JEE Main 2023 (Memory based)

31 January 2023 - Shift 2

Answer & Solutions

PHYSICS

1. Match the radiation listed in column-I with their uses listed in column-II correctly.

Column-I	Column-II
A) UV rays	P) Physiotherapy
B) Infra Red rays	Q) Treatment of cancer
C) X-Rays	R) Lasik eye surgery
D) Microwave rays	S) Aircraft navigation

A. $A-S,\ B-P, C-R, D-Q$
B. $A-R,\ B-P, C-Q, D-S$
C. $A-Q,\ B-P, C-S, D-R$
D. $A-R,\ B-P, C-S, D-Q$

Answer (B)

Solution:

UV rays are used for Lasik eye surgery.
IR is used for physiotherapy.
X-Rays are used for cancer treatment.
And Microwaves are used for aircraft navigation.

2. During an adiabatic process performed on a diatomic gas $725\,J$ of work is done on the gas. The change in internal energy of the gas is equal to

A. $495\,J$
B. $725\,J$
C. $225\,J$
D. $Zero$

Answer (B)

Solution:

For adiabatic process, $Q = 0$
So,
$\Delta U + W = 0$
Work done on gas will be negative
$\Delta U - 725 = 0 \Rightarrow \Delta U = 725\,J$

3. Two balls are projected with equal speed $(40\ m/s)$, one at an angle of $30°$ and other at $60°$ with horizontal. Find the ratio of maximum heights of both the balls.

A. $1/4$
B. $3/1$
C. $1/3$
D. $4/1$

Answer (C)

Solution:

Maximum height of projectile can be given as:

$$H_{max} = \frac{u^2 \sin^2 \theta}{2g}$$

Ratio of Maximum heights for same velocity:

$$Ratio = \frac{\sin^2 30°}{\sin^2 60°} = \frac{1}{3}$$

4. Find ionization energy of 2^{nd} excited state of Li^{2+}. It is given that ionization energy of ground state of hydrogen atom is $13.6\ eV$.

A. $20.4\ eV$
B. $27.2\ eV$
C. $6.8\ eV$
D. $13.6\ eV$

Answer (D)

Solution:

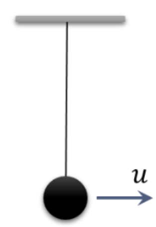
For Li^{+2} ion in 2^{nd} excited state, $Z = 3\ and\ n = 3$.

Ionisation energy can be calculated as:

$$E = 13.6(3)^2\left[\frac{1}{3^2} - 0\right] = 13.6\ eV$$

5. A ball of mass $1\ kg$ is hanging from $1\ m$ long inextensible string which can withstand maximum tension of $400\ N$. Find the maximum speed u that should be given to the ball.

A. $\sqrt{390}\ m/s$
B. $\sqrt{410}\ m/s$
C. $20\ m/s$
D. $22\ m/s$



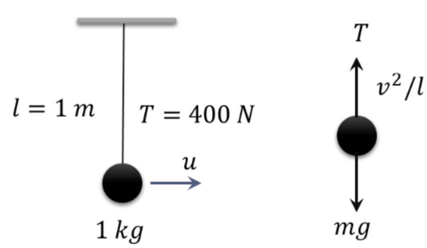
Answer (A)

Solution:

$$T = mg + m\left(\frac{v^2}{l}\right)$$

$$400 = 10 + v^2/1$$

$$v = \sqrt{390}\ \ m/s$$



6. Match the physical quantities given in Column−I with the physical dimensions in column-II

Column-I	Column-II
(A) Torque	(P) $ML^{-1}T^{-2}$
(B) Stress	(Q) ML^2T^{-2}
(C) Pressure Gradient	(R) $ML^{-2}T^{-2}$
(D) Angular momentum	(S) ML^2T^{-1}

A. $A-S,\ B-P, C-R, D-Q$
B. $A-Q,\ B-P, C-R, D-S$
C. $A-P,\ B-S, C-R, D-Q$
D. $A-Q,\ B-P, C-S, D-R$

Answer (B)

Solution:

$$[\tau]=[r][F]=[L][MLT^{-2}]=[ML^2T^{-2}]$$
$$[Stress]=\frac{[F]}{[A]}=\frac{[MLT^{-2}]}{[L^2]}=[ML^{-1}T^{-2}]$$
$$[Pressure\ Gradient]=[P]/[Z]=\frac{[ML^{-1}T^{-2}]}{[L]}=[ML^{-2}T^{-2}]$$
$$[Angular\ Momentum]=[\tau][t]=[ML^2T^{-2}][T]=[ML^2T^{-1}]$$

7. A lens of refractive index 1.5 and focal length $15\ cm$ in air is submerged in water. Change in focal length of lens is $(\mu=4/3)$

A. $45\ cm$
B. $60\ cm$
C. $30\ cm$
D. $10\ cm$

Answer (A)

Solution:

When lens is placed in air,

$$\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$
$$\frac{1}{15}=\left(\frac{1.5}{1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\ldots\ldots..(1)$$

When submerged in water $(\mu=4/3)$

$$\frac{1}{f'}=\left(\frac{1.5}{4/3}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\ldots\ldots..(2)$$

Diving equation (1) and (2)

$$\frac{f'}{15}=\left(\frac{0.5\times 4}{0.5}\right)\Rightarrow f'=60\ cm$$

$$\Delta f=f'-f=60-15=45\ cm$$

8. In a moving coil galvanometer, number of turns in the coil are increased to increase the current sensitivity by 50%. Find percentage change in voltage sensitivity.

A. $-50\ \%$
B. $50\ \%$
C. No change
D. 25%

Answer (C)

Solution:

Current sensitivity:
$$\frac{\theta}{I} = \frac{nAB}{K}$$

Voltage sensitivity $= \frac{naB}{KR}$

As current sensitivity increases by 50% so, number of turns increases by $50\ \%$

Resistance increases by $50\ \%$

Therefore, voltage sensitivity remains constant.

