JEE Main 2023 (Memory based)

24 January 2023 - Shift 1

Answer & Solutions

PHYSICS

1. The equation of wave is given as $y = 0.05\sin(2x - 4t)$, where x in meters and t is time in seconds. The velocity of the wave is equal to

A. 2
B. 4
C. 0.5
D. 0.25

Answer (A)

Sol. Comparing it with the general equation of wave; $y = A\sin(kx - \omega t + \phi)$

$k = 2, \omega = 4$

$V = \frac{\omega}{k} = \frac{4}{2} = 2\, m/s$

2. Two charges q_1 and q_2 separated by a distance d are placed in a medium of dielectric constant k, if they are placed in the air then find equivalent distance at which they experience same force.

A. $d\sqrt{k}$
B. $k\sqrt{d}$
C. $2d\sqrt{k}$
D. $1.5d\sqrt{k}$

Answer (A)

Sol.

Without dielectric placed in between, $F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$

With dielectric placed in between, $F' = \frac{q_1 q_2}{4\pi k\epsilon_0 d^2}$

At equivalent distance (r_{eq})

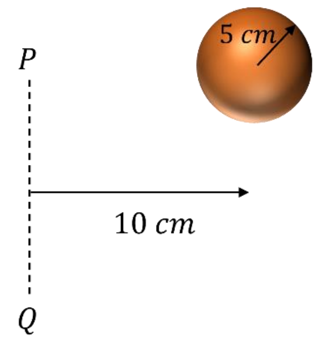
$$\frac{q_1 q_2}{4\pi\epsilon_0 r_{eq}^2} = \frac{q_1 q_2}{4\pi k\epsilon_0 d^2}$$

$$r_{eq}^2 = kd^2$$

$$r_{eq} = d\sqrt{k}$$

3. Find the radius of gyration for the uniform solid sphere of radius $5\ cm$ about the axis PQ, as shown in the figure.

A. $5\ cm$
B. $10\ cm$
C. $\sqrt{110}\ cm$
D. $\sqrt{90}\ cm$



Answer (C)

Sol.

Applying parallel axis - theorem

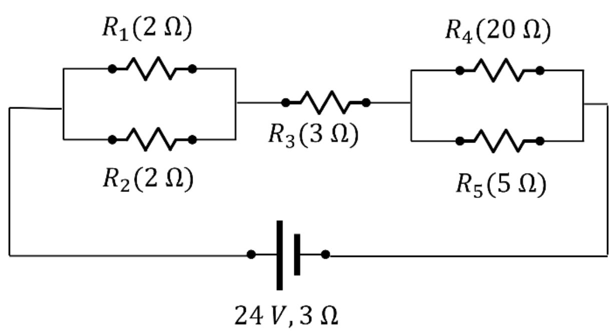
$MK^2 = M\left(\frac{2}{5}R^2 + d^2\right)$

$K^2 = \frac{2}{5} \times 25 + 100$

$K = \sqrt{110}\ cm$

4. In the circuit shown, Find the current through $R_4(I_4)$ and $R_5(\mathrm{I}_5)$

A. $\mathrm{I}_4 = \frac{24}{55}\ \mathrm{A}, \mathrm{I}_5 = \frac{96}{55}\ \mathrm{A}$
B. $\mathrm{I}_4 = \frac{96}{55}\ \mathrm{A}, \mathrm{I}_5 = \frac{24}{55}\ \mathrm{A}$
C. $\mathrm{I}_4 = \frac{24}{37}\ \mathrm{A}, \mathrm{I}_5 = \frac{96}{37}\ \mathrm{A}$
D. $\mathrm{I}_4 = \frac{96}{37}\ \mathrm{A}, \mathrm{I}_5 = \frac{24}{37}\ \mathrm{A}$



Answer (A)

Sol.

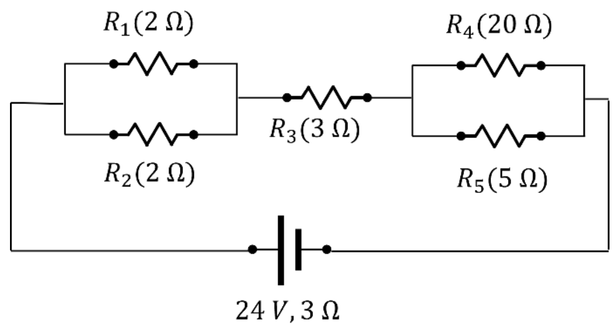
$R_{eq} = 2 \times \frac{2}{4} + 20 \times \frac{5}{25} + 3 + 3$

$R_{eq} = 11\ \Omega$

$Current, I_T = \frac{24}{11}\ A$

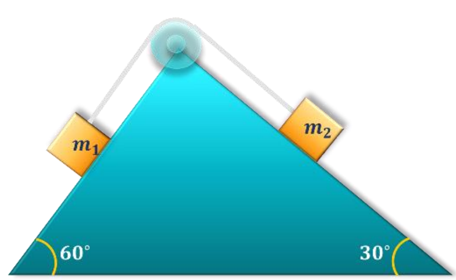
$I_4 = \frac{5}{25} \times \frac{24}{11}\ A = \frac{24}{55}\ A$

$I_5 = \frac{20}{25} \times \frac{24}{11}\ A = \frac{96}{55}$



5. In the figure shown two blocks of masses $m_1 = 4\ kg$ and $m_2 = 1\ kg$ are placed over a smooth fixed wedge, connected by an ideal string over a smooth pulley. As the system is released the tension in the string is

A. $4(\sqrt{3}+1)\ N$
B. $10\left(1 - \frac{1}{\sqrt{3}}\right)\ N$
C. $10(\sqrt{3}-1)\ N$
D. $\frac{10}{3}(\sqrt{3}-1)\ N$



Answer (A)

Sol.

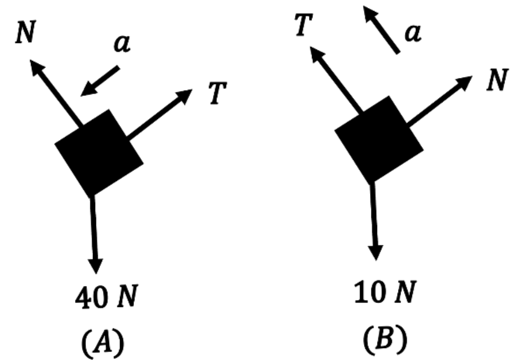
Equation for A parallel to the surface

$$40 \sin 60^0 - T = 4a$$

Equation for B parallel to the surface

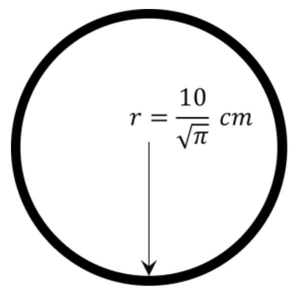
$$T - 10 \sin 30^0 = a$$

On solving: $4(\sqrt{3}+1)\ N$



6. A circular loop of radius $\frac{10}{\sqrt{\pi}}\ cm$ is placed in a linearly varying perpendicular magnetic field which has magnitude $0.5\ T$ at time $t = 0$. The magnetic field reduces to zero at $t = 0.5\ sec$. Find the emf induced in the loop at $t = 0.25\ sec$.

A. $0.01\ V$
B. $0.005\ V$
C. $0.02\ V$
D. $0.03\ V$



Answer: (A)

Sol.

$B = 0.5\ T$

$B = 0,\ at\ t = 0.5\ sec$

Assuming linear graph between B & t

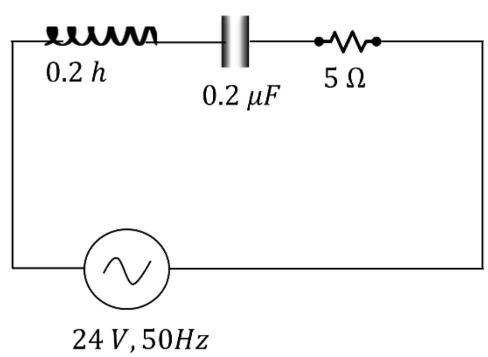
$$\varepsilon_{ind} = \frac{\Delta\phi}{\Delta t} = \frac{\Delta(BA)}{\Delta t} = A\frac{\Delta(B)}{\Delta t}$$

$$= \pi \times \left(\frac{10}{\sqrt{\pi}}\right)^2 \times 10^{-4} \times \left(\frac{0.25}{0.25}\right)$$

$$= 10^{-2} \times 1\ = 0.01\ V$$

7. Calculate the ratio between bandwidth and quality factor for the following circuit

A. 1/3
B. $1/8$
C. $1/16$
D. $1/4$



Answer (B)

Sol.

