JEE Main 2023 (Memory based)

29 January 2023 - Shift 2

Answer & Solutions

PHYSICS

1. A force $F = -40x$ acts on a mass of $1\ kg$. x is the position of the mass. If maximum speed of the mass is $4\ m/s$, find the amplitude. All parameters are in SI units.

A. $\frac{1}{\sqrt{10}} m$
B. $\frac{2}{\sqrt{10}} m$
C. $\frac{3}{\sqrt{10}} m$
D. $\frac{4}{\sqrt{10}} m$

Answer (B)

Solution:

For SHM:

$$F = -kx \Rightarrow k = 40$$

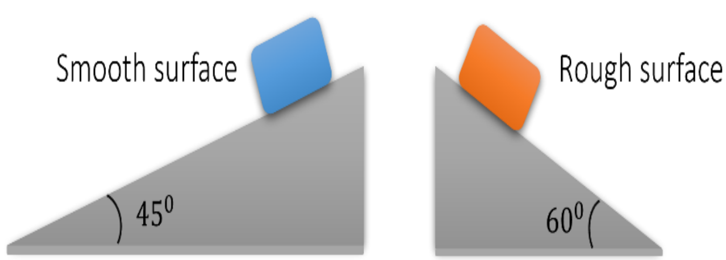
$$v_{max} = A\omega = A\sqrt{\frac{k}{m}}$$

$$4 = A\sqrt{\frac{40}{1}}$$

$$A = \frac{2}{\sqrt{10}}\ m$$

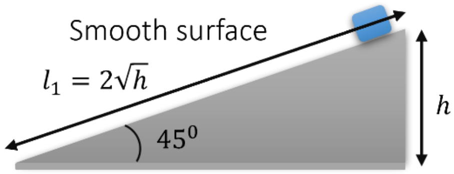
2. Consider 2 inclined plane of same height. 1^{st} has a smooth surface & angle of inclination is 45^0. Other has a rough surface & angle of inclination is 60^0. If the ratio of time taken to slide on them is $'n'$. Find coefficient of friction of rough inclined plane.

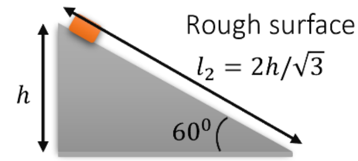
A. $3n^2$
B. $\mu = \frac{3-2n^2}{\sqrt{3}}$
C. $\mu = \frac{3-\sqrt{3}n^2}{2}$
D. $\mu = \frac{2n^2}{\sqrt{3}}$



Answer (B)

Solution:





$$a = g\sin\theta = \frac{g}{\sqrt{2}}$$

$$t = \sqrt{\frac{2l_1}{a}}$$

$$t = \sqrt{\frac{2\sqrt{2}h}{\frac{g}{\sqrt{2}}}}$$

$$t = \sqrt{\frac{4h}{g}}$$

$$a = g\sin\theta - \mu g\sin\theta$$

$$a = \left(\frac{g\sqrt{3}}{2} - \frac{\mu g}{2}\right) = g\left(\frac{\sqrt{3}}{2} - \frac{\mu}{2}\right)$$

$$t = \sqrt{\frac{l_2}{a}}$$

$$t = \sqrt{\frac{8h}{g(3-\sqrt{3}\mu)}}$$

So,

$$\frac{t_1}{t_2} = \sqrt{\frac{3-\sqrt{3}\mu}{2}} = n$$

$$3 - \sqrt{3}\mu = 2n^2$$

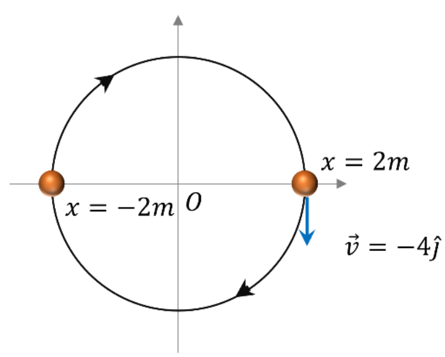
$$\mu = \frac{3-2n^2}{\sqrt{3}}$$

3. A particle undergoing uniform circular motion about origin. At certain instant $x = 2\,m$ and $\vec{v} = -4\hat{\jmath}\,m/s$, find velocity and acceleration of particle when at $x = -2\,m$.

A. $\vec{v} = -4\hat{\jmath},\ \vec{a} = 8\hat{\imath}$
B. $\vec{v} = 4\hat{\jmath},\ \vec{a} = 8\hat{\imath}$
C. $\vec{v} = -4\hat{\jmath},\ \vec{a} = -8\hat{\imath}$
D. $\vec{v} = 4\hat{\jmath},\ \vec{a} = -8\hat{\imath}$

Answer (B)

Solution:



For uniform circular motion:

At $x = -2\,m, v = 4\hat{\jmath}$

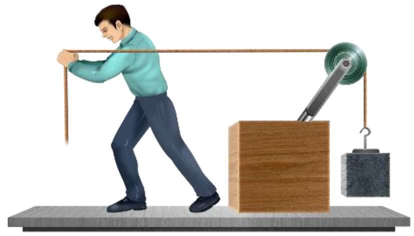
Acceleration towards the center is:

$$a = \frac{v^2}{r}$$

$$a = \frac{4^2}{2} = 8\,m/s^2$$

$$\vec{a} = 8\,m/s^2\hat{\imath}$$

4. A man pulls a block as shown:
Consider the following statements:
1: Work done by the gravity on block is positive.
2: Work done by the gravity on block is negative.
3: If man pulls block with constant speed, then tension in the string equals to weight of the block.
4: None of the above.



A. 2 and 3 only
B. 4 only
C. 4 only
D. 1 only

Answer (A)

Solution:

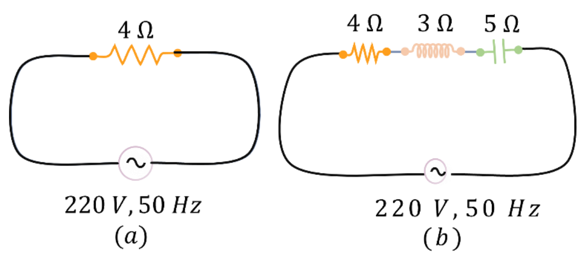
Weight acts down and displacement is up, so work done by gravity is negative.

If speed is constant, acceleration is zero, hence tension is equal to weight.

⇒Statement 3 is correct.