9. The equation of two simple harmonic motions are given by $y_1 = 10\sin\left(\omega t + \frac{\pi}{3}\right)$ and $y_2 = 5\left[\sin(\omega t) + \sqrt{3}\cos(\omega t)\right]$. The amplitude of resultant $\mathrm{S.H.M.}$ is

A. $10\ m$
B. $20\ m$
C. $5\ m$
D. $15\ m$

Answer (B)

Solution:

$$y_1 = 10\sin\left(\omega t + \frac{\pi}{3}\right)$$
$$y_2 = 5\left[\sin(\omega t) + \sqrt{3}\cos(\omega t)\right] = 10\, sin\left(\omega t + \frac{\pi}{3}\right)$$
Resultant of the SHM
$$y_{resultant} = y_1 + y_2$$
$$= 10\sin\left(\omega t + \frac{\pi}{3}\right) + 10\sin\left(\omega t + \frac{\pi}{3}\right)$$
$$= 20\sin\left(\omega t + \frac{\pi}{3}\right)$$
Amplitude $= 20\ m$

10. A body has weight W on the surface of earth. Find the weight at a height $9\ times$ the radius of earth.

A. $W/100$
B. $W/81$
C. $W/64$
D. $W/121$

Answer (A)

Solution:

$$W = \frac{GM_e m}{R_e^2} \;\ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

$$W' = \frac{GM_e m}{(R_e + 9R_e)^2} \ldots\ldots\ldots\ldots.(2)$$

From (1) and (2),

$$W' = \frac{W}{100}$$

11. A wire is first coiled in n circular turns and current I is run through it. Now the same wire is coiled in N circular turns and same current I is run through it. If B_1 and B_2 are the magnetic field at centre of two coil respectively then $\frac{B_1}{B_2}$ is equal to

A. $\sqrt{\frac{n}{N}}$

B. $\left(\frac{n}{N}\right)^2$

C. $\frac{n}{N}$

D. $\frac{n^3}{N^3}$

Answer (B)

Solution:

Let the length of wire is l,

Radius of the first coil $R_1 = \frac{l}{2\pi n}$

Radius of the second coil $R_2 = \frac{l}{2\pi N}$

$$B_1 = \frac{\mu_0 n I}{2R_1} = \frac{\mu_0 n I}{\frac{2l}{2\pi n}} = \frac{\mu_0 \pi n^2 I}{l}$$

$$B_2 = \frac{\mu_0 N I}{2R_2} = \frac{\mu_0 N I}{\frac{2l}{2\pi N}} = \frac{\mu_0 \pi N^2 I}{l}$$

$$\frac{B_1}{B_2} = \left(\frac{n}{N}\right)^2$$

12. For a medium, it is given that: Young's modulus $= 3.2 \times 10^{10}\ N/m^2$, Density $= 8000\ kg/m^3$. Find the speed of sound in this medium.

A. $1000\ m/s$

B. $2000\ m/s$

C. $500\ m/s$

D. $4000\ m/s$

Answer (B)

Solution:

$$v_s = \sqrt{\frac{Y}{\rho}}$$

$$= \sqrt{\frac{3.2 \times 10^{10}}{8000}}$$

$$= 2000\ m/s$$

13. When current of $4\ Amperes$ is made to run through a resistance of $R\ ohms$ for $10\ seconds$, it produces heat energy of $H\ units$. Now if $16\ Amperes$ of current is made to flow through same resistance for $10\ seconds$ then heat energy produced will be:

A. $16\ H$
B. $4\ H$
C. $8\ H$
D. $2\ H$

Answer (A)

Solution:

$$H = i^2Rt = 4^2 \times R \times 10 = 160R$$
$$H' = I^2Rt = 16^2 \times R \times 10 = 2560R = 16H$$

14. Across an inductor of $5mH$, an AC source with potential given as $268\sin(200\pi t)$ volts is used. The value of inductive reactance provided by inductor is equal to

A. $2\pi\ \Omega$
B. $\pi/2\ \Omega$
C. $20\pi\ \Omega$
D. $\pi\ \Omega$

Answer (D)

Solution:

$$\chi_L = \omega L = 200\pi \times 5 \times 10^{-3} = \pi\ \Omega$$

15. In a series RLC circuit, $R = 80\ \Omega$, $\mathrm{X_L} = 100\ \Omega$, $\mathrm{X_C} = 40\ \Omega$. If the source voltage is $2500\cos(628t)\ Volts$, Find peak current $(in\ Amperes)$

Answer (25)

Solution:

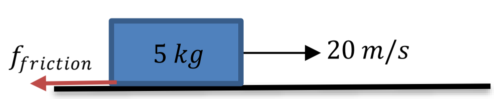
$$Z = \sqrt{R^2 + (X_L - X_c)^2}$$
$$= \sqrt{80^2 + (100 - 40)^2}$$
$$= 100\ \Omega$$

$$\Rightarrow I_0 = \frac{V_0}{Z} = \frac{2500}{100}\ A\ = 25\ A$$

16. A body moving horizontally has an initial speed of $20\ m/s$. Due to friction, body stops after $5\ sec$. If mass of body is $5\ kg$, coefficient of friction is $x/5$. Find x. (Take $g = 10\ m/s^2$)

Answer (0.4)

Solution:

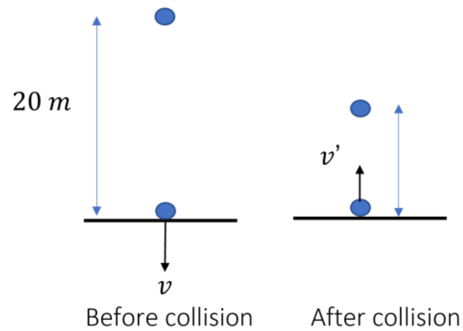


$$u = 20m/s$$
$$t = 5\ s$$
$$f_{friction} = \mu mg$$
$$a = \frac{f_{friction}}{m} = -\mu g$$
$$v = u + at = 20 + (-\mu g)(5)$$
$$0 = 20 - 50\mu$$
$$\mu = 0.4$$

17. A ball was dropped from $20\ m$ height from ground. Find the height $(in\ m)$ up to which it rises after the collision. (Use $e = \frac{1}{2}$, $g = 10\ m/s^2$)

Answer (5)

Solution:



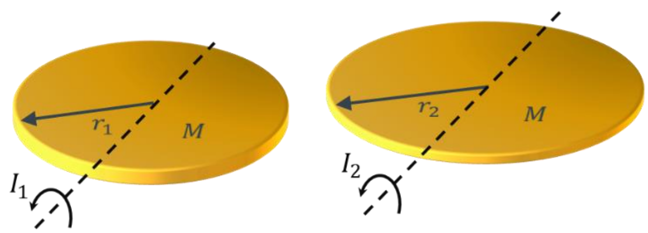
$$h = \frac{v^2}{2g}$$
$$v' = ev$$
$$h' = \frac{(v')^2}{2g} = \frac{e^2v^2}{2g} = e^2h = 0.5^2 \times 20 = \frac{20}{4} = 5\ m$$
$$h' = 5\ m$$

18. Two discs of same mass, radii r_1, r_2, thickness $1\ mm$ and $0.5\ mm$, have densities in the ratio $3:1$. the ratio of their moment of inertia about diameter is $1:x$. Find x.