For an RLC circuit

Band width $= R/L$

$$= \frac{5}{0.2}\ Hz$$

$$= 25\ Hz$$

For an RLC circuit quality factor

$= \frac{\sqrt{L}}{R\sqrt{C}}$

$= \frac{\sqrt{0.2}}{5 \times \sqrt{(0.2 \times 10^{-6})}}$

$= 200$

$\frac{BW}{Q} = \frac{25}{200} = 1/8$

8. If a ball is thrown from ground in vertical plane, it attains maximum height of $360\ m$. Find the maximum distance, the ball can cover on ground keeping the projection speed constant.

A. $360\ m$
B. $720\ m$
C. $1440\ m$
D. $180\ m$

Answer (B)

Sol.

For ground projectile, Range = $2\ \times$ Maximum height $= 2 \times 360 = 720\ m$

9. Which statement is correct about photoelectric effect?

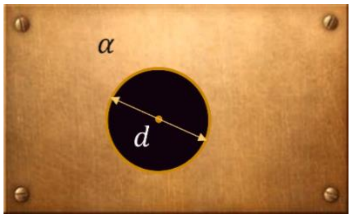
A. Maximum kinetic energy depends upon intensity of light.
B. Stopping potential is dependant only on work function of metal.
C. Photoelectric effect can be explained by wave nature of light.
D. Photoelectric effect can be explained by particle nature of light.

Answer (D)

Sol.

We know that photoelectric effect is supported by particle nature of light, so option D is correct.

10. A uniform rectangular plate has a circular hole of diameter 'd' as shown. The coefficient of linear expansion of the plate is α. Find the change in diameter of the hole, if temperature of the plate is increased by ΔT.



A. $2d\alpha\Delta T$
B. $d\alpha\Delta T$
C. $\frac{d}{2}\alpha\Delta T$
D. $3d\alpha\Delta T$

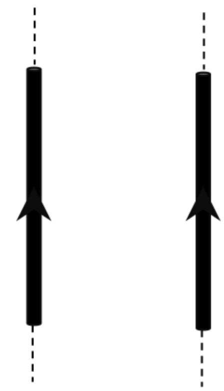
Answer (B)

Sol.

As we know that $\frac{\Delta d}{d} = \alpha\Delta T \implies \Delta d = d\alpha\Delta T$

11. Two parallel infinite wires carry equal currents as shown. If both the currents are doubled and separation is halved, the force on a $10\ cm$ section of one of the wires becomes:

A. 4 times
B. $1/4$ times
C. 8 times
D. $1/8$ times



Answer: (C)

Sol.

Magnetic force on length l of either wire

$F = \frac{\mu_0 I_1 I_2 l}{2\pi d}$

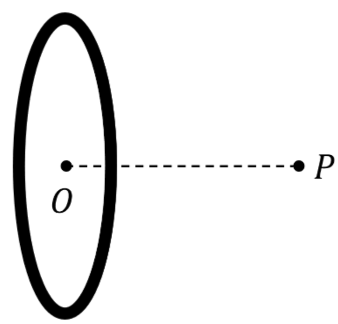
Original force, $F = \frac{\mu_0 I_0^2 l}{2\pi d}$

New force, $F' = \frac{\mu_0 \times 4 I_0^2 l}{2\pi\left(\frac{d}{2}\right)} = \frac{8\mu_0 I_0^2 l}{2\pi d}$

$F' = 8F$

12. A coil of radius R centred at O carries a current i. Point P is on the axis of coil at a distance R from the centre O as shown. Ratio of magnetic field at point O to magnetic field at point P is equal to

A. 2
B. $2\sqrt{2}$
C. $1/\sqrt{2}$
D. $1/2\sqrt{2}$



Answer (B)

Sol.

$B_0 = \frac{\mu i}{2R}$

$B_p = \frac{\mu i R^2}{2(R^2+R^2)^{3/2}} = \frac{\mu i}{4\sqrt{2}R}$

$\frac{B_0}{B_p} = 2\sqrt{2}$

13. Statement 1: Photodiodes are operated in reverse biased.

Statement 2 : Current in forward biased is more than current in reverse bias in $p-n$ diode.

A. Both the statements are true.
B. Statement 1 is true and statement 2 is false.
C. Statement 1 is true and statement 2 is false.
D. Both the statements are false.

Answer (A)

Sol.

Statement 1 is true as photodiode is used in reverse bias to increase the sensitivity of diode current.

Statement 2 is true as diode provides greater resistance in reverse bias.

14. Weight of an object at earth's surface is $18\ N$. If the object is taken $3200\ km$ above the surface, then the weight of the object (in N) is_______

(Given; radius of Earth =$6400\ km$)

Answer (8)

Sol.

As we know that;

$g = \frac{GM}{r^2}$

$g_{new} = \frac{GM}{\left(R+\frac{R}{2}\right)^2} = \frac{4}{9} \times \frac{GM}{R^2} = \frac{4}{9} g_{surface}$

$New\ weight = \frac{4}{9} \times 18\ N = 8\ N$

15. A block of mass $2\ kg$ is attached with two identical spring of force constant $20\ N/m$ as shown in the figure. If the time period of the oscillation of the block is $2\pi\sqrt{\frac{1}{x}}\ sec$. Find x.

Answer (20)

Sol.

Equivalent spring constant

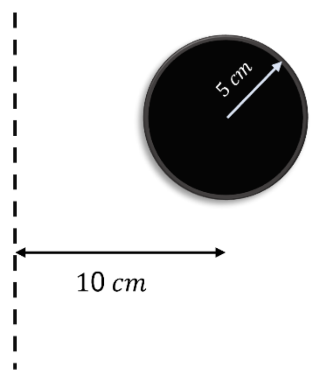
$k_{eq} = k_1 + k_2 = 40\ N/m$

Time period of system is:

$$T = 2\,\pi\sqrt{\frac{m}{k_{eq}}} = 2\,\pi\sqrt{\frac{2}{40}} = 2\,\pi\sqrt{\frac{1}{20}}$$

$x = 20$

16. A ring of uniform wire and radius $5\ cm$ is made to rotate about a coplanar axis which is at a distance of $10\ cm$ from the centre of the ring as shown. The radius of gyration of ring about the axis is equal to $\frac{15}{\sqrt{K}}\ cm$. The value of K is equal to



Answer : 2

Sol.

Moment of inertia is given as.

$$I_{axis} = \frac{m\ \times 5^2}{2} + m\ \times 10^2$$

$$= \frac{225}{2}\ m$$

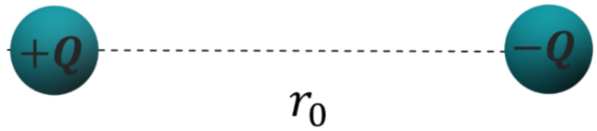
Let radius of gyration is K so,

$$mK^2 = \frac{225}{2}\ m$$

$K = \frac{15}{\sqrt{2}} cm$

So, the answer is **2**

17. Two charges (both at rest initially), having a charge Q and $-Q$ are released from the situation shown. If the kinetic energy of the system when the separation between them becomes half is $\frac{1}{4\pi\epsilon_0}\frac{Q^2}{nr_0}$, find n?



Answer: 1

Sol.

Initial potential energy $U_i = -\frac{1}{4\pi\epsilon_0} \times \frac{Q^2}{r_0}$

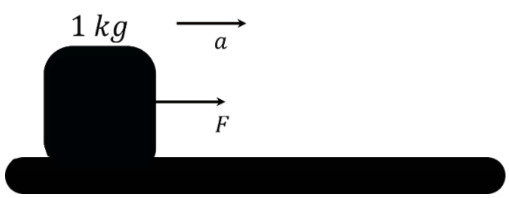
Final potential energy $U_f = -\frac{1}{4\pi\epsilon_0} \times \frac{Q^2}{r_0/2}$

Loss in potential energy ,$U_i - U_f = -\frac{1}{4\pi\epsilon_0} \times \frac{Q^2}{r_0} + \frac{1}{4\pi\epsilon_0} \times \frac{Q^2}{\frac{r_0}{2}} = \frac{1}{4\pi\epsilon_0} \times \frac{Q^2}{r_0}$

Kinetic energy$= \frac{1}{4\pi\epsilon_0} \times \frac{Q^2}{r_0}$

$n = 1$

18. A constant force acting on a body of mass $1\ kg$ provides it a kinetic energy of $1800\ J$ by the end of 5^{th} second. If the body was initially at rest at the beginning of action of force then magnitude of force is equal to ________ N.