$T - mg = ma$
If $a = 0$, $T = mg$

5. RMS current in circuit (a) is I_a while RMS current in circuit (b) is I_b then:



A. $I_a > I_b$
B. $I_a < I_b$
C. $I_a = I_b$
D. $none\ of\ the\ above$

Answer (A)

Solution:

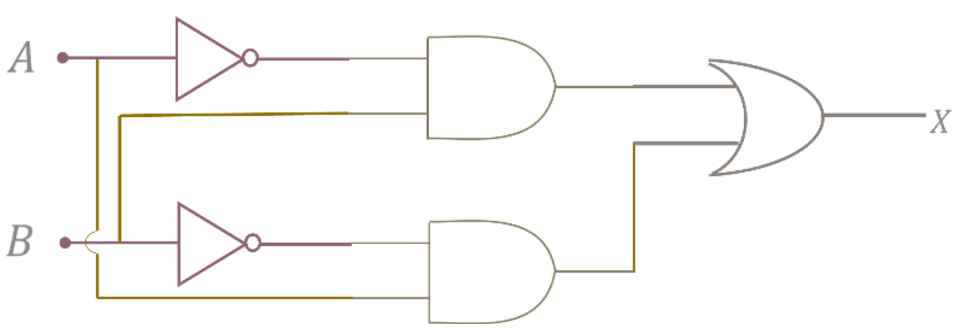
Impedance for circuit (a) and (b):

$Z_a = 4\ \Omega$ and $Z_b = \sqrt{(4^2 + (5-3)^2}\ \Omega = \sqrt{20}\ \Omega$

$I_a = \frac{220}{4} \quad and \quad I_b = \frac{220}{\sqrt{5}}$

$I_a > I_b$

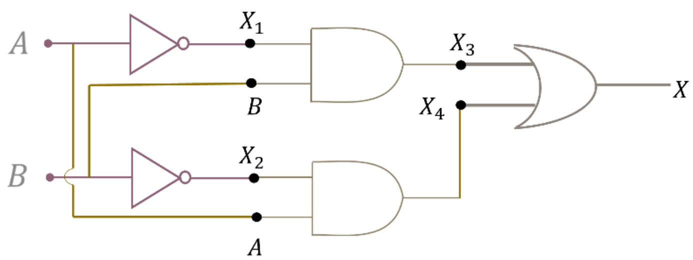
6. Find truth table:



A.	B.	C.	D.
$\begin{array}{ccc} A & B & X \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{array}$	$\begin{array}{ccc} A & B & X \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{array}$	$\begin{array}{ccc} A & B & X \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{array}$	$\begin{array}{ccc} A & B & X \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{array}$

Answer (D)

Solution:



$X_1 = \bar{A}$

$X_3 = B.\bar{A}$

$X_2 = \bar{B}$

$X_4 = (A\bar{B})$

$X = X_3 + X_4$

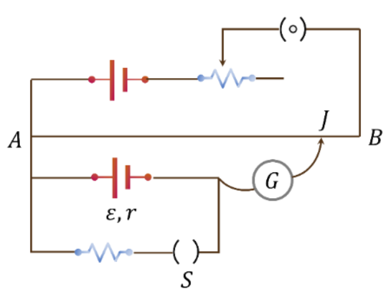
$X = A\bar{B} + B\bar{A}$

Y=output=XOR Gate

So, the correct answer is:

$$\begin{array}{ccc} A & B & X \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{array}$$

7. Consider the following potentiometer circuit. When switch S is open, length AJ is $300\ cm$. When switch S is closed, length AJ is $200\ cm$. If $R = 5\Omega$, then find internal resistance r of the cell.



A. $4\ \Omega$

B. $2\ \Omega$

C. $5\ \Omega$

D. $2.5\ \Omega$

Answer (D)

Solution:

For both the cases:

$C \times 300 = \epsilon \ldots\ldots(1)$

$C \times 200 = \dfrac{\epsilon}{R+r} \times R \ldots\ldots(2)$

$\dfrac{300}{200} = \dfrac{R+r}{R}$

$r = \dfrac{R}{2} = 2.5\ \Omega$

8. In a communication system, maximum voltage is $14\ mV$ and minimum voltage is $6\ mV$. Find out the modulation index.

A. 0.2
B. 0.6
C. 0.4
D. 0.3

Answer (C)

Solution:

$$Index = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

$$= \frac{14-6}{14+6}$$

$$= 0.4$$

9. The gravitational potential due to a solid uniform sphere of mass M and radius R at a point at radial distance $r\ (r > R)$ from its centre is equal to

A. $-\frac{GM}{r}$
B. $-\frac{GM}{2r}$
C. $-\frac{GMR}{r^2}$
D. $-\frac{GM(R+r)}{r^2}$

Answer (A)

Solution:

For outside point of solid sphere, $V = -\dfrac{GM}{r}$

10. Resolving power of compound microscope will increase with

A. Decrease in wavelength of light and increase in numerical aperture.
B. Increase in wavelength of light and decrease in numerical aperture.
C. Increase in both wavelength numerical aperture.
D. Decrease in both wavelength numerical aperture.

Answer (A)

Solution:

Resolving power of compound microscope $\propto \left(\frac{2n\sin\theta}{\lambda}\right)$

$\lambda =$ Wavelength of used light
$n \sin\theta =$ Numerical aperture
$n =$ Refractive index of medium separating object and aperture

11. It is given that $x^2 + y^2 = a^2$ where,a: radius.
Also, it is given that $(x - \alpha t)^2 + \left(y - \frac{t}{\beta}\right)^2 = a^2$, where, t: time
Then dimensions of α and β are

A. $[M^0LT^{-1}]$ & $[M^0L^{-1}T]$
B. $[M^0LT\,]$ & $[M^0L^{-1}T^{-1}]$
C. $[M^0LT\,]$ & $[M^0LT^{-1}]$
D. $[M^0L^{-1}T\,]$ & $[M^0LT\,]$

Answer (A)

Solution:

$$x = \alpha t = \frac{t}{\beta}$$
$$\Rightarrow L' = \alpha T' = \frac{T'}{\beta}$$
$$\Rightarrow \alpha = [LT^{-1}] \,\&\, \beta = [L^{-1}T]$$

12. Assertion (A): EM waves are not deflected by electric field and magnetic field.
Reason (R): EM waves don't carry any charge, so they are not deflected by electric field and magnetic field.

A. $Both\ (A)\ and\ (R) are\ true\ and\ (R)\ is\ the\ correct\ explanation\ of\ (A)$
B. $Both\ (A)\ and\ (R) are\ true\ and\ (R)\ is\ not\ the\ correct\ explanation\ of\ (A)$
C. $(A)\ is\ true\ but\ (R)\ is\ false.$
D. $(A)\ is\ false\ but\ (R)\ is\ true.$

Answer (A)

Solution:

EM wave does not have charge therefore they are not deflected by electric or magnetic field.

13. De-Broglie wavelength of a body of mass m and kinetic energy E is given by:

A. $\lambda = h/mE$
B. $\lambda = \sqrt{2mE}/h$
C. $\lambda = h/\sqrt{2mE}$
D. $\lambda = \sqrt{h/2mE}$

Answer (C)

Solution:

$$\lambda_d = \frac{h}{p}\ \ and\ \ p = \sqrt{2mE}$$
$$\lambda_d = \frac{h}{\sqrt{2mE}}$$

14. In a region with electric field $30\,\hat{\imath}\,V/m$ a charge particle of charge $q = 2 \times 10^{-4}\,C$ is displaced slowly from $(1,2)$ to origin. The work done by external agent is equal to

A. $1\,mJ$
B. $6\,mJ$
C. $2\,mJ$
D. $3\,mJ$

Answer (B)

Solution:

$F = qE$

$= 2 \times 10^{-4} \times 30\ N$

Work Done $= 6 \times 10^{-3} \times 1\ J$

Work Done $= 6\ mJ$

15. At $300\ K$, RMS speed of an ideal gas molecules is $\sqrt{\frac{\alpha+5}{\alpha}}$ times the average speed of gas molecules, then value of α is equal to (take $\pi = 22/7$)

Answer (28)

Solution:

$$v_{rms} = \sqrt{\frac{3RT}{M_0}}$$

$$v_{avg} = \sqrt{\frac{8RT}{\pi M_0}}$$

$$\frac{v_{rms}}{v_{avg}} = \sqrt{\frac{3\pi}{8}} = \sqrt{\frac{33}{28}} = \sqrt{\frac{28+5}{28}}$$

So, $\alpha = 28$

16. An α particle and a proton are accelerated through same potential difference. The ratio of $de - Broglie$ wavelength of α particle to proton is equal to $1/\sqrt{x}$. Value of x is
Take $m_\alpha = 4m_{proton}$

Answer (8)

Solution:

$$\lambda = \frac{h}{p}$$

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}}$$

$$\frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{m_p q_p}{m_\alpha q_\alpha}} = \sqrt{\frac{1}{4} \times \frac{1}{2}} = \frac{1}{\sqrt{8}}$$

$x = 8$

17. Time period of rotation of a planet is $24\ hours$. If the radius decreases to $\frac{1}{4}th$ of the original value, then the new time period is x hours. Find $2x$.