Answer (6)

Solution:

Mass of both disc is equal:



So, $M_1 = M_2$
$$\pi r_1^2 h_1 \rho_1 = \pi r_2^2 h_2 \rho_2$$
$$r_1^2 \times \frac{h_1}{h_2} \times \frac{\rho_1}{\rho_2} = r_2^2$$
$$\Rightarrow r_1^2 \times 2 \times \frac{\rho_1}{\rho_2} = r_2^2$$
$$\Rightarrow \frac{r_1^2}{r_2^2} = \frac{\rho_2}{2\rho_1} = \frac{1}{6} \qquad \left(\because \frac{\rho_2}{\rho_1} = \frac{1}{3}\right)$$

Ratio of MOI:

$$\frac{\frac{1}{4}Mr_1^2}{\frac{1}{4}Mr_2^2} = \frac{r_1^2}{r_2^2} = \frac{1}{6}$$

19. Two wavelengths $\lambda_1 = 600\ nm$ and $\lambda_2 = 800\ nm$ are used in a $YDSE$ experiment. Their maxima coincide at certain locations on the screen. Find the minimum separation $(in\ mm)$ between such a location and central maxima. It is given that $d = 0.35\ mm$ and $D = 7\ m$.

Answer (48)

Solution:

$$n_1 \times \frac{\lambda_1 D}{d} = n_2 \times \frac{\lambda_2 D}{d}$$
$$\Rightarrow 6n_1 = 8n_2$$
$$\Rightarrow Maximum, n_1 = 4 \quad and\ n_2 = 3$$

So, first coincidence is the 4^{th} maxima of $\lambda = 600\ nm$ with third maxima of wavelength $800\ nm$

$$Min.separation = 4 \times \frac{600\ nm \times 7\ m}{0.35\ mm} = 48 \times 10^{-3}\ m \Rightarrow Min.separation = 48\ mm$$

20. A particle is in uniform circular motion with time period $4\ s$ and radius $\sqrt{2}\ m$. Find the magnitude of displacement $(in\ m)$ is $3\ s$.

Answer (2)

Solution:

$$\theta = \frac{3}{4} \times 2\pi = \frac{3\pi}{2}$$

$$\Rightarrow |Displacement| = \sqrt{2}\, R = 2\ m$$

CHEMISTRY

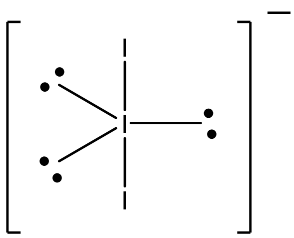
1. Which one of the following species is linear in shape?

A. I_3^-
B. I_3^+
C. ICl_3
D. ICl_2^+

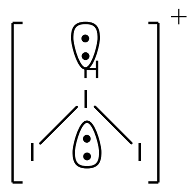
Answer (A)

Solution:

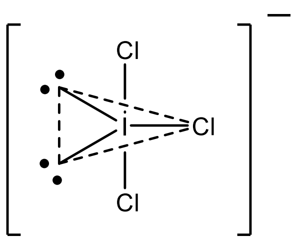
The shapes of the given species are



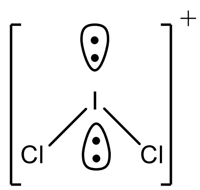
I_3^- - Linear



I_3^+- Angular (or) bent



ICl_3 - T - Shaped



ICl_2^+ - Angular (or) bent

2. For a given hydrocarbon, 11 moles of O_2 is used and produces 4 moles of H_2O. Then, the formula for hydrocarbon is:

A. $C_{11}H_8$
B. C_9H_8
C. $C_{11}H_{16}$
D. C_6H_{14}

Answer (B)

Solution:

$$C_xH_y + \left(x + \frac{y}{4}\right) O_2 = xCO_2 + \frac{y}{2} \times (H_2O)$$

$$\frac{y}{2} = 4$$

$$y = 8$$

$$x + \frac{8}{4} = 11$$

$$x = 9$$

Hence, hydrocarbon will be C_9H_8.

3. Which of the following plays an important role in neuromuscular functions.

A. Ca
B. Mg
C. Be
D. Li

Answer (A)

Solution:

Calcium plays an important role in neuromuscular functions.

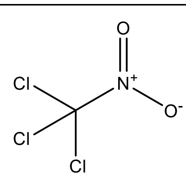
4. Which of the following compound contain maximum number of chlorine atoms?

A. Chloropicrin
B. Chloral
C. Gammaxene
D. Freon-12

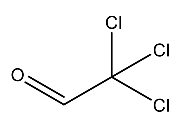
Answer (C)

Solution:

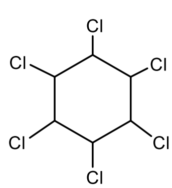
Compounds	Number of chlorine atoms
A. Chloropicrin	3
B. Chloral	3
C. Gammaxene	6
D. Freon-12	2



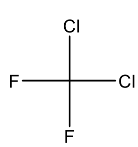
Chloropicrin



Chloral



Gammaxene



Freon-12

5. Decreasing order of Lewis acid character is:

A. $BF_3 > BCl_3 > BBr_3 > BI_3$
B. $BI_3 > BBr_3 > BCl_3 > BF_3$
C. $BF_3 > BCl_3 > BI_3 > BBr_3$
D. $BI_3 > BCl_3 > BF_3 > BBr_3$

Answer (B)

Solution:

Lewis acid charecter α Tendency to accept electrons

And due to backbonding the peripheral atom donates electron to the central atom there by the tendency of the central atom to accept electron decreases.

Thus lewis acid character is inversely proportional to extent of back bonding.

Extent of back bonding

$\frac{BF_3}{2p-2p} > \frac{BCl_3}{2p-3p} > \frac{BBr_3}{2p-4p} > \frac{BI_3}{2p-5p}$

Hence the correct answer is option B.

6. pH of acid rain is 5.6 Which of the following reaction is involved in acid rain.

A. $H_2O + SO_2 + O_2 \rightarrow H_2SO_4$

B. $N_2 + O_2 + H_2O \rightarrow HNO_3$

C. $N_2O + O_2 + H_2O \rightarrow HNO_3$

D. None of these

Answer (A)

Solution:

The correct answer to this question is option (A).