Answer: 12 N

Sol.

$a = F/m$

As force is constant so block is moving with constant acceleration

$S = ½\ at^2 = Ft^2/2m$

From work energy theorem

$W = \Delta KE = \vec{F}.\vec{s}$

$$\frac{F^2t^2}{2m} = KE$$

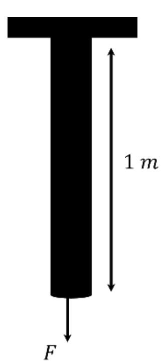
$$F = \sqrt{\frac{2mKE}{t^2}} = \sqrt{\frac{2 \times 1 \times 1800}{25}} = 12\ N$$

19. A light rod of cross-sectional area A and Young's Modulus Y is arranged as shown:

The applied force $F = 250\ N$. If length of rod is $1\ m$, the extension comes out to be $x \times 10^{-6}\ metres$. Find x.

Given that : $A = 6.25 \times 10^{-4}\ m^2$

$Y = 10^{10}\ N/m^2$



Answer: 40.00

Sol.

$$\Delta l = \frac{Fl}{AY}$$

$$\Delta l = \frac{250 \times 1}{6.25 \times 10^{-4} \times 10^{10}}$$

$$\Delta l = 40 \times 10^{-6}\ m$$

$$x = 40.00$$

20. **Statement 1:** If the weight of the lift is equal to the tension force of the cable wire, then it moves with uniform velocity.

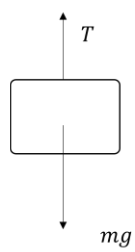
Statement 2: If the lift moves downward with an acceleration, then the contact force between the boy's feet and lift floor is more than the weight of boy.

A. Both the statements are true and (2) is the correct explanation of (1)
B. Both the statements are true and (2) is not the correct explanation of (1)
C. Statement 1 is true and statement 2 is false.
D. Statement 2 is true and statement 1 is false.

Answer (C)

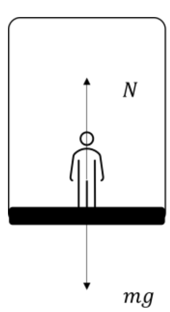
Sol.

Statement 1:



$a = 0,$ as lift is moving with constant velocity.
So, $T = mg$
Statement 1 is correct

Statement 2:



$mg - N = ma$
$N = m(g - a)$
So, $N < mg$
Statement 2 is incorrect.

21. Statement 1: Noradrenaline is one of the neurotransmitters

Statement 2: Deficiency of noradrenaline causes depression.

E. Statement 1 and 2 both are correct
F. Statement 1 is correct but statement 2 is incorrect
G. Statement 1 is incorrect but statement 2 is correct
H. Statement 1 and 2 both are incorrect.

Answer (A)

Sol.

Noradrenaline is one of the neurotransmitters that plays a role in mood changes. If the level of noradrenaline is low for some reason, then the signal-sending activity becomes low, and the person suffers from depression. So, statement 1 and 2 both are correct.

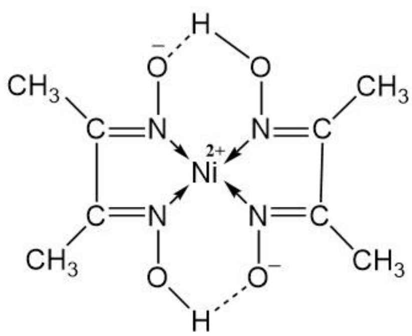
22. The cation gives bright red color with dimethyl glyoxime. Which is that cation?

A. Cu^{2+}
B. Ni^{2+}
C. Zn^{2+}
D. Co^{2+}

Answer (B)

Sol.

Ni^{2+} gives bright red color with dimethylglyoxime in an alkaline medium.



23. A 25 mL buffer solution is prepared by mixing CH_3COOH of concentration 0.1 M and CH_3COONa of concentration 0.01 M. If the P^H of the solution is 5, then calculate the pK_a of CH_3COOH

A. 4
B. 5
C. 6
D. 7

Answer (C)

Sol.

$[CH_3COOH] = 0.1$

$[CH_3COONa] = 0.01$

$p^H = pK_a + log \frac{[\mathrm{CH_3}COONa]}{[CH_3COOH]}$

$= p^H - log \frac{[0.01]}{[0.1]}$

$= p^H - \log[10^{-1}] = p^H + log10$

$pK_a = 5 + 1 = 6$

24. The number of unpaired electrons on cobalt in the complex ion $[CoCl_4]^{2-}$ is

A. 2
B. 3
C. 4
D. 5

Answer (B)

Sol.

$[\boldsymbol{CoCl_4}]^{2-} = \boldsymbol{Co^{2+}}(\boldsymbol{Td}) = \boldsymbol{e^4t_2^3}$

Hence number of unpaired electrons is 3

25. Magnetic moment of a metal ion is 3.87 B.M. Identify the metal ion?

A. V^{3+}
B. Cr^{3+}
C. Mn^{2+}
D. Ti^{2+}

Answer (B)

Sol.

Magnetic moment (μ) = $\sqrt{\boldsymbol{n}(\boldsymbol{n}+\boldsymbol{2})}$

n – number of unpaired electrons

The given Magnetic moment is 3.87 B.M.
This magnetic moment corresponds to 3 unpaired electrons.
V^{3+} , Cr^{3+} , Mn^{2+}, Ti^{2+}, has 2, 3, 5, 2 unpaired electrons respectively.

Hence Option B is the correct answer.

26. The Correct statement about freons is

A. They are used as a cancer medicine
B. They are chlorofluorocarbon compounds
C. These are toxic organic compounds
D. These are flammable compounds

Answer (B)

Sol.

The chlorofluorocarbons (CFC'S) are known as Freons. These are non-reactive, non-flammable, non-toxic organic molecules.

27. Statement 1: Freezing point of a solution decreases with decrease in the amount of non-volatile solute.

Statement 2: Freezing point of the solution is less than that of solvent.

A. Statement 1 and 2 both are correct.
B. State 1 is correct but statement 2 is incorrect
C. Statement 1 is incorrect but statement 2 is correct
D. Statement 1 and 2 both are incorrect

Answer (3)

Sol.

Freezing point of a solution decreases with increase in the amount of non-volatile solute.

Freezing point of the solution is less than that of solvent, in case of non-volatile solvent

28. Consider the following reaction given below $\boldsymbol{BeO + HF + NH_3 \rightarrow A \xrightarrow{\Delta} BeF_2 + (NH_4)F}$

Identify the missing compound A

A. $Be(OH)_2$
B. $(NH_4)_2(BeF_4)$
C. $(NH_4)_2(BeF_3)$
D. D. $(NH_4)_2(BeF_6)$

Answer (B)

Sol.

$$BeO + HF + NH_3 \rightarrow \underset{\textbf{(A)}}{(NH_4)_2(BeF_4)} \xrightarrow{\Delta} BeF_2 + NH_4F$$

Option (2) is the correct answer.

29.

List - I	List - II
A. Soda Ash	P. NaF
B. Chlorophyll	Q. $Ca(OH)_2$
C. White washing	R. Na_2CO_3
D. Tooth paste	S. Mg^{2+} ions

Match the compounds given in List – II with the compounds which are present in List – I

A. A – P, B – Q, C – R, D – S.
B. A – R, B – S, C – Q, D – P.
C. A – R, B – S, C – P, D – Q.
D. A – P, B – Q, C – S, D – R.

Answer (B)

Sol.

Soda Ash - Na_2CO_3

Chlorophyll - Mg^{2+} ions

White washing - $Ca(OH)_2$

Tooth paste - NaF

30. Calculate the percentage of Fe^{2+} ions in $Fe_{0.93}O$

A. 15 %
B. 85 %
C. 65 %
D. 35 %

Answer (B)

Sol.