Answer (3)

Solution:

We know, $I\omega = constant$

$$\Rightarrow I_1\omega_1 = \frac{I_1}{16}\omega_2$$

$$\Rightarrow \omega_2 = 16\omega_1$$

$$\Rightarrow T_2 = \frac{T_1}{16} = 1.5\ hours$$

$$\Rightarrow 2x = 3\ hours$$

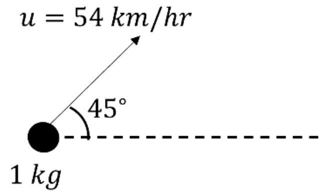
18. A projectile is fire with velocity $54\ km/hr$ making an angle $45°$ with horizontal. Angular momentum of this particle of mass $1\ kg$ about the point of projection one second into the motion will be $\frac{5N}{\sqrt{2}}$ in SI units ($g = 10\ m/s^2$). Find the value of N.

Answer (3)

Solution:

$u = 54\ km/hr = 15\ m/s$

Torque at time t is $\tau = mgu\cos\theta\ t$



$$\frac{dL}{dt} = \tau$$

$$\int_0^L dL = \int_0^1 mgu\cos\theta\ tdt$$

$$L = \frac{mgu\cos\theta}{2} = \frac{10 \times 15}{2\sqrt{2}} = \frac{75}{\sqrt{2}}\ kg\ m^2/sec$$

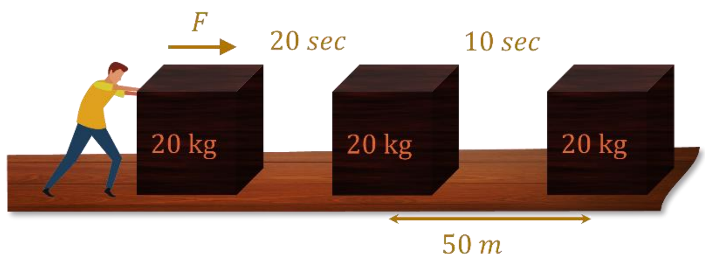
Comparing with $\frac{5N}{\sqrt{2}} \Rightarrow N = 15$

19. A block of mass $20\ kg$ is moved with a constant force 'F' for $20\ seconds$ starting from rest and then 'F' is removed. It is then observed that block moves $50\ m$ in next $10\ seconds$. Find $F\ (in\ N)$.

Answer (5)

Solution:

When Force is removed, let velocity of block is v.



$$v = \frac{50}{10} = 5m/s$$

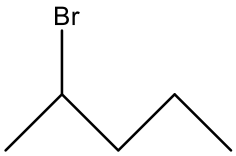
For first $20\ seconds$,

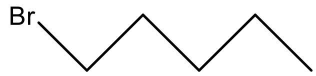
Impulse, $Ft = mv$

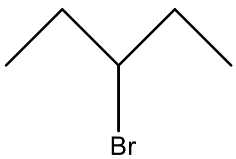
$F \times 20 = 20 \times 5$

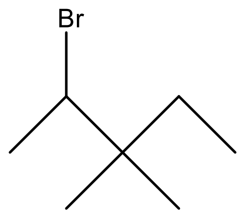
$F = 5\ N$

1. In which of the given molecules, dehydrohalogenation forms maximum number of isomers (excluding rearrangement)

A. 

B. 

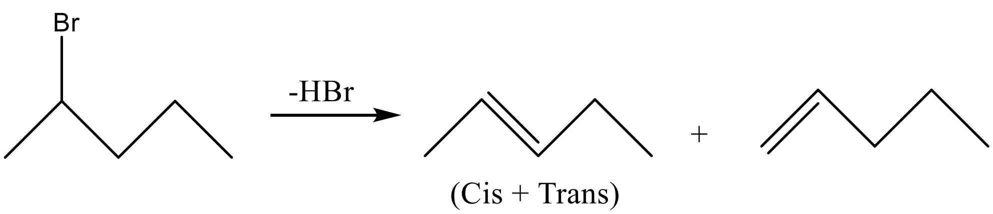
C. 

D. 

Answer (A)

Solution:

A.



B. Only 1 Product

C. 2 Products

D. Only 1 Product

2. If Bohr's radius of H atom in ground state is O.6 A^o, find out the Bohr's radius of He^+ ion in 3rd orbit of He^+ion

A. $2.7\ A^O$

B. $0.9\ A^O$

C. $5.4\ A^O$

D. $1.8\ A^O$

Answer (A)

Solution:

$$r \,\alpha \frac{n^2}{z}$$

$$r = 0.6 \times \frac{n^2}{z}$$

$$r = 0.6 \times \frac{(3)^2}{2}$$

$$r = 0.3 \times 9 = 2.7\ A^O$$

3. Which one of the following ones contains sulphide ions?

A. Malachite
B. Calamine
C. Sphalerite
D. Siderite

Answer (C)

Solution:

The chemical formulae of the given ores are
Malachite: $CuCO_3.Cu(OH)_2$
Calamine: $ZnCO_3$
Sphalerite: ZnS
Siderite: $FeCO_3$
Therefore, Sphalerite contains sulphide ions.

4. Match the correct column

List - I	List - II
A. Thermosetting	P. Neoprene
B. Thermoplastic	Q. Polyester
C. Elastomer	R. Polystyrene
D. Fiber	S. Urea formaldehyde resin

A. A – P, B – R, C – Q, D – S
B. A – S, B – R, C – P, D - Q
C. A – S, B – R, C – Q, D - P
D. A – P, B – R, C – S, D - Q

Answer (B)

Solution:

Urea formaldehyde resin is Thermosetting polymer
Polystyrene is Thermoplastic polymer
Neoprene is an Elastomer
Polyester is a Fiber

5. At 300 K the ratio of V_{rms} and V_{avg} of oxygen molecule is $\sqrt{\frac{\alpha\pi}{\alpha+5}}$, the value of α will be

A. 1
B. 2
C. 3
D. 4

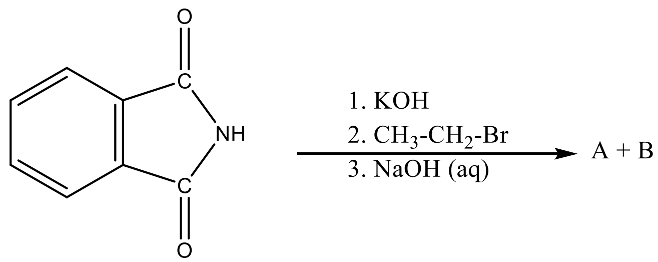
Answer (C)

Solution:

$$\frac{V_{rms}}{V_{avg}} = \sqrt{\frac{3\pi}{8}} = \sqrt{\frac{\alpha\pi}{\alpha+5}}$$

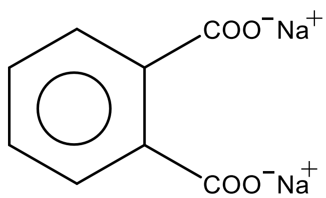
$\therefore \alpha = 3$

6.