7. Which of the following metals of f-block have half-filled f-subshell?

1. Samarium (Sm)
2. Gadolinium (Gd)
3. Europium (Eu)
4. Terbium (Tb)

[Atomic numbers : Sm = 62, Eu = 63, Gd = 64, Tb = 65]

A. 1 and 2

B. 2 and 3

C. 3 and 4

D. 1 and 3

Answer (B)

Solution:

The valence shell electronic configuration of the given f-block metals are

1. Sm: $4f^6 6s^2$
2. Gd: $4f^7 5d^1 6s^2$
3. Eu: $4f^7 6s^2$
4. Tb: $4f^9 6s^2$

Therefore, Gd and Eu have half-filled f-subshell.

8. If ionisation energy of H-atom is 13.6 eV. Find out ionisation energy of Li^{2+} ions.

A. 54.4 eV

B. 122.4 eV

C. 13.6 eV

D. 3.4 eV

Answer (B)

Solution:

$I.E = 13.6 \times z^2$

$= 13.6 \times (3)^2$

$= 13.6 \times 9$

$= 122.4$ eV

9. Which of the following compound is not a disinfectant?

A. Chloroxylenol
B. Bithionol
C. Terpineol
D. Peracetic acid

Answer (D)

Solution:

Chloroxylenol, bithionol, and terpineol are the disinfectants.

10. A reaction follows 1st order kinetics with rate constant (k) = 20 min^{-1}. Calculate the time required to reach the concentration to 1/32 times of initial concentration.

A. $0.17325\ min$
B. $1.7325\ min$
C. $17.325 min$
D. $173.25\ min$

Answer (A)

Solution:

$\boldsymbol{K = 20\ min^{-}}$

$\boldsymbol{t_{\frac{1}{2}} = \frac{0.693}{K} = \frac{0.693}{20} min}$

$\boldsymbol{C = \frac{C_o}{(C)^n} = \frac{C_o}{32}}$

C = Concentration at time t

C_o = Initial concentration

n = no of half life's

n = 5

$t = 5 \times t_{\frac{1}{2}}$

$= 5 \times \frac{0.693}{20} = 0.17325\ min$

11. If solubility of AgCl in aqueous solution is 1.434×10^{-3} M than find the value of [$-\log K_{sp}$] where K_{sp} is the solubility product of AgCl

A. 3.7
B. 5.7
C. 6.7
D. 7.7

Answer (B)

Solution:

Solubility of AgCl in water = $1.434 \times 10^{-3}\ M$

Solubility product (K_{sp}) of $AgCl$ = $(1.434 \times 10^{-3})^2$

Therefore $K_{sp} = 2 \times 10^{-6}$

$-\log K_{sp} = -log2 + 6 = 5.7$

12. Consider the following combination of n, l, and m values.
(i) n=3; l=0; m=0
(ii) n=4; l=0; m=0
(iii) n=3; l=1; m=0
(iv) n=3; l=2; m=0
The correct order of energy of the corresponding orbitals for multielectron species

A. (ii) > (i)> (iv)> (iii)
B. (iv) > (ii) > (iii) > (i)
C. (i) > (iii) > (iv) > (ii)
D. (iv) > (iii) > (i) > (ii)

Answer (B)

Solution:

In case of multielectron species energy of electron corresponding to an orbital is α (n + l).

If the value of n + l comes out to be same then the one having higher value if n has more energy.
(i) n=3; l=0; m=0
Here, n + l = 3 + 0 = 3

(ii) n=4; l=0; m=0
Here, n + l = 4 + 0 = 4

(iii) n=3; l=1; m=0
Here, n + l = 3 + 1 = 4

(iv) n=3; l=2; m=0
Here, n + l = 3 + 2 = 5

From above we can say that (iv) has maximum energy and (i) has minimum energy.
Among (ii) and (iii) since the value of (n + l) is same and (ii) has higher value of n therefore (ii) has more energy than (iii).

13. Two metals are given:
Metal – 1: Work function = 4.8 eV
Metal – 2: Work function = 2.8 eV
Photons of wavelength 350 nm are incident on both metals separately. Which metal will eject electrons at this wavelength?

A. Metal-1 only
B. Metal-2 only
C. Both metal -1 and metal -2
D. None of metal -1 and metal -2

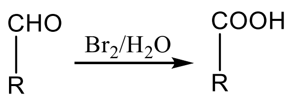
Answer (B)

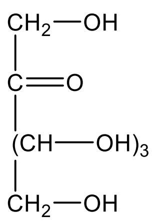
Solution:

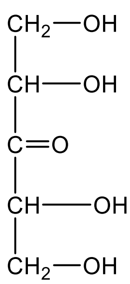
$$E_{\text{photon}} = \frac{12400}{3500} = 3.54 \text{ eV}$$
$$W_{metal-1} > E_{photon} > W_{metal-2}$$

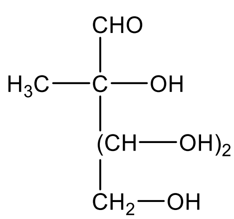
Only metal 2 will emit photons

14. A biomolecule gives the following observations
(i) with Br_2/H_2O, it gives monocarboxylic acid
(ii) with acetate, it gives tetraacetate
(iii) with HI/Red P, it gives isopentane
The correct structure of biomolecule is:

A. 

B. 

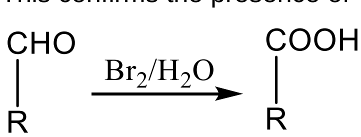
C. 

D. 

Answer (D)

Solution:

Br_2/H_2O Is a mild oxidising agent, it converts the carbonyl group present as aldehyde to carboxylic acid group. This confirms the presence of -CHO group in the reactant.



Acetylation of reactant with acetate is giving us tetraacetate this confirms the presence of 4 -OH groups in the reactant.

$Red\ P/HI$ behave as a strong reducing agent and is converting the reactant into isopentane which confirms the presence of 5 – carbons in the reactant with one methyl chain.

Thus, from the above options the correct answer is option (D).

15. Which of the following has more relative lowering in vapour pressure at the same temperature

A. 0.1 M urea
B. 0.1 M NaCl
C. 0.1 M sucrose
D. 0.1 M $CaCl_2$

Answer (D)

Solution:

Relative lowering of vapor pressure is a colligative property and colligative property depends only on the amount of solute.