Iron exists as Fe^{2+} and Fe^{3+} in $Fe_{0.93}O$

Let's take Fe^{2+} = x and Fe^{3+} = y

Thus, x+y = 0.93 -----(1)

Applying the concept of charge balancing

2x+3y =2 --------(2)

Upon solving (1) and (2)

y = 0.14

Percentage of $\boldsymbol{Fe^{3+}} = \frac{\mathbf{0.14}}{\mathbf{0.93}} \times \mathbf{100} = \mathbf{15}\%$

Percentage of $\boldsymbol{Fe^{2+}} = \mathbf{100 - 15 = 85}\%$

31. Following Reactions are given

A. $\Delta n = -25\ kJ/mol\ ;\ \Delta s = +30\ J/mol\ ; T = 300K$
B. $\Delta n = +30\ kJ/mol\ ;\ \Delta s = -50\ J/mol\ ; T = 300K$
C. $\Delta n = +30\ kJ/mol\ ;\ \Delta s = +500\ J/mol\ ; T = 300K$
D. $\Delta n = -30\ kJ/mol\ ;\ \Delta s = -1500\ J/mol\ ; T = 300K$

How many of the above reactions are non-Spontaneous under given conditions

Answer (2)

Sol.

Reaction (B)

Enthalpy change $(\Delta n) = +30000\ J/mol$

Entropy Change $(\Delta s) = -50 \frac{J}{mol}/K$

$\Delta G = \Delta n - T\Delta S$

For non- spontaneous reaction $\Delta G > 0$

$30000 + 15000 > 0$ (Non-Spontaneous)

Reaction (D)

Enthalpy change $(\Delta n) = -30000\, J/mol$

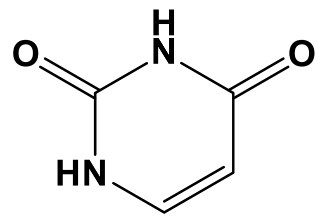
Entropy Change $(\Delta s) = -1500\,\frac{J}{mol}/K$

$\Delta G = \Delta n - T\Delta S$

$= -30000 - (300)(-1500)$

$= (-30000 + 45000) > 0$ (Non-Spontaneous)

32. The percentage of nitrogen in uracil is :



uracil

Answer (25)

Sol.

Molecular weight of uracil = 112 g

Weight of nitrogen = 28 g

% of Nitrogen = $\frac{28}{112}\, X\, 100 = 25\%$

33. Which of the following oxoacids can reduce $AgNO_3$?

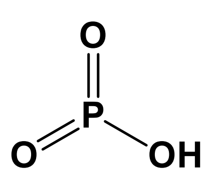
A. (HPO3)n
B. H4P2O7
C. H4P2O5
D. H3PO4

Answer (C)

Sol.

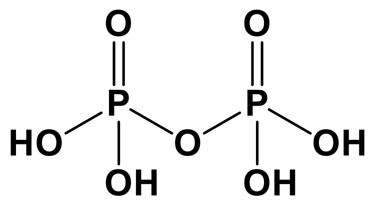
To reduce $AgNO_3$, oxoacid should behave as a reducing agent. Central atom with maximum oxidation state can act only as an oxidizing agent and in oxidation state less than the maximum can act as reducing agent.
The Maximum oxidation state of P is +5, thus

a. $(HPO_3)n$ – Metaphosphoric acid ; P is in +5 oxidation state. Thus, it behaves only as an oxidizing agent.



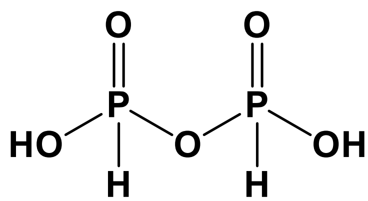
metaphosphoric acid

b. $H_4P_2O_7$ – Pyrophosphoric acid; P is in +5 oxidation state. Thus, it behaves only as an oxidizing agent.



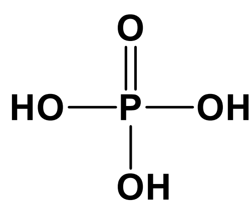
Pyrophosphoric acid

c. $H_4P_2O_5$ – Pyro phosphorous acid; P is in +3 oxidation state. $H_4P_2O_5$ can act as reducing agent due to the presence of P-H bond.



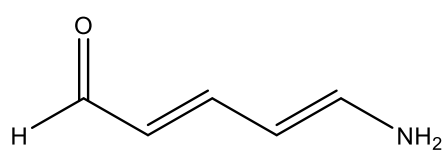
pyrophosphorous acid

d. H_3PO_4 – Orthophosphoric acid; P is in + 5 oxidation state. Thus, it behaves only as an oxidizing agent.

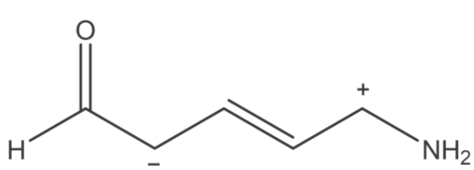


ortho phosphoric acid

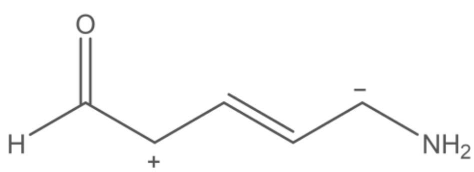
14. The correct stability order of the resonating structure of

 is ?

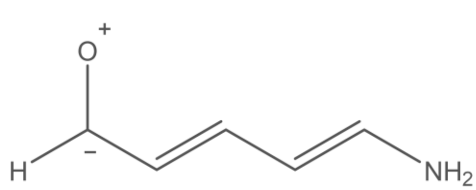
A)



B)



C)



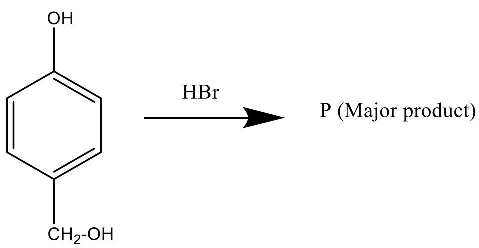
A. A > B > C
B. B > A > C
C. C > A > B
D. B > C > A

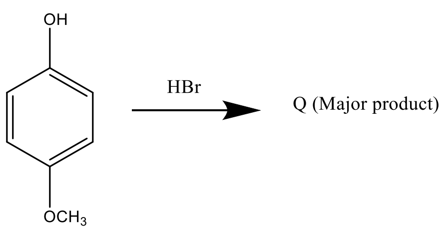
Answer (A)

Sol.

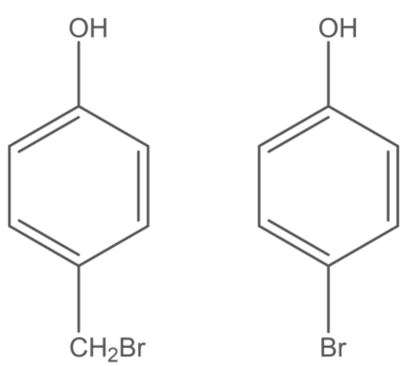
Positive charge on more electropositive atom and negative charge on electronegative atom is more stable. Thus, structure (A) and (B) are more stable than structure (C) because of the positive charge on oxygen which is more electronegative. Comparing (A) and (B) , in (B) lone pair of electron on $-NH_2$ and negative charge on carbon creates repulsion which makes less stable than (A).

15. Consider the following reactions.

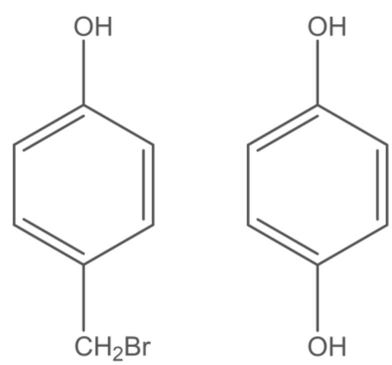




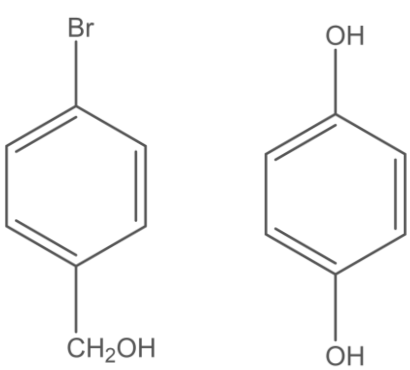
The products P & Q are respectively,

A.