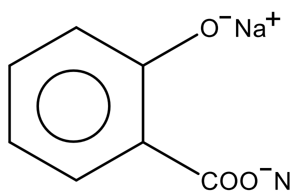


A and B are respectively are

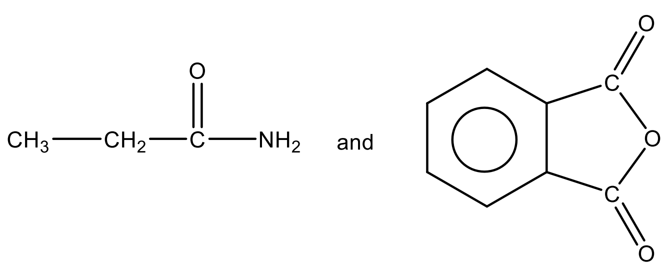
A. CH_3-CH_2-NH_2 and



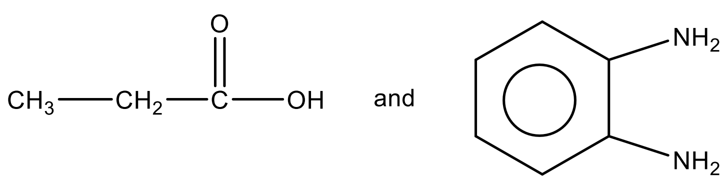
B. CH_3-CH_2-NH_2 and



C.

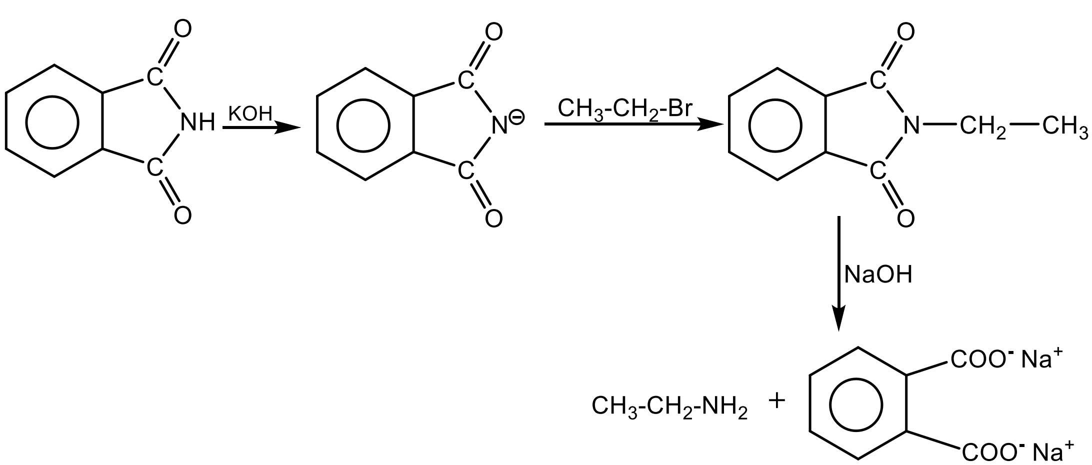


D.



Answer (A)

Solution:



7. Match List – I with List - II

List - I	List - II
A. Electroosmosis	P. Solvent moves from low concentration to high concentration of solution
B. Electrophoresis	Q. Solvent moves from high concentration to low concentration of solution
C. Reverse Osmosis	R. Dispersion medium (DM) moves towards oppositely charged electrode across semipermeable membrane
D. Osmosis	S. Colloidal particles move in the presence of electric field (DP & DM)

A. A – R, B – S, C – Q, D – P
B. A – Q, B – P, C – R, D - S
C. A – P, B – Q, C – R, D - S
D. A – P, B – R, C – Q, D - S

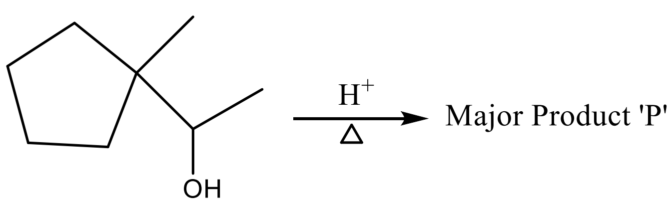
Answer (A)

Solution:

All options are definition based

A. Electroosmosis – Movement of dispersion medium across semipermeable membrane in an electric field.
B. Electrophoresis – Movement of DP & DM towards respective electrodes.
C. Reverse Osmosis – Movement of solvent from higher concentration to lower concentration of solution.
D. Osmosis - Movement of solvent from lower concentration to higher concentration of solution.

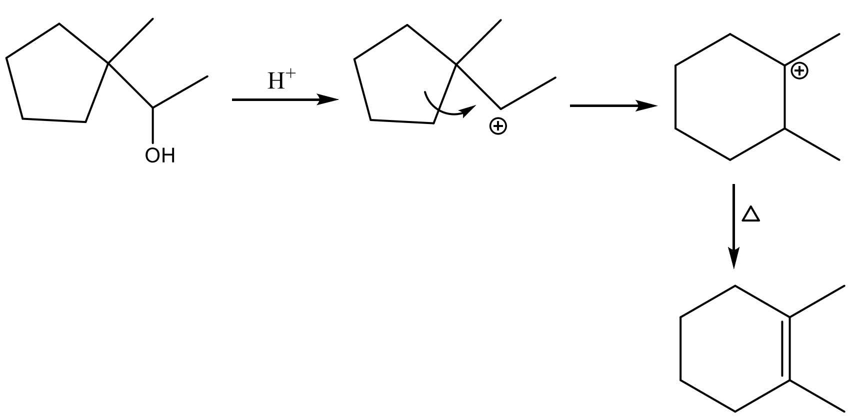
8. Consider the following reaction:



Find the number of $\propto\ -H$'s in the major product is?

Answer (10.00)

Solution:



Number of $\propto\ -H$'s in P =10

9. A 1:1 (by mole) mixture of A and B is passed to a container. Molar mass of A is 16 g, and molar mass of B is 32 g. And the half-life of A is 1 day and half-life of B is ½ day. Then find the average molar mass of the remained mixture after 2 days (Round off the nearest integer)

Answer (19)

Solution:

A: 1 mole $\xrightarrow{1\ day} \frac{1}{2}$ mole $\xrightarrow{1\ day} \frac{1}{4}$ mole

B: 1 mole $\xrightarrow{\frac{1}{2} day} \frac{1}{2}$ mole $\xrightarrow{\frac{1}{2} day} \frac{1}{4}$ mole $\xrightarrow{\frac{1}{2} day} \frac{1}{8}$ mole $\xrightarrow{\frac{1}{2} day} \frac{1}{16}$ mole

$$M_{avg} = \frac{\frac{1}{4}\times 16+\frac{1}{16}\times 32}{\frac{1}{4}+\frac{1}{16}} = \frac{6}{0.25+0.625} = \frac{6}{0.3125} = 19.2$$

10. How many of the oxides given are acidic.

NO, NO_2, N_2O_3, Cl_2O_7, CO, SO_2, SO_3, N_2O

Answer (5)

Solution:

NO_2, N_2O_3, Cl_2O_7, SO_2, SO_3 are acidic oxides

11. The colour of CrO_5 in ether is:

A. Yellow
B. Green
C. Blue
D. Orange

Answer (C)

CrO_5 in ether will exhibit blue color.