$\frac{\Delta P}{p^{O}_{solvent}} = i\chi_{solute}$

$\Delta P = \text{Lowering of vapor pressure}$

$p^{O}_{solvent} = \text{Pure state pressure of solvent}$

$\chi_{solute} = \text{Molefraction of solute}$

$i = \text{Van't hoff factor}$

From above we can say that $\frac{\Delta P}{p^{O}_{solvent}} \alpha\ i$

A. Urea → non electrolyte, therefore, i = 1
B. NaCl → electrolyte, therefore, i = 2
C. Sucrose → non electrolyte, therefore, i = 1
D. $CaCl_2$ → electrolyte, therefore, i = 3

Hence the $CaCl_2$ solution will show maximum relative lowering in vapor pressure.

16. Assertion: First ionization energy of 4d series element is always greater than those of 3d series element.
Reason: 4d series element has much more nuclear charge than those of 3d series element.

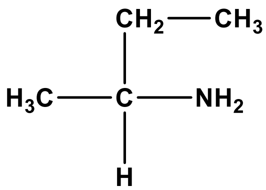
A. Assertion is correct, but reason is incorrect.
B. Assertion is incorrect, but reason is correct
C. Assertion is correct and reason are correct.
D. Assertion is incorrect and reason are incorrect.

Answer (B)

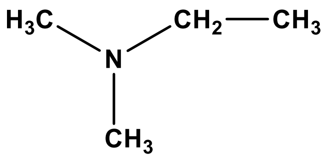
Solution:

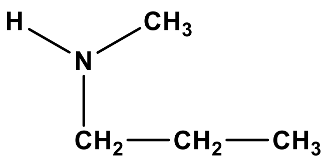
The first ionization energy of 4d series elements is not always greater than those of 3d series elements. So assertion is incorrect. The reason is a correct statement because 4d series elements have much more nuclear charge than those of 3d series elements.

17. What is the structural formula of compound $C_4H_{11}N$, which reacts with HNO_2 and is optically active?

A. 

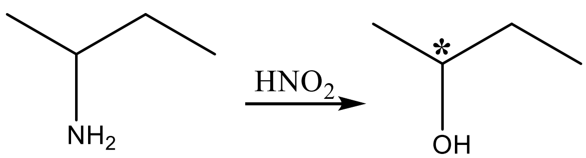
B. 

C. 

D. 

Answer (A)

Solution:



(Optically active)

18. Energy of a radiation given by $E = \frac{hc}{\lambda_{absorb}}$. If $E = 96\frac{KJ}{mole}$. Then find λ_{absorb} $(in\ A^o)$

A. 12471 A^o

B. 124.71 A^o

C. 1247.1 A^o

D. 1.2471 A^o

Answer (A)

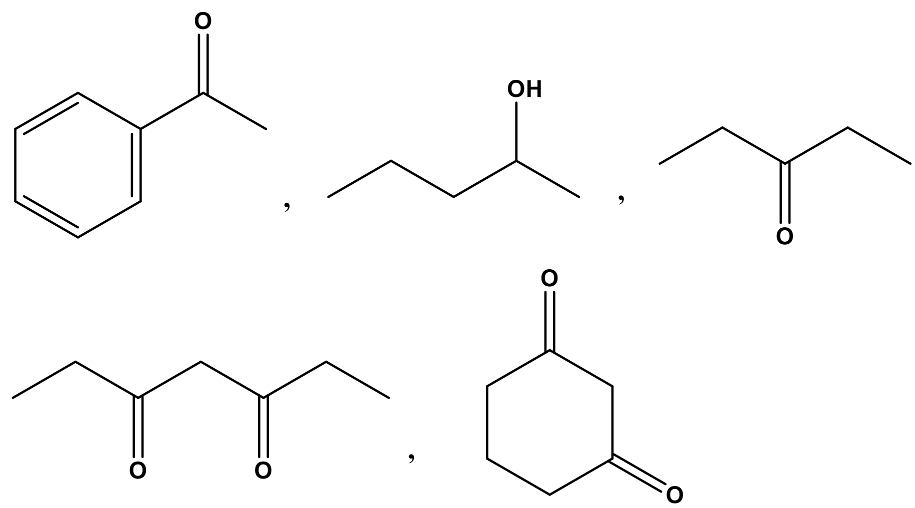
Solution:

$$\frac{6.626\times10^{-34}\times3\times10^{8}\times6.023\times10^{23}}{96\times10^{3}} = \lambda$$

$$\lambda = 1.2471 \times 10^{6} m = 12471\ A^o$$

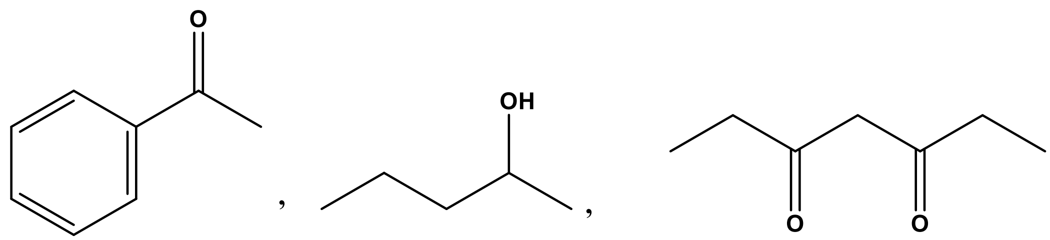
.

19. How many of the following compounds can give iodoform test?

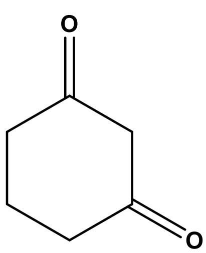


Answer (4)

Solution:



and



will give Iodoform Test.

20. For the given reaction

$C + O_2 \rightarrow CO_2(g)$

12 gm of C is reacted with 48 gm of O_2 to give CO_2. If volume of CO_2 gas produced at STP is t litres. Find out 2t

Given: Molar volume at STP = 22.4 Lit/mol

Answer (45)

Solution:

$C \quad + \quad O_2 \rightarrow CO_2$

12gm 48gm

1 mol 3/2 mol

Volume = t = 22.4 Lit

2t = 44.8 Lit ~ 45 Lit

21. In non-stoichiometry compound $M_{0.83}O$, M exists in 2 states +2 and +3 calculate the percentage of M^{2+} ion in the compound

Answer (59)

Solution:

Let M^{2+} is x

Let M^{3+} will be y

Therefore, x + y = 0.83 → Eq -1

Using charge balancing, $2x + 3y = 2$ → Eq -2

From Eq – 1 and Eq – 2

$x = 0.49$

$$\%M^{2+} = \frac{0.49}{0.83} X100 = 59\%$$

22. The resistivity of 0.8 M solution of an electrolyte is $5X10^{-3}\ \Omega\ cm$. If λ_m is $2.5\ X10^{x}$. Find x

Answer (5)