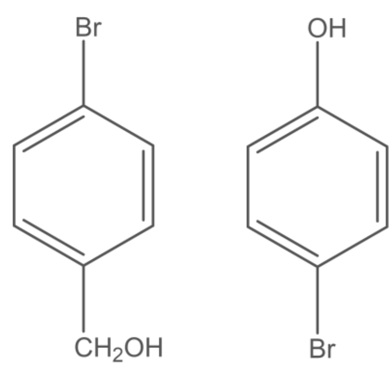
P **Q**

B. P


P **Q**

C.


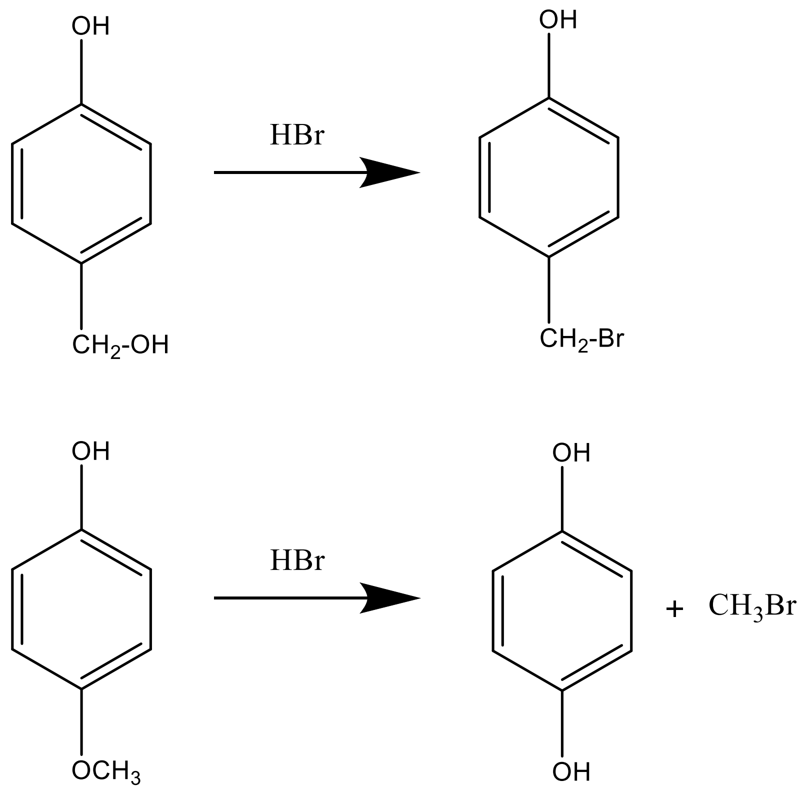
P **Q**

D


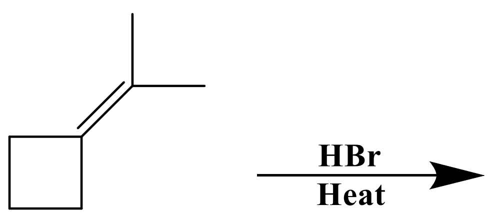
P **Q**

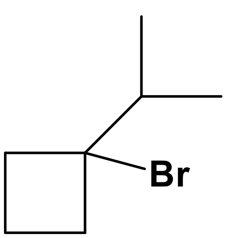
Answer (B)

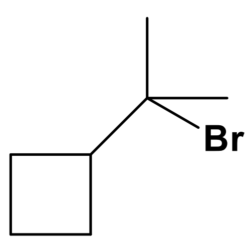
Sol.

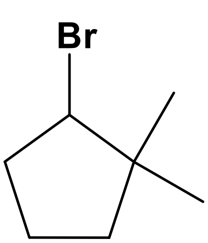


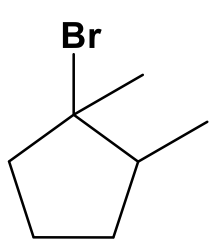
16. The correct product of the following reaction is :



A. 

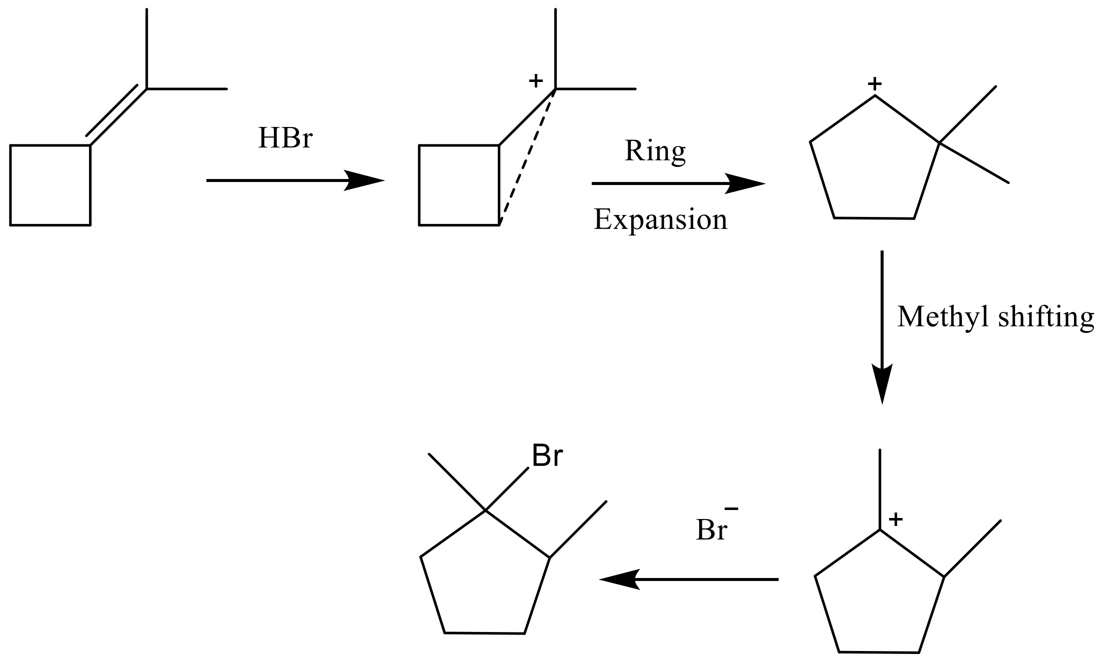
B. 

C. 

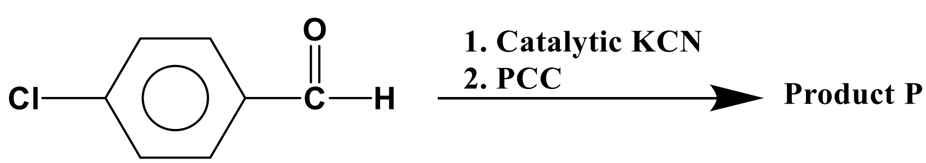
D. 

Answer (D)

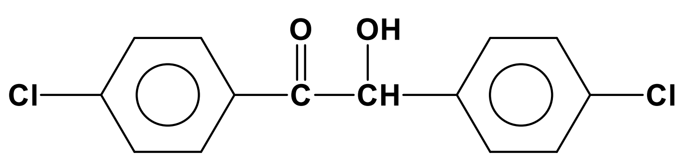
Sol.

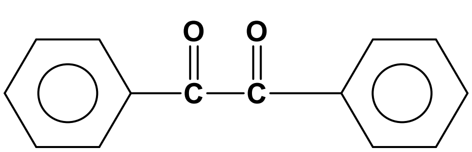


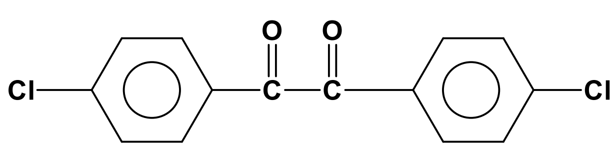
17. Consider the following sequence of reaction?

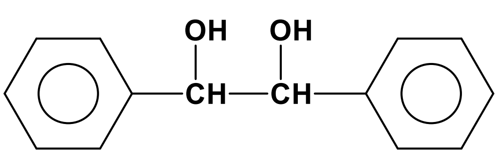


The correct structure of P is :

A. 