12. The number of voids in 0.02 moles of a solid which forms HCP lattice is given as : (Given $N_A = 6 \times 10^{23}$)

A. 3.6×10^{22}
B. 3.6×10^{24}
C. 7.2×10^{20}
D. 5.4×10^{26}

Answer (A)

Solution:

Total number of voids = $\frac{18}{6} \times 6 \times 10^{23} \times 0.02 = 3.6 \times 10^{22}$

13. Which of the following complex has zero spin only magnetic moment?

A. $[FeF_6]^{3-}$
B. $[CoF_6]^{3-}$
C. $[Co(C_2O_4)_3]^{3-}$
D. $[Fe(H_2O)_6]^{3+}$

Answer (C)

Solution:

$[Co(C_2O_4)_3]^{3-}$ has d^2sp^3 hybridisation and $3d^6$ electronic configuration and it has zero unpaired electrons.

14. Which of the following diseases can be cured by equanil drug.

A. Pain
B. Stomach ulcer
C. Depression
D. Hyperacidity

Answer (C)

Solution:

Depression can be cured by equanil drug.

15. Compare the bond order of the following molecules.

O_2^{2-}, NO, CO

A. $O_2^{2-} > NO > CO$
B. $O_2^{2-} > CO > NO$
C. $CO > NO > O_2^{2-}$
D. $NO > CO > O_2^{2-}$

Answer (C)

Solution:

The correct bond order:

$O_2^{2-} \rightarrow 1$
$NO \rightarrow 2.5$
$CO \rightarrow 3$

∴ The correct order is $CO > NO > O_2^{2-}$

16. Statement – I: Ionization enthalpy difference from B to Al is more than that of Al to Ga
Statement – II: Ga has completely filled d-orbital
Choose the correct option from the following.

A. Both statement – I and statement – II are correct
B. Statement – I is incorrect and statement – II is correct
C. Statement – I is correct and statement – II is incorrect
D. Both statement – I and II are incorrect

Answer (A)

Solution:

Ga has similar ionisation enthalpy as Al because of poor shielding effect of completely filled d-orbital in Ga.

17. Which of the following relation is correct.

A. $\Delta G = \Delta H - T\Delta S$ at constant T & P
B. $\Delta U = \Delta H + nR\Delta T$ (For n moles of an ideal gas)

C. $P\Delta V = (\Delta n)RT$
D. None of these

Answer (A)

Solution:

$\Delta G = \Delta H - T\Delta S$ → correct relation at constant T & P

$\Delta H = \Delta U + nR\Delta T$ (For n moles of an ideal gas)

$P\Delta V = (\Delta n)RT$ is only true for a chemical reaction at constant T & P.

So, correct answer is option (A).

18. Thermal decomposition products of $LiNO_3$ are

$$LiNO_3 \xrightarrow{\Delta} \text{Products}$$

A. $LiNO_2$ and O_2
B. $LiNO_2$, NO_2 and O_2
C. Li_2O, NO_2 and O_2
D. Li, NO and O_2

Answer (C)

Solution:

Thermal decomposition of $LiNO_3$ gives the following products

$$4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 4NO_2 + O_2$$

19. BOD value of drinking water ranges between:

A. 3 - 5
B. 10 -13
C. 14 – 17
D. 20 - 22

Answer (A)

Solution:

BOD value of drinking water ranges between 3 and 5.

20. The ratio of de Broglie wavelength of proton to that of **α**–particle, if they are accelerated through same potential is given as:

A. $2\sqrt{2}$: 1
B. 2 : 1
C. 1 : $2\sqrt{2}$
D. $\sqrt{2}$: 1

Answer (A)

Solution:

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha KE_\alpha}{m_p KE_p}} = \sqrt{\frac{4m_p \times 2v}{m_p \times v}} = \sqrt{8} \quad = 2\sqrt{2} : 1$$

21. Which of the following is produced when propanamide is treated with Br_2 in presence of KOH.

A. Ethyl nitrile
B. Propanamine
C. Ethyl amine
D. Propane nitrile

Answer (C)

Solution:

$$CH_3CH_2CONH_2 \xrightarrow{Br_2/KOH} CH_3CH_2NH_2$$

MATHEMATICS

1. The 3 digit numbers which are divisible by either 3 or 4 but not divisible by 48:

A. 414
B. 420
C. 429
D. 432

Answer (D)

Solution:

No's divisible by $3 = 300$

No's divisible by $4 = 225$

No's divisible by $12 = 75$

No's divisible by $48 = 18$

Total numbers $= 300 + 225 - 75 - 18$

$= 432$

2. The letters of word GHOTU is arranged alphabetically as in a dictionary. The rank of the word TOUGH is:

A. 84

B. 79

C. 74

D. 89

Answer (D)

Solution:

Number of words starting with

G _ _ _ _ $= 4! = 24$

H _ _ _ _ $= 4! = 24$

O _ _ _ _ $= 4! = 24$

T G _ _ _ $= 3! = 6$

T H _ _ _ $= 3! = 6$

T O G _ _ $= 2! = 2$

T O H _ _ $= 2! = 2$

T O U G H $= 1$

$\therefore$ Rank of word TOUGH is $= 24 \times 3 + 6 \times 2 + 2 \times 2 + 1 = 89$

3. $\int_{\frac{1}{2}}^{2} \frac{\tan^{-1} x}{x} dx$ equals:

A. $\frac{\pi}{2} \ln 2$
B. $\frac{\pi}{4} \ln 2$
C. $\pi \ln 2$
D. $\ln 2$

Answer (A)

Solution:

$I = \int_{\frac{1}{2}}^{2} \frac{\tan^{-1} x}{x} dx \quad \cdots (1)$

Put $x = \frac{1}{t}$

$dx = -\frac{1}{t^2} dt$

$I = \int_{2}^{\frac{1}{2}} \frac{\tan^{-1}\left(\frac{1}{t}\right)}{\frac{1}{t}} \left(-\frac{1}{t^2}\right) dt$

$I = \int_{\frac{1}{2}}^{2} \frac{\tan^{-1}\left(\frac{1}{t}\right)}{t} dt$

$I = \int_{\frac{1}{2}}^{2} \frac{\cot^{-1} t}{t} dt \quad \cdots (2)$

By adding eq. (1) and eq. (2)

$2I = \int_{\frac{1}{2}}^{2} \frac{\pi}{2} \frac{dt}{t} \qquad \cdots \left(\because \tan^{-1} t + \cot^{-1} t = \frac{\pi}{2}\right)$

$\Rightarrow I = \frac{\pi}{2} \ln 2$

4. Shortest distance between lines: $\frac{x-1}{2} = \frac{2y-2}{3} = \frac{z-3}{1}$ and $\frac{x-2}{3} = \frac{y-1}{2} = \frac{z+2}{4}$ is:

A. $\frac{13}{\sqrt{165}}$

B. $\frac{15}{\sqrt{165}}$

C. $\frac{18}{\sqrt{165}}$

D. $\frac{19}{\sqrt{165}}$

Answer (A)

Solution:

For line $\frac{x-1}{2} = \frac{2y-2}{3} = \frac{z-3}{1}$

$\overrightarrow{a_1} = \hat{\imath} + \hat{\jmath} + 3\hat{k}$

$\overrightarrow{b_1} = 2\hat{\imath} + \frac{3}{2}\hat{\jmath} + \hat{k}$

For line $\frac{x-2}{3} = \frac{y-1}{2} = \frac{z+2}{4}$

$\overrightarrow{a_2} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$

$\overrightarrow{b_2} = 3\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & \frac{3}{2} & 1 \\ 3 & 2 & 4 \end{vmatrix} = 4\hat{\imath} - 5\hat{\jmath} - \frac{\hat{k}}{2}$$

$$(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2}) = (\hat{\imath} - 5\hat{k}) \cdot \left(4\hat{\imath} - 5\hat{\jmath} - \frac{\hat{k}}{2}\right)$$

$$= 4 + \frac{5}{2} = \frac{13}{2}$$

Shortest distance $= \left|\frac{[\overrightarrow{a_2} - \overrightarrow{a_1} \quad \overrightarrow{b_1} \quad \overrightarrow{b_2}]}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right| = \left|\frac{(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right|$

$$= \frac{\frac{13}{2}}{\sqrt{16+25+\frac{1}{4}}} = \frac{13}{\sqrt{165}}$$

5. $R = \{(a, b) : 2a + 3b$ is divisible by 5 and $a, b \in N\}$ is:

A. Transitive but not symmetric
B. Equivalence Relation
C. Symmetric but not Transitive
D. Not Equivalence

Answer (B)

Solution:

$f(a, b) = 2a + 3b$
For reflexive
$f(a, a) = 2a + 3a = 5a$ i.e, divisible by 5
$\Rightarrow (a, a) \in R$
For symmetric
$f(b, a) = 2b + 3a = \underbrace{5a + 5b}_{\text{Divisible by } 5} - \underbrace{(2a + 3b)}_{\text{Divisible by } 5}$
$f(b, a)$ is divisible by $5 \Rightarrow (b, a) \in R$
For transitive
$f(a, b) = 2a + 3b$ is divisible by 5
$f(b, c) = 2b + 3c$ is divisible by 5
$\Rightarrow \ 2a + 5b + 3c$ is divisible by 5
So, $2a + 3c$ is divisible by 5
$\Rightarrow (a, c) \in R$

6. $(\sim A) \vee B$ is equivalent to:

A. $A \to B$
B. $A \leftrightarrow B$
C. $\sim A \wedge B$
D. $B \to A$

Answer (A)

Solution:

Making truth table,

A	B	$\sim A$	$(\sim A) \vee B$	$A \to B$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

The truth table clearly shows $(\sim A) \vee B \equiv A \to B$

7. The value of $\int_{\frac{1}{2}}^{2}\left(\frac{t^4+1}{t^6+1}\right)dt$:

A. $\tan^{-1}2+\tan^{-1}8+\frac{2\pi}{3}$

B. $2\tan^{-1}2+\frac{2}{3}\tan^{-1}8-\frac{2\pi}{3}$

C. $2\tan^{-1}2+\frac{2}{3}\tan^{-1}8+\frac{2\pi}{3}$

D. $2\tan^{-1}2-\frac{2}{3}\tan^{-1}8+\frac{2\pi}{3}$

Answer (B)

Solution:

$$\int_{\frac{1}{2}}^{2}\left(\frac{t^4+1}{t^6+1}\right)dt=\int_{\frac{1}{2}}^{2}\frac{(t^4+1)(t^2+1)}{(t^6+1)(t^2+1)}dt$$

$$=\int_{\frac{1}{2}}^{2}\frac{t^6+1+t^2(t^2+1)}{(t^6+1)(t^2+1)}dt$$

$$=\int_{\frac{1}{2}}^{2}\frac{dt}{(t^2+1)}+\frac{1}{3}\int_{\frac{1}{2}}^{2}\frac{3t^2dt}{t^2+1}$$

$$=\tan^{-1}t\,\Big|_{\frac{1}{2}}^{2}+\frac{1}{3}\tan^{-1}t^3\,\Big|_{\frac{1}{2}}^{2}$$

$$=\left(\tan^{-1}2-\tan^{-1}\left(\frac{1}{2}\right)\right)+\frac{1}{3}\left(\tan^{-1}8-\tan^{-1}\left(\frac{1}{8}\right)\right)$$

$$=(\tan^{-1}(2)-\cot^{-1}(2)\quad)+\frac{1}{3}(\tan^{-1}(8)-\cot^{-1}(8))$$

$$=\left(\tan^{-1}2-\left(\frac{\pi}{2}-\tan^{-1}(2)\right)\right)+\frac{1}{3}\left(\tan^{-1}(8)-\left(\frac{\pi}{2}-\tan^{-1}(8)\right)\right)$$

$$=2\tan^{-1}2+\frac{2}{3}\tan^{-1}(8)-\frac{\pi}{2}-\frac{\pi}{6}$$

$$=2\tan^{-1}2+\frac{2}{3}\tan^{-1}8-\frac{2\pi}{3}$$

8. Area of region $|\cos x-\sin x|\le y\le\sin x$ for $x\in\left(0,\frac{\pi}{2}\right)$ is:

A. $\left(-1+2\sqrt{2}\right)$ sq. units

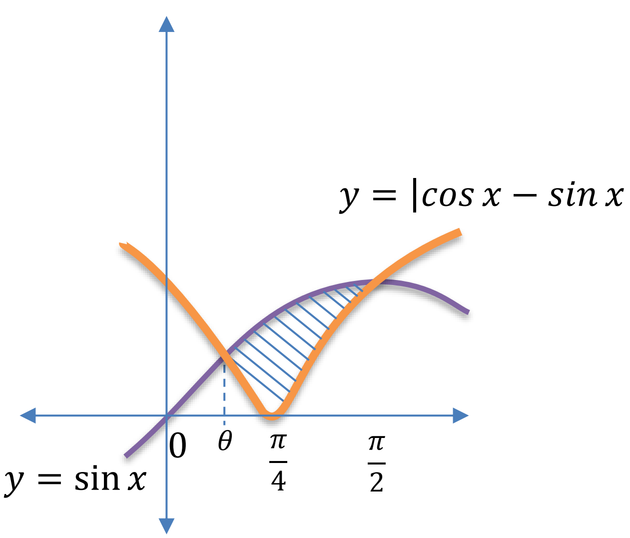
B. $\left(1-\frac{1}{\sqrt{2}}\right)$ sq. units

C. $\left(\sqrt{5}+1-2\sqrt{2}\right)$ sq. units

D. $\left(\sqrt{5}-\sqrt{2}\right)$ sq. units

Answer:(C)

Solution:



$$A=\int_{\theta}^{\frac{\pi}{2}}(\sin x-|\cos x-\sin x|)dx \text{ where } \theta=\tan^{-1}\frac{1}{2}$$

$$A=\int_{\theta}^{\frac{\pi}{4}}(\sin x-\cos x+\sin x)dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\sin x+\cos x-\sin x)dx$$

$$A=-2\cos x-\sin x|_{\theta}^{\frac{\pi}{4}}+\sin x|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$A=-\left(\sqrt{2}+\frac{1}{\sqrt{2}}-2\cos\theta-\sin\theta\right)\ +\left(1-\frac{1}{\sqrt{2}}\right)$$

$$A=-\sqrt{2}-\frac{1}{\sqrt{2}}+(2\cos\theta+\sin\theta)+\left(1-\frac{1}{\sqrt{2}}\right)$$

$$A=1-2\sqrt{2}+2.\frac{2}{\sqrt{5}}+\frac{1}{\sqrt{5}}\ (since\tan\theta=2)$$

$$A=\sqrt{5}+1-2\sqrt{2}$$

9. For solution of differential equation $x \ln x \frac{dy}{dx} + y = x^2 \ln x$, $y(2) = 2$, then $y(e)$ is equal to:

A. $1 + \frac{e^2}{4}$

B. $1 - \frac{e^2}{4}$

C. $\frac{e^2}{2}$

D. $1 + \frac{e^2}{2}$

Answer (A)