Solution:

$$\kappa = \frac{10^3}{5}\ S\ cm^{-1}$$

$$\lambda_m = \kappa\,\mathrm{X}\frac{1000}{\mathrm{m}} = \frac{\frac{10^3}{5} X1000}{0.8} = \frac{200X\ 10^3}{0.8}$$

$$= \frac{2X10^5}{0.8} = 2.5\ X10^5$$

$x = 5$

MATHEMATICS

1. Let $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{c} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$. There is a vector $\vec{u}$ such that $\vec{u} \times \vec{a} = \vec{b} \times \vec{c}$ and $\vec{u} \cdot \vec{a} = 0$. find $25|\vec{u}|^2$

A. 560
B. $\frac{925}{7}$
C. 446
D. 330

Answer (B)

Solution:

$|\vec{u} \times \vec{a}|^2 + |\vec{u} \cdot \vec{a}|^2 = |\vec{u}|^2|\vec{a}|^2$

$\left|\vec{b} \times \vec{c}\right|^2 + 0 = |\vec{u}|^2 \times 14$

$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 3 \\ 1 & 2 & 2 \end{vmatrix}$

$\vec{b} \times \vec{c} = \hat{\imath}(-8) - \hat{\jmath}(-1) + \hat{k}(3)$

$\vec{b} \times \vec{c} = -8\hat{\imath} + \hat{\jmath} + 3\hat{k}$

$|\vec{b} \times \vec{c}| = \sqrt{74}$

$\Rightarrow \left(\sqrt{74}\right)^2 = |\vec{u}|^2 \times 14$

$\Rightarrow 25|\vec{u}|^2 = \frac{74}{14} \times 25$

$= \frac{925}{7}$

2. The range of $y = \frac{x^2+2x+1}{x^2+8x+1}$ in the domain of function is :

A. $\left(-\infty, -\frac{2}{3}\right] \cup [2, \infty)$

B. $(-\infty, 0] \cup \left[\frac{2}{5}, \infty\right)$

C. $(-\infty, \infty)$

D. $\left(-\infty, -\frac{2}{5}\right] \cup [1, \infty)$

Answer (B)

Solution:

$y = \frac{x^2+2x+1}{x^2+8x+1}$

$\Rightarrow x^2(y-1) + x(8y-2) + y - 1 = 0,\ x \in R$

If $y \neq 1$

$D \geq 0$

$\Rightarrow 4(4y-1)^2 - 4(y-1)(y-1) \geq 0$

$\Rightarrow (4y-1)^2 - (y-1)^2 \geq 0$

$\Rightarrow \big(4y - 1 - (y-1)\big)\big(4y - 1 + (y-1)\big) \geq 0$

$\Rightarrow (3y)(5y-2) \geq 0$

$y \in (-\infty, 0] \cup \left[\frac{2}{5}, \infty\right) - \{1\}$

If $y = 1$

$6x = 0 \Rightarrow x = 0$

$\therefore y \in (-\infty, 0] \cup \left[\frac{2}{5}, \infty\right)$

3. If $a,\ b \in I$ and relation R_1 is defined as $a^2 - b^2 \in I$ and relation R_2 is defined as $2 + \frac{a}{b} > 0$, then:

A. R_1 is symmetric but R_2 is not
B. R_2 is symmetric but R_1 is not
C. R_1 and R_2 are both symmetric
D. R_1 and R_2 are both transitive

Answer (A)

Solution:

$R_1 \rightarrow a^2 - b^2 \in I$

As $a,\ b \in Z$ if $a^2 - b^2 \in Z$ then $b^2 - a^2 \in Z$

Also $a^2 - b^2 \in Z$ & $b^2 - c^2 \in Z \Rightarrow a^2 - c^2 \in Z$

$\therefore R_1$ is symmetric as well as transitive.

$R_2 \rightarrow 2 + \frac{a}{b} > 0$

Let $a = -1$ and $b = 10$

So, for $(a,\ b) \in R_2 \Rightarrow 2 + \frac{a}{b} > 0$

Now, for $(b, a) \in R_2 \Rightarrow 2 + \frac{b}{a} = 2 - 10 < 0$

$2 + \frac{b}{a} \notin R_2 \Rightarrow R_2$ is NOT symmetric.

4. If $\int \frac{xdx}{\sqrt{x^2+x+2}} = Af(x) + Bg(x) + C$ where C is constant of integration, then $A + 2B$ is equal to:

A. 1
B. 0
C. -1
D. -2

Answer (B)

Solution:

Let $x = \alpha(2x+1) + \beta$

$\alpha = \frac{1}{2},\ \beta = -\frac{1}{2}$

$\int \frac{xdx}{\sqrt{x^2+x+2}} = \frac{1}{2}\int \frac{2x+1}{\sqrt{x^2+x+2}}dx - \frac{1}{2}\int \frac{dx}{\sqrt{x^2+x+2}}$

$= \frac{1}{2} \times 2\sqrt{x^2+x+2} - \frac{1}{2}\int \frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2 + \frac{7}{4}}}dx + C$

$= \sqrt{x^2+x+2} - \frac{1}{2}\ln\left|x + \frac{1}{2} + \sqrt{x^2+x+2}\right| + C$

$I = Af(x) + Bg(x) + C$
$\Rightarrow A = 1,\ B = -\frac{1}{2}$
$\therefore A + 2B = 0$

5. $\lim_{x\to\infty} \frac{\left(\sqrt{3x^2+1}+\sqrt{3x^2-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}$ is equal to:

A. 27
B. $\frac{27}{2}$
C. 18
D. 6

Answer (A)

Solution:

$$\lim_{x\to\infty} \frac{\left(\sqrt{3x^2+1}+\sqrt{3x^2-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6} = \lim_{x\to\infty} \frac{x^6\left(\sqrt{3+\frac{1}{x^2}}+\sqrt{3-\frac{1}{x^2}}\right)^6}{x^6\left[\left(1+\sqrt{1-\frac{1}{x^2}}\right)^6+\left(1-\sqrt{1-\frac{1}{x^2}}\right)^6\right]}$$
$$= \frac{\left(\sqrt{3}+\sqrt{3}\right)^6}{\left(1+\sqrt{1}\right)^6+\left(1-\sqrt{1}\right)^6}$$
$$= \frac{\left(2\sqrt{3}\right)^6}{2^6}$$
$$= \left(\sqrt{3}\right)^6$$
$$= 27$$

6. Foot of perpendicular from origin to a plane which cuts the coordinate axes at A, B, C is $(2,0,4)$. Volume of tetrahedron $OABC$ is $144\ m^3$. Which of the following point does not lie on plane?