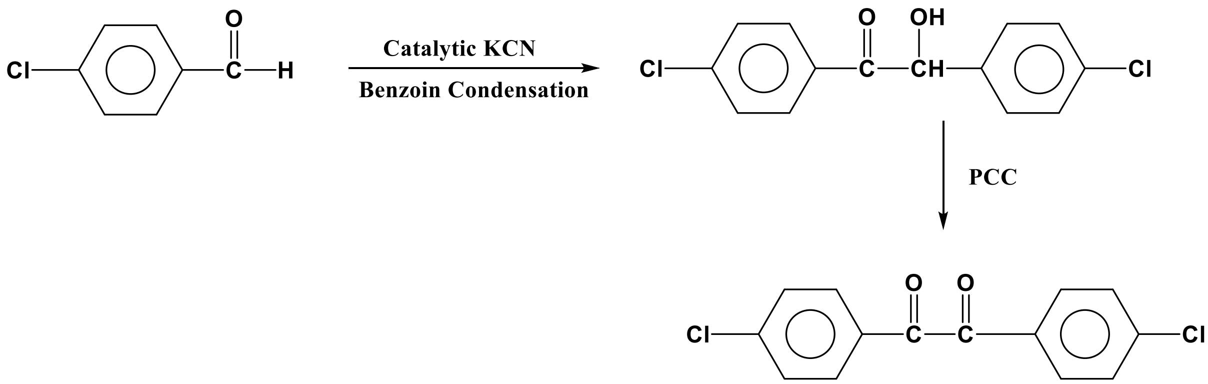
B. 

C. 

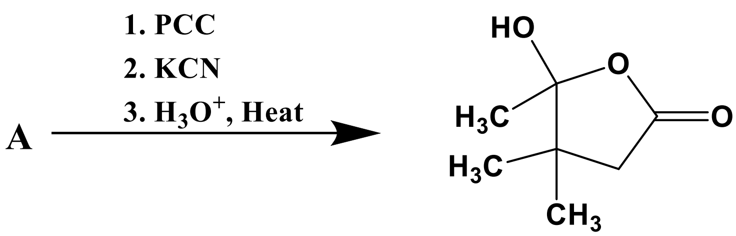
D. 

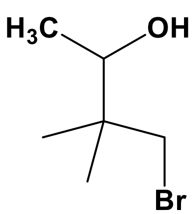
Answer (C)

Sol.

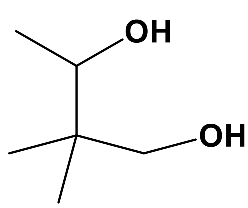


18. The correct structure of A is :

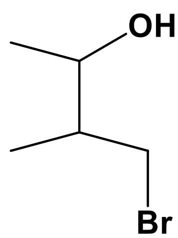




A.

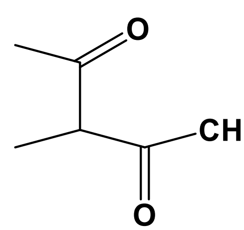


B.



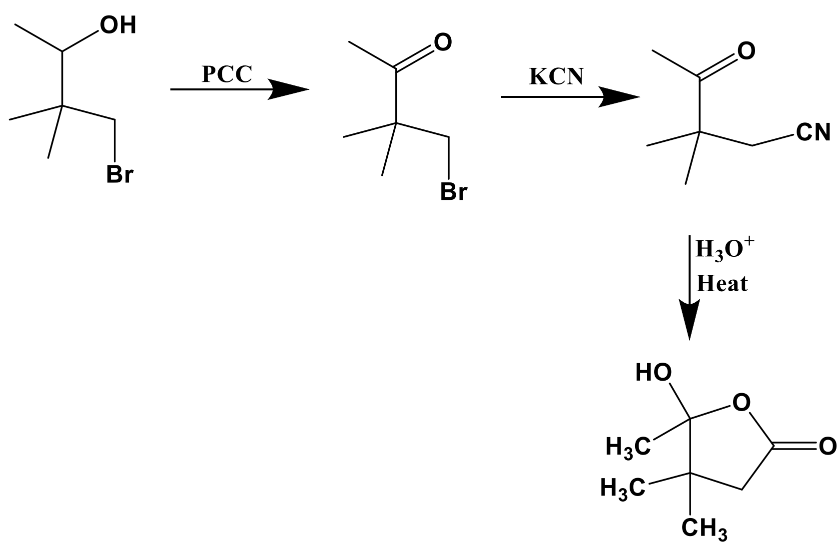
C.

D.



Answer (A)

Sol.



19. Match the following :

Column I	Column II
(A). Aluminium	1. Electrolysis
(B). Iron	2. Reverberatory Furnace
(C). Silicon	3. Blast furnace
(D). Copper	4. Zone refining

A. (A – 1) ; (B – 3) ; (C – 4) ; (D – 2)
B. (A – 2) ; (B – 3) ; (C – 4) ; (D – 1)
C. (A – 3) ; (B – 2) ; (C – 1) ; (D – 4)
D. (A – 2) ; (D – 4) ; (C – 1) ; (D – 3)

Answer (A)

Sol.

Aluminum is extracted by electrolysis of bauxite (Al_2O_3)
Iron is extracted in blast furnace
Silicon is extracted using zone refining
Copper is extracted in reverberatory furnace

20. 5.0 g of NaOH is dissolved in water to get 450 mL solution. What volume of the solution is required to prepare 500 Ml of 0.1 M NaOH solution ?

Answer (180)

Sol.

Molarity of NaOH solution prepared = $\frac{5\ x\ 1000}{40\ x\ 450} = \frac{5}{18} M$
Let V mL of the solution is required to prepare 500 mL of 0.1 M NaOH
Hence, V x $\frac{5}{18} = 500 \text{ X } 0.1$
V = 180 mL

21. If the primary valency of central metal ion in the complex$[Co(NH_3)_5Cl]Cl_2$ is n and its secondary valency is y, then find the value if x + y.

Answer (9)

Sol.

Primary valency(oxidation number) of Co ion in the given complex is 3 and its secondary valency (coordination number) is 6. Hence, x = 3 and y = 6

Thus x + y = 3 + 6 = 9

MATHEMATICS

1. Two lines are given as $\frac{x-2}{3}=\frac{y-1}{3}=\frac{z-0}{2}$ and $\frac{x-1}{3}=\frac{y-2}{2}=\frac{z-1}{3}$ Then the shortest distance between the lines is:

A. $\frac{6}{\sqrt{43}}$
B. $\frac{11}{\sqrt{43}}$
C. $\frac{3}{\sqrt{43}}$
D. $\frac{5}{\sqrt{43}}$

Answer (B)

Sol.

Shortest distance $= \left|\frac{[\overrightarrow{a_2}-\overrightarrow{a_1}\quad \overrightarrow{b_1}\quad \overrightarrow{b_2}]}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}\right|$

From given data

$\overrightarrow{a_2}-\overrightarrow{a_1}=(2-1)\hat{\imath}+(1-2)\hat{\jmath}+(0-1)\hat{k}=\hat{\imath}-\hat{\jmath}-\hat{k}$

$\overrightarrow{b_1}=3\hat{\imath}+3\hat{\jmath}+2\hat{k}$

$\overrightarrow{b_2}=3\hat{\imath}+2\hat{\jmath}+3\hat{k}$

$$\overrightarrow{b_1}\times\overrightarrow{b_2}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 3 & 3 & 2\\ 3 & 2 & 3\end{vmatrix}=5\hat{\imath}-3\hat{\jmath}-3\hat{k}$$

$$\text{Shortest Distance} = \frac{\begin{vmatrix}1 & -1 & -1\\ 3 & 3 & 2\\ 3 & 2 & 3\end{vmatrix}}{|5\hat{\imath}-3\hat{\jmath}-3\hat{k}|}=\frac{11}{\sqrt{43}}$$

2. Tangent is drawn at a point on the parabola $y^2=24x$. It intersects the hyperbola $xy=2$ at points A and B. Locus of mid point of AB is:

A. $y^2=3x$
B. $y^2=-3x$
C. $y^2=6x$
D. $y^2=-6x$

Answer (B)

Sol.

