

$\Delta OQP \cong \Delta ORP$

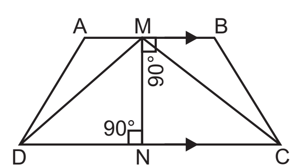
$\therefore \quad \angle 3 = \angle 4$ [CPCT]

So, n is bisector of $\angle QOR$.

Hence, proved.

16. Line-segment joining the mid-points M and N of parallel sides AB and DC respectively of trapezium ABCD is perpendicular to both the sides AB and DC. Prove that AD = BC.

Sol. Join MD and CM.



We have, $\angle DNM = \angle NMB$ [Alt. $\angle$s] $\because AB \parallel CD$

Now, in ΔDMN and ΔCNM,

$CN = DN$ [$\because$ N is the mid-point of DC]

$\angle DNM = \angle CNM$ [Each = 90°]

$NM = NM$ [Common side]

$\therefore \quad \Delta DMN \cong \Delta CNM$ [By SAS congruence rule]

$\therefore \quad DM = CM$ and $\angle NMC = \angle NMD$ ...(1)[CPCT]

Now, $\angle AMN = \angle BMN$ [Each = 90°]

and $\angle NMD = \angle NMC$ [Proved above]

$\therefore \quad \angle AMN - \angle NMD = \angle BMN - \angle NMC$ [on subtraction]

$\Rightarrow \quad \angle AMD = \angle BMC$ ...(2)

$AM = BM$ [Given]

$DM = CM$ [From (1)]

$\angle AMD = \angle BMC$ [From (2)]

$\Delta AMD \cong \Delta MBC$ [By SAS congruence rule]

$\therefore \quad AD = BC$ [CPCT]

17. ABCD is a quadrilateral such that diagonal AC bisects the angles A and C. Prove that AB = AD and CB = CD.

Ans. Given : A quadrilateral ABCD such that $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$.

To prove : AB = AD and CB = CD

Proof : In ΔABC and ΔADC, we have

$\angle 1 = \angle 2$ [Given]

$AC = AC$ [Common side]

$\angle 3 = \angle 4$ [Given]

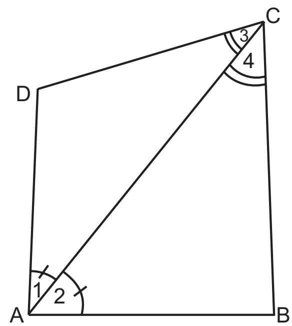
So, by ASA criterion of congruence, we have

$\Delta ABC \cong \Delta ADC$

$\therefore \quad AB = AD$ [CPCT]

and $\quad CB = CD$ [CPCT]

Hence, proved.



18. ABC is a right triangle such that AB = AC and bisector of angle C intersects the side AB at D. Prove that AC + AD = BC.

Sol. Given : A right triangle ABC, AB = AC and CD is the bisector of $\angle C$.

To prove : AC + AD = BC

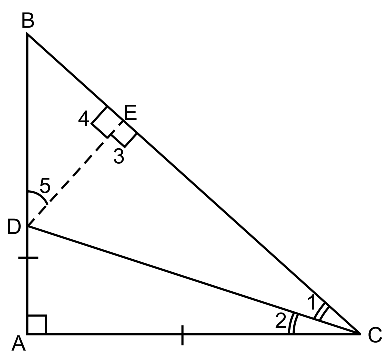
Construction: Draw $DE \perp BC$.

Proof : In right triangle ABC, we have

$AB = AC$ [Given]

$\therefore$ BC is hypotenuse

$\Rightarrow \quad \angle A = 90°$



In ΔDAC and ΔDEC, we have

$\angle A = \angle 3$ [$\because$ Each equal to 90°]

$\angle 1 = \angle 2$ [Given]

$DC = DC$ [Common side]

So, by AAS criterion of congruence, we have

$\Delta DAC \cong \Delta DEC$

$\therefore \quad DA = DE$ …(1) [CPCT]

and $\quad CA = CE$ …(2) [CPCT]

In ΔBAC, we have

$AB = AC$ [Given]

$\Rightarrow \angle C = \angle B$ [$\because$ Angles opposite to equal sides of a triangle are equal]

Now, $\angle A + \angle B + \angle C = 180°$ [Angle sum property of a triangle]

$\Rightarrow 90° + \angle B + \angle B = 180°$ [$\because \angle B = \angle C$]

$\Rightarrow 2\angle B = 90°$

$\Rightarrow \angle B = \dfrac{90°}{2} = 45°$

Now, in $\angle BED$, we have

$\Rightarrow \angle 4 + \angle 5 + \angle B = 180°$ [Angle sum property of a triangle]

$\Rightarrow 90° + \angle 5 + 45° = 180°$

$\Rightarrow \angle 5 = 180° - 135°$

$\Rightarrow \angle 5 = 45°$

$\therefore \angle B = \angle 5$

$\Rightarrow DE = BE$ ...(3) [$\because$ Sides opposite to equal angles of a triangle are equal]

From (1) and (3), we get

$DA = DE = BE$ ...(4)

Now, $BC = CE + BE$

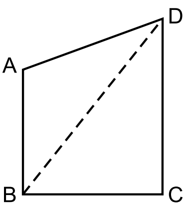
$\Rightarrow BC = CA + DA$ [Using (2), (3) and (4)]

$\Rightarrow BC = AC + AD$

$\Rightarrow AC + AD = BC$

Hence, proved.