Solution:

$x \ln x \frac{dy}{dx} + y = x^2 \ln x$

$\frac{dy}{dx} + \frac{y}{x \ln x} = x$

I.F $= e^{\int \frac{1}{x \ln x} dx} = e^{\ln|\ln x|} = |\ln x|$

Solution of equation is,

$y \cdot (I.F) = \int x \cdot |\ln x| dx$

$y \cdot |\ln x| = |\ln x| \frac{x^2}{2} - \frac{x^2}{4} + C$

Put $x = 2$

$\Rightarrow 2|\ln 2| = |\ln 2| \cdot 2 - 1 + C$

$\Rightarrow C = 1$

Put $x = e$

$y = \frac{e^2}{2} - \frac{e^2}{4} + 1$

$y(e) = 1 + \frac{e^2}{4}$

10. Let $f(x) = x^2 + 2x + 5$ and α, β be roots of $f\left(\frac{1}{t}\right) = 0$, then $\alpha + \beta =$

A. $-\frac{2}{5}$

B. -2

C. $\frac{5}{2}$

D. $-\frac{5}{2}$

Answer (A)

Solution:

$f(x) = x^2 + 2x + 5$

$f(t) = 0$

$\Rightarrow \frac{1}{t^2} + \frac{2}{t} + 5 = 0$

$\Rightarrow 5t^2 + 2t + 1 = 0 \quad (t \neq 0)$

This equation has roots α and β

$\therefore \alpha + \beta = -\frac{2}{5}$

11. If the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+3}{1}$ and $\frac{x-11}{4}=\frac{y-9}{2}=\frac{z-4}{3}$ intersects at point P, then perpendicular distance of P from plane $3x+2y+6z=10$ is:

A. $\frac{2}{7}$
B. $\frac{3}{7}$
C. $\frac{4}{7}$
D. $\frac{5}{7}$

Answer (B)

Solution:

$L_1 \equiv \frac{x-1}{2}=\frac{y-2}{3}=\frac{z+3}{1}=\lambda$

$L_2 \equiv \frac{x-11}{4}=\frac{y-9}{2}=\frac{z-4}{3}=\mu$
$x=2\lambda+1=4\mu+11 \quad \cdots(1)$
$z=\lambda-3=3\mu+4 \quad \cdots(2)$
By solving eq.(1) and eq.(2)
We get $\lambda=1,\ \mu=-2$
∴ Point of intersection of the two lines
$x=3, y=5, z=-2$
$\Rightarrow P\equiv(3,5,-2)$
Distance from given plane $=\left|\frac{9+10-12-10}{\sqrt{9+4+36}}\right|=\frac{3}{7}$

12. If $\cos^2 2x-\sin^4 x-2\cos^2 x=\lambda$ has a solution $\forall\, x\in\mathbb{R}$, then the range of λ is:

A. $\left[-\frac{1}{2},\ 1\right]$
B. $\left[-\frac{4}{3},\ 0\right]$
C. $(0,\ 2)$
D. None of these

Answer (B)

Solution:
$\cos^2 2x-\sin^4 x-2\cos^2 x=\lambda$
$\Rightarrow (\cos^2 x-\sin^2 x)^2-\sin^4 x-2\cos^2 x=\lambda$
$\Rightarrow 3\cos^4 x-4\cos^2 x=\lambda$
$\Rightarrow 3\left(\left(\cos^2 x-\frac{2}{3}\right)^2-\frac{4}{9}\right)=\lambda$
$\Rightarrow \lambda_{\min}=-\frac{4}{3}\ \&\ \lambda_{\max}=0$
$\Rightarrow \lambda\in\left[-\frac{4}{3},\ 0\right]$

13. $\vec{a}=9\hat{\imath}+2\hat{k}, \vec{b}=\hat{\imath}+\hat{\jmath}+\hat{k}$ and $\vec{c}=7\hat{\imath}-3\hat{\jmath}+2\hat{k}$ are three given vectors. Let there be a $\vec{r}$ such that $(\vec{r}\times\vec{b})+(\vec{b}\times\vec{c})=0$ and $\vec{r}\cdot\vec{a}=0$ then $\vec{r}\cdot\vec{c}$ is ______.

A. $\frac{280}{11}$
B. 28
C. $\frac{279}{13}$
D. $\frac{290}{11}$

Answer (A)

Solution:

$(\vec{r}\times\vec{b})+(\vec{b}\times\vec{c})=0$

$(\vec{r}\times\vec{b})-(\vec{c}\times\vec{b})=0$

$(\vec{r}-\vec{c})\times\vec{b}=0$

$\Rightarrow \vec{r}=\vec{c}+\lambda\vec{b}$

$\vec{r}\cdot\vec{a}=0 \quad \cdots$ (given)

$\vec{c}\cdot\vec{a}+\lambda\vec{b}\cdot\vec{a}=0$

$67+\lambda(11)=0$

$\lambda=-\frac{67}{11}$

$\vec{r}\cdot\vec{c}=(\vec{c}+\lambda\vec{b})\cdot\vec{c}$

$\vec{r}\cdot\vec{c}=|\vec{c}|^2+\lambda\vec{b}\cdot\vec{c}$

$=62-\frac{67}{11}(7-3+2)$

$=62-\frac{67}{11}(6)$

$\vec{r}\cdot\vec{c}=\frac{682-402}{11}=\frac{280}{11}$

14. For observation set x data obtained is $x_i=\{11,12,13,\ldots,41\}$

For another observation set y data obtained is $y_i=\{61,62,63,\ldots,91\}$

Then value of $\bar{x}+\bar{y}+\sigma^2$ where $\bar{x},\bar{y}$ are means of respective data set while σ^2 is the variance of combined data is :

A. 801
B. 754
C. 807
D. 774

Answer (C)

Solution:

$\bar{x}=\frac{\frac{31}{2}(11+41)}{31}=\frac{1}{2}\times 52=26$

$\bar{y}=\frac{\frac{31}{2}(61+91)}{31}=\frac{1}{2}\times 152=76$

$\sigma^2=\frac{\sum x_i^2+\sum y_i^2}{62}-\left(\frac{\sum x_i+\sum y_i}{62}\right)^2$

$\sigma^2=\frac{(11^2+12^2+13^2+\cdots 41^2)+(61^2+62^2+\cdots 91^2)}{62}-51^2$

$\sigma^2=\frac{\left(\frac{41\times42\times83}{6}-\frac{10\times11\times21}{6}\right)+\left(\frac{91\times92\times183}{6}-\frac{60\times61\times121}{6}\right)}{62}-(51)^2$

$\sigma^2=\frac{(41\times7\times83-11\times35)+(91\times46\times61-10\times61\times121)}{62}-(51)^2$

$\sigma^2=\frac{23436+181536}{62}-(51)^2$

$\sigma^2=3306-2601=705$

$\therefore \bar{x}+\bar{y}+\sigma^2=26+76+705=807$

15. If the curve represented by $y = \frac{(x-a)}{(x-3)(x-2)}$ passes through $(1, -3)$ then equation of normal at $(1, -3)$ to the curve is given by