Let a point on $y^2=24x$ be $(6t^2,12t)$

Equation of tangent $\equiv 12yt=12(x+6t^2)$

$\Rightarrow ty=x+6t^2 \quad \cdots(1)$

Let midpoint of chord AB be (h,k)

Equation of chord bisect at this point is:

$\frac{xk+hy}{2}=hk \quad \cdots(2)$

Comparing (1) and (2) we get,

$\frac{t}{h/2}=\frac{-1}{k/2}=\frac{6t^2}{hk}$

$\Rightarrow t = \frac{-h}{k}$ and $-2h = 6(\frac{-h}{k})^2$

$\Rightarrow -2hk^2 = 6h^2$

$\Rightarrow k^2 = -3h$

The required locus is $y^2 = -3x$

3. $\lim_{t \to 0} [(1^{\frac{1}{\sin^2 t}} + 2^{\frac{1}{\sin^2 t}} + 3^{\frac{1}{\sin^2 t}} + \cdots + n^{\frac{1}{\sin^2 t}})^{\sin^2 t}$

A. 0
B. n
C. $\frac{n^2-n}{2}$
D. $n^2 + n$

Answer (B)

Sol.

$$\lim_{t \to 0} n\left[\left(\frac{1}{n}\right)^{cosec^2 t} + \left(\frac{2}{n}\right)^{cosec^2 t} + \left(\frac{3}{n}\right)^{cosec^2 t} + \cdots + \left(\frac{n}{n}\right)^{cosec^2 t}\right]^{\sin^2 t}$$

$\Rightarrow \quad n[0 + 0 + \cdots 1]$

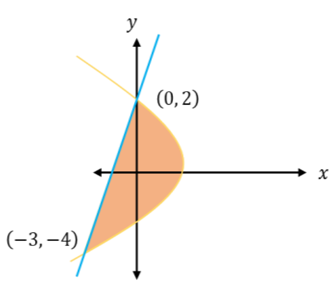
$\Rightarrow \quad n \times 1 = n$

4. The area enclosed between $y^2 = -4x + 4$ and $y = 2x + 2$ is:

A. 3
B. 6
C. 9
D. 12

Answer (C)

Sol.



Required area $= \int_{-4}^{2} \left(\frac{4-y^2}{4} - \frac{y-2}{2}\right) dy$

$= \left[2y - \frac{y^3}{12} - \frac{y^2}{4}\right]_{-4}^{2}$

$= \left(4 - \frac{8}{12} - 1\right) - \left(-8 + \frac{16}{3} - 4\right)$

$= \left(3 - \frac{2}{3}\right) + 12 - \frac{16}{3}$

$= 9$

5. $\sum_{r=0}^{22} \ {}^{22}C_r \ {}^{23}C_r$ is equal to:

A. ${}^{44}C_{22}$
B. ${}^{45}C_{23}$
C. ${}^{45}C_{24}$
D. ${}^{44}C_{23}$

Answer (B)

Sol.

$$\sum_{r=0}^{22} \ {}^{22}C_r \ {}^{23}\sum_{r=0}^{22} {}^{22}C_r \ {}^{23}C_r = \sum_{r=0}^{22} \ {}^{22}C_r \ {}^{23}\sum_{r=0}^{22} {}^{22}C_r \ {}^{23}C_{23-r}$$

$$= {}^{22}C_0 \ {}^{23}C_{23} + {}^{22}C_1 \ {}^{23}C_{22} + \cdots + {}^{22}C_{21} \ {}^{23}C_2 + {}^{22}C_{22} \ {}^{23}C_1$$

$$(1+x)^{22} = {}^{22}C_0 + {}^{22}C_1 x + \cdots + {}^{22}C_{21} x^{21} + {}^{22}C_{22} x^{22}$$

$$(1+x)^{23} = {}^{23}C_0 + {}^{23}C_1 x + \cdots + {}^{23}C_{22} x^{22} + {}^{23}C_{23} x^{23}$$

coefficient of x^{23} in $(1+x)^{22}(1+x)^{23} = \sum_{r=0}^{22} \ {}^{22}C_r \ {}^{23}\sum_{r=0}^{22} {}^{22}C_r \ {}^{23}C_{23-r}$

$= {}^{45}C_{23}$

6. $\sim(\sim p \wedge q) \Rightarrow (\sim p \vee q)$ is equivalent to:

A. $\sim p \vee q$
B. $\sim p \wedge q$
C. $p \wedge q$
D. $p \vee q$

Answer (A)

Sol.

$\sim(\sim p \wedge q) \Rightarrow (\sim p \vee q)$ $\quad [p \rightarrow q \Leftrightarrow \sim p \vee q]$
$= (\sim p \wedge q) \vee (\sim p \vee q)$
$= (\sim p \vee (\sim p \vee q)) \wedge (q \vee (\sim p \vee q))$
$= (\sim p \vee q) \wedge (q \vee \sim p)$
$= \sim p \vee q$

7. There are 12 languages. One can choose at most 2 from 5 particular languages. The number of ways in which one can select 5 languages is:

A. 540
B. 535
C. 546
D. 525

Answer (C)

Sol.

Case -1: If no language is selected from given 5 particular languages

$\Rightarrow {}^{7}C_5$

Case -2: If 1 language is chosen from the given 5 and 4 from other 7 languages

$\Rightarrow {}^{7}C_4 \ {}^{5}C_1$

Case -3: If 2 languages are chosen from the given 5 and 3 from other 7 languages

$\Rightarrow\ {}^7C_3\ {}^5C_2$

$\therefore$ Total ways $= {}^7C_5 + {}^7C_4\ {}^5C_1 + {}^7C_3\ {}^5C_2$

$= 21 + 175 + 350$

$= 546$

8. The solution of differential equation $\frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^3}$ is:

A. $y = \left(1 + \frac{1}{x}\right) + ce^{\frac{1}{x}}$
B. $y = \left(1 - \frac{1}{x}\right) + ce^{\frac{1}{x}}$
C. $y = \left(x + \frac{1}{x}\right) + ce^{\frac{1}{x}}$
D. $y = \left(x - \frac{1}{x}\right) + ce^{\frac{1}{x}}$

Answer (A)

Sol.

Given equation is linear differential equation

$\therefore I.F = e^{\int \frac{1}{x^2}dx} = e^{-\left(\frac{1}{x}\right)}$

$\Rightarrow \int d(ye^{-\frac{1}{x}}) = \int \frac{e^{-\frac{1}{x}}}{x^3}dx$

$\Rightarrow ye^{-\frac{1}{x}} = \frac{1}{x}e^{-\frac{1}{x}} + e^{-\frac{1}{x}} + c$

$\Rightarrow y = (\frac{1}{x} + 1) + ce^{\frac{1}{x}}$

9. The equation $x^2 - [x] + x + 3 = x[x]^2$ has (where $[.]$ represents greatest integer function)

A. No solution
B. 1 solution in $(-\infty, 1)$
C. 2 solution in $(-\infty, \infty)$
D. 1 solution in $(-\infty, \infty)$

Answer (D)

Sol.

$x^2 - [x] + x + 3 = x[x]^2$

$x^2 + \{x\} + 3 = x[x]^2$

Case 1: $x < 0$

L.H.S> 0, R.H.S< 0

$\therefore$ No solution

Case 2: $x \in [0,1)$

L.H.S≥ 3, R.H.S$= 0$

No solution

Case 3: $x \in [1,2)$

L.H.S$\in [4,8)$, R.H.S$\in [1,2)$

No solution

Case 4: $x \in [2,3)$

L.H.S$\in [7,13)$, R.H.S$\in [8,12)$

$\therefore\ x^2 + x - 2 + 3 = 4x$

$\Rightarrow x^2 - 3x + 1 = 0$

$\Rightarrow x = \frac{3+\sqrt{5}}{2}$ one solution

Case 5: $x \in [3,4)$

L.H.S$\in [12,20)$, R.H.S$\in [27,36)$

Similarly, For $x > 4$, R.H.S > L.H.S Always

$\therefore$ one solution in $[1,2)$ or only one solution in $(-\infty, \infty)$

10. The function $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$ is:

A. Continuous but non- differentiable at $x = 0$
B. Discontinuous at $x = 0$
C. $f'(x)$ is differentiable but not continuous
D. $f'(x)$ is continuous but non- differentiable

Answer (D)

Sol.

$f(x) = x^2 \, sin\,(\frac{1}{x})$

At $x = 0$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = f(0)$

$f(x)$ is continuous at $x = 0$

LHD at $x = 0$ is $\lim_{h \to 0} \frac{f(0-h)-f(0)}{-h}$

$\lim_{h \to 0} \frac{-h^2 sin(\frac{1}{h})}{-h} = 0$

RHD at $x = 0$ is $\lim_{h \to 0} \frac{f(0+h)-f(0)}{h} = 0$

$\therefore$ $f(x)$ is continuous as well as differentiable hence Option A and B are wrong.

A function cannot be differentiable unless continuous hence Option C is wrong.