19. AB and CD are the smallest and largest sides of a quadrilateral ABCD. Out of $\angle B$ and $\angle D$ decide which is greater.



Sol. Given: A quadrilateral ABCD in which AB and CD are the smallest and largest sides of quadrilateral ABCD.

To prove: $\angle B > \angle D$

Construction: Join BD.

Proof: In ΔABD, we have

$\Rightarrow AB < AD$

[$\because$ AB is the smallest side of quadrilateral ABCD]

$\Rightarrow AD > AB$

$\Rightarrow \angle ABD > \angle ADB$ ...(1)[$\because$ Angle opposite to longest side is greater]

Again, in ΔCBD, we have

$CD > BC$ [$\because$ CD is the longest side of quadrilateral ABCD]

$\Rightarrow \angle CBD > \angle BDC$ ...(2) [$\because$ Angle opposite to longest side is greater]

Adding (1) and (2), we get

$\angle ABD + \angle CBD > \angle ADB + \angle BDC$

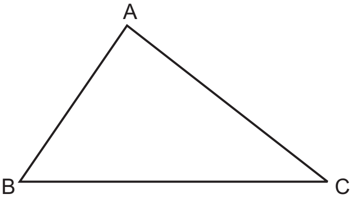
$\Rightarrow \angle ABC > \angle ADC$

$\Rightarrow \qquad \angle B > \angle D$

Hence, proved.

20. Prove that in a triangle, other than an equilateral triangle, angle opposite the longest side is greater than $\frac{2}{3}$ of a right angle.

Sol. Given : A triangle ABC, other than an equilateral triangle.



To prove : $\angle A > \frac{2}{3}$ rt. $\angle$

Proof : In ΔABC, we have

$BC > AB$

$\Rightarrow \qquad \angle A > \angle C$ ...(1) [$\because$ In a triangle, angle opposite to the longer side is larger]

$BC > AC$

$\Rightarrow \qquad \angle A > \angle B$ ...(2) [$\because$ In a triangle, angle opposite to the longer side is larger]

Adding (1) and (2), we get

$A + \angle A > \angle B + \angle C$

$\Rightarrow \qquad 2\angle A > \angle B + \angle C$

Now, adding $\angle A$ on both sides, we get

$2\angle A + \angle A > \angle A + \angle B + \angle C$

$\Rightarrow \qquad 3\angle A > \angle A + \angle B + \angle C$

$\Rightarrow \qquad 3\angle A > 180^\circ$ [Angle sum property of a triangle]

$\Rightarrow \qquad \angle A > \frac{180^\circ}{3}$

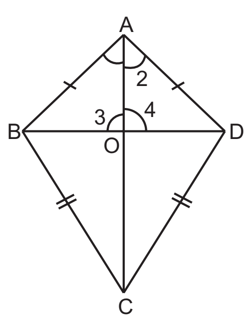
$\Rightarrow \qquad \angle A > \frac{2}{3} \times 90^\circ$

$\Rightarrow \qquad \angle A > \frac{2}{3}$ rt. $\angle$

Hence, proved.

21. ABCD is quadrilateral such that AB = AD and CB = CD. Prove that AC is the perpendicular bisector of BD.

Sol. Given : A quadrilateral ABCD in which AB = AD and CB = CD.



To prove : AC is the perpendicular bisector of BD.

Proof : In ΔABC and ΔADC, we have

$AB = AD$ [Given]

$BC = CD$ [Given]

$AC = AC$ [Common side]

So, by SSS criterion of congruence, we have

$\Delta ABC \cong \Delta ADC$

$\therefore \quad \angle 1 = \angle 2$ [CPCT]

Now, in ΔAOB and ΔAOD, we have

$AB = AD$ [Given]

$\angle 1 = \angle 2$ [Proved above]

$AO = AO$ [Common side]

So, by SAS criterion of congruence, we have

$\Delta AOB \cong \Delta AOD$

$\therefore \quad BO = DO$ [CPCT]

and $\angle 3 = \angle 4$ [CPCT]

But, $\angle 3 + \angle 4 = 180°$ [Linear pair axiom]

$\Rightarrow \quad \angle 3 + \angle 3 = 180°$ [$\because \angle 3 = \angle 4$]

$\Rightarrow \quad 2\angle 3 = 180°$

$$\Rightarrow \quad \angle 3 = \frac{180°}{2} = 90°$$

$\therefore$ AC is perpendicular bisector of BC [$\because \angle 3 = 90°$ and $BO = DO$]

Hence, proved.