A. $2x + 3y = -7$
B. $3x - 2y = 9$
C. $3x - 4y = 21$
D. $x - 4y = 13$

Answer (D)

Solution:

Curve $y = \frac{(x-a)}{(x-3)(x-2)}$ passes through $(1, -3)$

$\Rightarrow -3 = \frac{(1-a)}{(-2)(-1)}$

$\Rightarrow a = 7$

$f(x) = \frac{(x-7)}{(x-3)(x-2)}$

$f'(x) = \frac{(x-3)(x-2)-(x-7)(2x-5)}{((x-3)(x-2))^2}$

$f'(1) = \frac{2-18}{2^2} = -4$

Slope of normal $= \frac{-1}{-4} = \frac{1}{4}$

Equation of normal:

$y + 3 = \frac{1}{4}(x - 1)$

$\Rightarrow 4y + 12 = x - 1$

$\Rightarrow x - 4y = 13$

16. The number of four-digit numbers N such that $GCD(N, 54) = 2$ is ______.

Answer (3000)

Solution:

N should be divisible by 2 but not by 3.

$N =$ (number of numbers divisible by 2) $-$ (number of number divisible by 6)

$N = \frac{9000}{2} - \frac{9000}{6} = 3000$

17. If $f(1) + 2f(2) + 3f(3) + \cdots + nf(n) = n(n+1)f(n)$ and $f(1) = 1$, then $\frac{1}{f(2022)} + \frac{1}{f(2028)}$ is equal to ______.

Answer (4050)

Solution:

We have,

$f(1) + 2f(2) + 3f(3) + \cdots + nf(n) = n(n+1)f(n) \cdots (i)$

Replacing n by $n+1$ in (i)

$f(1) + 2f(2) + 3f(3) + \cdots + nf(n) + (n+1)f(n+1) = (n+1)(n+2)f(n+1) \cdots (ii)$

Using (i) in (ii) we have:

$n(n+1)f(n) + (n+1)f(n+1) = (n+1)(n+2)f(n+1)$

$\Rightarrow f(n+1) = \left(\frac{n}{n+1}\right) f(n)$

$\because f(1) = 1$

$\Rightarrow f(2) = \frac{1}{2}$

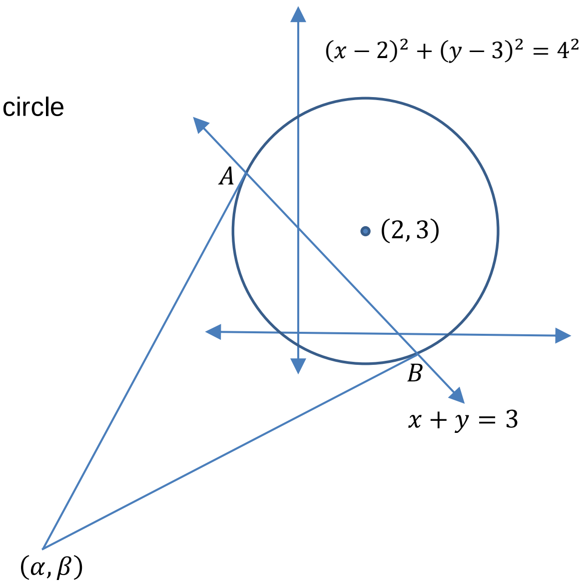
$\Rightarrow f(3) = \frac{1}{3}$
$\dots$
$\Rightarrow f(n) = \frac{1}{n}$

$\frac{1}{f(2022)} + \frac{1}{f(2028)} = 2022 + 2028 = 4050$

18. A line $x + y = 3$ cuts the circle having centre $(2,\ 3)$ and radius 4 at two points A and B. Tangents drawn at A and B intersect at $(\alpha,\ \beta)$. Then the value of $4\alpha - 7\beta$ is __________.

Answer (11)

Solution:



The given line $x + y = 3$ is the chord of contact of $(\alpha,\ \beta)$ w.r.t given circle
Circle Equation: $(x-2)^2 + (y-3)^2 = 4^2$
$\Rightarrow x^2 + y^2 - 4x - 6y - 3 = 0$
Chord of contact of $(\alpha,\ \beta)$ w.r.t circle is
$\alpha x + \beta y - 2(x + \alpha) - 3(\beta + y) - 3 = 0$
$(\alpha - 2)x + (\beta - 3)y - (2\alpha + 3\beta + 3) = 0$
But equation of chord of contact is $x + y - 3 = 0$
Comparing the coefficients,
$x + y - 3 = 0$
$\frac{\alpha - 2}{1} = \frac{\beta - 3}{1} = -\frac{2\alpha + 3\beta + 3}{-3}$
$\Rightarrow \alpha = -6,\ \beta = -5$
$\therefore 4\alpha - 7\beta = 11$

19. Consider a sequence $a_1,\ a_2,\ \cdots,\ a_n$ given by $a_n = a_{n-1} + 2^{n-1}$, $a_1 = 1$ and another sequence given by $b_n = b_{(n-1)} + a_{n-1}$, $b_1 = 1$. Also $P = \sum_{n=1}^{10} \frac{b_n}{2^n}$ and $Q = \sum_{n=1}^{10} \frac{n}{2^{n-1}}$, then $2^7(P - 2Q)$ is __________.

Answer (7.5)

Solution:

$a_2 - a_1 = 2^1$
$a_3 - a_2 = 2^2$
$\dots$
$a_n - a_{n-1} = 2^{n-1}$

$a_n = 2^n - 1$

$b_2 - b_1 = a_1$
$b_3 - b_2 = a_2$
$\dots$
$b_n - b_{n-1} = a_{n-1}$

$b_n = 2^n - n$

$P - 2Q = \sum_{n=1}^{10} \frac{2^n - n}{2^n} - \frac{2n}{2^{n-1}}$
$= \sum_{n=1}^{10} \left(1 - \frac{5n}{2^n}\right) = 10 - 5\sum_{n=1}^{10} \frac{n}{2^n}$
Let $S_n = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \cdots + \frac{n}{2^n} \quad \cdots (1)$
$\frac{S_n}{2} = \frac{1}{2^2} + \frac{2}{2^3} + \frac{3}{2^4} + \cdots + \frac{n}{2^{n+1}} \quad \cdots (2)$

By subtracting eq.(2) from eq.(1) we get,

$\frac{S_n}{2} = \left(\frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^n}\right) - \frac{n}{2^{n+1}}$

$\frac{S_n}{2} = \left(\frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^n}\right) - \frac{n}{2^{n+1}}$

$\Rightarrow \frac{S_n}{2} = \frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^n\right)}{\frac{1}{2}} - \frac{n}{2^{n+1}}$

$\Rightarrow S_n = 2\left(1 - \left(\frac{1}{2}\right)^n - \frac{n}{2^{n+1}}\right)$

$\Rightarrow S_{10} = 2\left(1 - \left(\frac{1}{2}\right)^{10} - \frac{10}{2^{11}}\right)$

$= 2\left(1 - \frac{12}{2^{11}}\right)$

$P - 2Q = 10 - 5 \times 2\left(1 - \frac{12}{2^{11}}\right)$

$P - 2Q = 10 - 10 + \frac{120}{2^{11}} = \frac{60}{2^{10}}$

$\therefore 2^7(P - 2Q) = 7.50$