11. The distance of the point $P(1,2,3)$ from the plane containing the points A(1, 4, 5), B(2, 3, 4) and C(3, 2, 1) is equal to:

A. $5\sqrt{2}$
B. $3\sqrt{2}$
C. 1
D. $\sqrt{2}$

Answer (D)

Sol.

Direction ratio of $\overrightarrow{AB}$ $=< 1, -1, -1 >$

Direction ratio of $\overrightarrow{AC}$ $=< 2, -2, -4 >$

Vector normal to $\overrightarrow{AB}$ and $\overrightarrow{AC}$ $= (1,1,0)$

Equation of plane $(x-1, y-4, z-5) \cdot (1, 1, 0) = 0$

$\Rightarrow x + y - 5 = 0$

Distance from $(1,2,3) = |\frac{1+2-5}{\sqrt{(1^2+1^2)}}| = \sqrt{2}$

12. If $\left(1+\sqrt{3}i\right)^{200} = 2^{199}(p+iq)$, then $p+q+q^2$ and $p-q+q^2$ are roots of the equation:

A. $x^2-4x+1=0$
B. $x^2-4x-1=0$
C. $x^2+4x+1=0$
D. $x^2+4x-1=0$

Answer (A)

Sol.

$(1+\sqrt{3}i)^{200} = 2^{199}(p+iq)$

$\Rightarrow 2^{200}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^{200} = 2^{199}(p+iq)$

$\Rightarrow 2^{200}\left(cos\left(\frac{\pi}{3}\right)+i\ sin\left(\frac{\pi}{3}\right)\right)^{200} = 2^{199}(p+iq)$

$\Rightarrow 2^{200}(e^{i\frac{\pi}{3}})^{200} = 2^{199}(p+iq)$

$\Rightarrow 2^{200}\left(cos\left(\frac{200\pi}{3}\right)+i\ sin\left(\frac{200\pi}{3}\right)\right) = 2^{199}(p+iq)$

$\Rightarrow 2^{200}\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) = 2^{199}(p+iq)$

$\Rightarrow 2^{199}(-1+\sqrt{3}i) = 2^{199}(p+iq)$
$\Rightarrow p=-1, q=\sqrt{3}$
$p+q+q^2 = 2+\sqrt{3}$
$p-q+q^2 = 2-\sqrt{3}$
So, the equation whose roots are $2+\sqrt{3}$ and $2-\sqrt{3}$ is
$x^2-(2+\sqrt{3}+(2-\sqrt{3})x+(2+\sqrt{3})(2-\sqrt{3})=0$
$\Rightarrow x^2-4x+1=0$

13. $\sum_{r=0}^{2023} r^2 \cdot {}^{2023}C_r = \alpha\cdot 2023\cdot 2^{2022}$ then α is equal to:

A. 1012
B. 1011
C. 1020
D. 1022

Answer (A)

Sol.

$\sum_{r=0}^{2023} r^2 \cdot {}^{2023}C_r = \sum_{r=0}^{2023}(r^2-r)\cdot {}^{2023}C_r + \sum_{r=0}^{2023} r\cdot {}^{2023}C_r$

$= 2023\cdot 2022\cdot \sum_{r=2}^{2023} {}^{2021}C_{r-2} + 2023\sum_{r=1}^{2023} {}^{2022}C_{r-1}$

$= 2023\cdot 2022\cdot 2^{2021} + 2023\cdot 2^{2022}$

$= 2^{2022}\cdot 2023\cdot 1012$

Then $\alpha = 1012$

14. If $y^2 + ln\,(cos^2 x) = y, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ then

A. $y''(0) = 0$
B. $|y''(0)| = 2$
C. $y'(0) = 3$
D. $y'(0) = -3$

Answer (B)

Sol.

Differentiating both sides we have
$2yy' + \frac{1}{cos^2 x} \cdot (2\,cos\,x) \cdot (-\,sin\,x) = y'$
$2yy' - 2\,tan\,x = y' \cdots (i)$

Differentiating both sides again we have
$\Rightarrow 2(y')^2 - 2yy'' - 2\,sec^2\,x = y'' \cdots (ii)$
From (i) substituting $x = 0$
$y'(0) = 0$
and substituting $x = 0$ and $y'(0) = 0$ in (ii)
$y''(0) = -2$
$|y''(0)| = 2$

15. if $\vec{v}.\vec{w} = 2, \vec{u} \times \vec{w} = \vec{v} + \alpha\vec{u}, \vec{u} = 2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}, \vec{v} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k}$ then $\vec{u} \cdot \vec{w}$

A. $\frac{28}{12}$
B. $\frac{12}{29}$
C. $-\frac{29}{12}$
D. $\frac{29}{12}$

Answer (D)

Sol.

If $\vec{u} \times \vec{w} = \vec{v} + \alpha\vec{u}$

$\vec{v}.\vec{w} + \alpha\vec{u} \cdot \vec{w} = \vec{0}$ (taking dot product with $\vec{v}$)

$\alpha(\vec{u} \cdot \vec{w}) = -2 \cdots (i)$

$0 = \vec{u} \cdot \vec{v} + \alpha|\vec{u}|^2$

$\Rightarrow 29\alpha + 24 = 0$

$\Rightarrow \alpha = -\frac{24}{29}$

From (i)

$\vec{u} \cdot \vec{w} = \frac{-2}{\left(-\frac{24}{29}\right)} = \frac{58}{24} = \frac{29}{12}$

16. If $R = \{(a,b): g.c.d(a,b) = 1, a, b \in Z\}$. Then relation R is:

A. Reflexive
B. Symmetric
C. Transitive
D. None of the above

Answer (B)

Sol.

Reflexive relation:-
$(a, a) \in R \; \forall a \in A$
Let $a = 5, g.c.d(5,5) = 5 \neq 1$
$\Rightarrow R$ is not reflexive relation

Symmetric Relation:-
If $(a, b) \in R \;\Rightarrow (b, a) \in R$
If $g.c.d(a, b) = 1 \Rightarrow g.c.d(b, a) = 1$

Transitive Relation:-
If (a, b) and $(c, c) \in R \Rightarrow (a, c) \in R$
$(2,3)$ and $(3,4) \in R$ but $(2,4) \notin R$ (because $g.c.d(2,4) = 2$)
$\therefore$ R is symmetric relation

17. The sum of all the values of $\boldsymbol{x}$ satisfying $\boldsymbol{cos^{-1} x - 2\, sin^{-1} x = cos^{-1} 2x}$ is:

A. 0
B. 1
C. $\frac{1}{2}$
D. $-\frac{1}{2}$

Answer (A)

Sol.

$cos^{-1} x - 2\, sin^{-1} x = cos^{-1} 2x$

$\Rightarrow \frac{\pi}{2} - 3\, sin^{-1} x = cos^{-1} 2x$

$\Rightarrow cos\left(\frac{\pi}{2} - 3\, sin^{-1} x\right) = cos\,(cos^{-1} 2x)$

$\Rightarrow sin\,(3\, sin^{-1} x) = cos\,(cos^{-1} 2x)$

$\Rightarrow 3x - 4x^3 = 2x$

$\Rightarrow 4x^3 = x$

$\Rightarrow x = 0, \pm\frac{1}{2}$

Sum of values of $x = 0$

18. The value of $\boldsymbol{12\int_0^3 |x^2 - 3x + 2| dx}$ is equal to:

Answer (22)

Sol.

$I = \int_0^3 |x^2 - 3x + 2| dx = \int_0^3 |(x-1)(x-2)| dx \int_0^3 |x^2 - 3x + 2| dx = \int_0^3 |(x-1)(x-2)| dx$
$= \int_0^1 (x-1)(x-2) dx - \int_1^2 (x-1)(x-2) dx + \int_2^3 (x-1)(x-2) dx$
$= \int_0^1 (x-1)(x-2) dx - \int_1^2 (x-1)(x-2) dx + \int_2^3 (x-1)(x-2) dx$
$= \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_0^1 - \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_1^2 + \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_2^3$
$= \left(\frac{1}{3} - \frac{3}{2} + 2\right) - \left\{\left(\frac{8}{3} - 6 + 4\right) - \left(\frac{1}{3} - \frac{3}{2} + 2\right)\right\} + \left\{\left(9 - \frac{27}{2} + 6\right) - \left(\frac{8}{3} - 6 + 4\right)\right\}$
$= \frac{11}{6}$
Hence $12\int_0^3 |x^2 - 3x + 2| dx = \int_0^3 |x^2 - 3x + 2| dx = 12I = 12 \times \frac{11}{6} = 22$