A. $(2,2,4)$
B. $(0,3,4)$
C. $(1,1,5)$
D. $(5,5,1)$

Answer (B)

Solution:

Equation of required plane :

$2(x-2) + a(y-a) + 4(z-4) = 0$

$\Rightarrow 2x + ay + 4z = 20 + a^2$

$A\left(10 + \frac{a^2}{2}, 0,0\right),\ B\left(0, \frac{20+a^2}{a}, 0\right), C\left(0,0, \frac{20+a^2}{4}\right)$

Volume of tetrahedron $\frac{1}{6}[\vec{a}\quad \vec{b}\quad \vec{c}] = 144$

$\Rightarrow \frac{1}{6}\left(\frac{20+a^2}{2}\right)\left(\frac{20+a^2}{a}\right)\left(\frac{20+a^2}{4}\right) = 144$

$\Rightarrow (20 + a^2)^3 = 144 \times 48a$

$\Rightarrow a = 2$

$\therefore$ equation of plane $: 2x + 2y + 4z = 24$

$\Rightarrow x + y + 2z = 12$

$(0,3,4)$ does not lie on the plane.

7. If $z = \frac{i-1}{\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}}$, then z is:

A. $\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

B. $\frac{1}{\sqrt{2}}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

C. $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

D. $\frac{1}{\sqrt{2}}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

Answer (C)

Solution:

$z = \frac{i-1}{\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}}$

$= \frac{i-1}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}$

$= \frac{2(i-1)}{(1+i\sqrt{3})} \times \frac{(1-i\sqrt{3})}{(1-i\sqrt{3})}$

$= \frac{1}{2}\left(\sqrt{3}-1\right) + \frac{i}{2}\left(\sqrt{3}+1\right)$

$\therefore Arg\ (z) = \tan^{-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) = \frac{5\pi}{12}$

$|z| = \sqrt{2}$

$z = \sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

8. Given that $\theta \in [0,\ 2\pi]$, the largest interval of values of θ which satisfy the inequation $\sin^{-1}\sin\theta - \cos^{-1}\sin\theta \geq 0$ is:

A. $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

B. $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

C. $[0,\ \pi]$

D. $\left[\frac{\pi}{2}, \frac{5\pi}{4}\right]$

Answer (A)

Solution:

$\sin^{-1}\sin\theta - \left(\frac{\pi}{2} - \sin^{-1}\sin\theta\right) \geq 0$

$\Rightarrow \sin^{-1}\sin\theta \geq \frac{\pi}{4}$

$\Rightarrow \frac{1}{\sqrt{2}} \leq \sin\theta \leq 1$

$\Rightarrow \frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$

9. If $\left((p \wedge q) \Rightarrow (r \vee q)\right) \wedge \left((p \wedge r) \Rightarrow q\right)$ is tautology, where $r \in \{p,\ q,\ \sim p,\ \sim q\}$, then the number of values of r is

A. 1

B. 2

C. 3

D. 4

Answer (B)

Solution:

$\big((p \wedge q) \Rightarrow (r \vee q)\big) \wedge \big((p \wedge r) \Rightarrow q\big)$

$\Rightarrow \big((\sim p \vee \sim q) \vee (r \vee q)\big) \wedge (\sim p \vee \sim r \vee q)$

$\Rightarrow \big(\sim p \vee r \vee (q \vee \sim q)\big) \wedge (\sim p \vee \sim r \vee q)$

$\Rightarrow T \wedge (\sim p \vee \sim r \vee q)$

$\Rightarrow (\sim p \vee \sim r \vee q)$

For the above statements to be tautology

r can be $\sim p$ or q

∴ Two values of r is possible.

10. If $|\vec{a}| = \sqrt{31}$, $4|\vec{b}| = |\vec{c}| = 2$, Given that $2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$,then find $\frac{|\vec{a}\times\vec{c}|}{\vec{a}\cdot\vec{b}}$

A. 3
B. $-\sqrt{3}$
C. 1
D. -3

Answer (B)

Solution:

$2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$

$2(\vec{a} \times \vec{b}) - 3(\vec{c} \times \vec{a}) = 0$

$2(\vec{a} \times \vec{b}) + 3(\vec{a} \times \vec{c}) = 0$

$\vec{a} \times (2\vec{b} + 3\vec{c}) = 0$

$\Rightarrow \vec{a} = \lambda(2\vec{b} + 3\vec{c})$

$\Rightarrow |\vec{a}|^2 = \lambda^2 \left(4|\vec{b}|^2 + 9|\vec{c}|^2 + 12\vec{b} \cdot \vec{c}\right)$

$\Rightarrow 31 = 31\lambda^2 \Rightarrow \lambda = \pm 1$

$\Rightarrow \vec{a} = \pm(2\vec{b} + 3\vec{c})$

$|\vec{b} \times \vec{c}|^2 = |\vec{b}|^2 |\vec{c}|^2 - (\vec{b} \cdot \vec{c})^2$

$= \frac{1}{4} \times 4 - \left(1 \times \left(-\frac{1}{2}\right)\right)^2$

$|\vec{b} \times \vec{c}|^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$|\vec{b} \times \vec{c}| = \frac{\sqrt{3}}{2}$

$\frac{|\vec{a} \times \vec{c}|}{\vec{a} \cdot \vec{b}} = \frac{|(2\vec{b} + 3\vec{c}) \times \vec{c}|}{(2\vec{b} + 3\vec{c}) \cdot \vec{b}}$

$\frac{|\vec{a} \times \vec{c}|}{\vec{a} \cdot \vec{b}} = \frac{\sqrt{3}}{\frac{1}{2} + 3 \times 2 \times \frac{1}{2} \times \left(-\frac{1}{2}\right)}$

$\Rightarrow \frac{|\vec{a} \times \vec{c}|}{\vec{a} \cdot \vec{b}} = -\sqrt{3}$

11. Number of 7 digit odd no's formed using 7 digits $1,\ 2,\ 2,\ 2,\ 3,\ 3,\ 5$ will be:

A. 80
B. 420
C. 240
D. 140

Answer (C)

Solution:

Even number formed
$\_\_\ \_\_\ \_\_\ \_\_\ \_\_\ \_\_\ 2$
Number of ways $= \frac{6!}{2!2!} = 180$
Total number of ways $= \frac{7!}{3!2!} = \frac{720\times7}{12} = 420$
Odd numbers $= 420 - 180$
$= 240$

12. The minimum value of the function $f(x) = |x^2 - x + 1| + [x^2 - x + 1]$ where $[.]$ denotes greatest integer function, is:

A. $\frac{3}{4}$
B. $\frac{5}{4}$
C. $\frac{1}{4}$
D. 0

Answer (A)

Solution:
$g(x) = x^2 - x + 1$
$= x^2 - 2 \cdot x \cdot \frac{1}{2} + \frac{1}{4} - \frac{1}{4} + 1$
$= \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$
$g(x)$ attains minimum value when $x = \frac{1}{2}$
So $f(x)$ is minimum when $x = \frac{1}{2}$
$f\left(\frac{1}{2}\right) = \frac{3}{4} + 0$
$= \frac{3}{4}$

13. The foci and eccentricity of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $(\pm4,\ 0)$ and $\sqrt{3}$ respectively. Then the length of latus rectum of the hyperbola is:

A. 8
B. $\frac{16}{\sqrt{3}}$
C. 4
D. $2\sqrt{3}$

Answer (B)

Solution:

$ae = 4$

$\Rightarrow a = \frac{4}{\sqrt{3}}$

Length of LR= $\frac{2b^2}{a}$

$= \frac{2}{a}a^2(e^2 - 1)$

$= 2a(e^2 - 1)$

$= \frac{8}{\sqrt{3}}(3 - 1)$

$= \frac{16}{\sqrt{3}}$

14. If $[\alpha\ \beta\ \gamma]\begin{bmatrix} 5 & 6 & 8 \\ 6 & 3 & 8 \\ -1 & 3 & 0 \end{bmatrix} = [0\ 0\ 0]$ where $(\alpha,\ \beta,\ \gamma)$ be a point on $2x + 5y + 3z = 5$, then $6\alpha + 5\beta + 9\gamma$ is equal to:

A. 20

B. $\frac{20}{3}$

C. 21

D. 7

Answer (B)

Solution:

$5\alpha + 6\beta - \gamma = 0$

$6\alpha + 3\beta + 3\gamma = 0$

$8\alpha + 8\beta = 0$

$\Rightarrow \alpha = -\beta\ \&\ \beta = \gamma$

Let $\alpha = k,\ \beta = -k,\ \gamma = -k$

$(\alpha,\ \beta,\ \gamma)$ lie on $2x + 5y + 3z = 5$

$\Rightarrow 2(k) + 5(-k) + 3(-k) = 5$

$\Rightarrow k = -\frac{5}{6}$

$\Rightarrow \alpha = -\frac{5}{6},\ \beta = \frac{5}{6},\ \gamma = \frac{5}{6}$

$\therefore 6\alpha + 5\beta + 9\gamma = -5 + \frac{25}{6} + \frac{45}{6} = \frac{20}{3}$

15. Coefficient of x^{-6} in expansion of $\left(\frac{4x}{5} - \frac{5}{2x^2}\right)^9$ is:

Answer (−5040)

Solution:

$T_{r+1} = {}^9C_r\left(\frac{4x}{5}\right)^{9-r}\left(-\frac{5}{2x^2}\right)^r$

$\Rightarrow T_{r+1} = {}^9C_r\left(\frac{4}{5}\right)^{9-r}\left(-\frac{5}{2}\right)^r x^{9-r-2r}$

For coefficient of x^{-6}: $9 - r - 2r = -6$

$\Rightarrow r = 5$

$\therefore$ Coefficient of $x^{-6} = {}^9C_5\left(\frac{4}{5}\right)^4\left(-\frac{5}{2}\right)^5$

$= \frac{9!}{5!4!} \times \frac{4^4}{5^4} \times \frac{(-5)^5}{2^5}$
$= -5040$

16. The value of sum $1 \cdot 1^2 - 2 \cdot 3^2 + 3 \cdot 5^2 - 4 \cdot 7^2 + \cdots + 15 \cdot (29)^2$ is ______.

Answer (6952)

Solution:

$S = 1 \cdot 1^2 - 2 \cdot 3^2 + 3 \cdot 5^2 - 4 \cdot 7^2 + \cdots + 15 \cdot (29)^2$
$S = 1 \cdot 1^2 + 2 \cdot 3^2 + 3 \cdot 5^2 + 4 \cdot 7^2 + \cdots + 15 \cdot (29)^2 - 2(2 \cdot 3^2 + 4 \cdot 7^2 + \cdots + 14 \cdot (27)^2)$
$S = \sum_{n=1}^{15} n(2n-1)^2 - 2\sum_{n=1}^{7}(2n)(4n-1)^2$
$= \sum_{n=1}^{15}(4n^3 - 4n^2 + n) - 4\sum_{n=1}^{7}(16n^3 - 8n^2 + n)$
$= \left[4 \times \left(\frac{15\times16}{2}\right)^2 - 4 \times \frac{(15)(16)(31)}{6} + \frac{15\times(16)}{2}\right] - \left[64 \times \left(\frac{7\times8}{2}\right)^2 - 32 \times \frac{7(8)(15)}{6} + 4 \times \frac{7(8)}{2}\right]$
$= (4(14400) - 4(1240) + 120) - (64(28)^2 - 140(32) + 112)$
$= 52760 - 45808 = 6952$

17. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a 2×2 matrix such that $a, b, c, d \in \{0, 1, 2, 3, 4\}$. The number of matrices A such that sum of elements of A is a prime number lying between 2 and 13 is __________.

Answer (204)

Solution:

As given $a + b + c + d = 3$ or 5 or 7 or 11
If $a + b + c + d = 3$
x^3 coefficient in $(1 + x + x^2 + \cdots + x^4)^4$
x^3 coefficient in $(1 - x^5)^4(1 - x)^{-4}$
$= {}^{4+3-1}C_3 = {}^6C_3 = 20 \quad [\because x^r \text{ coefficient in } (1-x)^{-n} = {}^{n+r-1}C_r]$
If $a + b + c + d = 5$
x^5 coefficient in $(1 - x^5)^4(1 - x)^{-4} = (1 - 4x^5 + \cdots)(1 - x)^{-4} \qquad = {}^{4+5-1}C_5 - 4^{4+0-1}C_0 = {}^8C_5 - 4 = 52$
If $a + b + c + d = 7$
x^7 coefficient in $(1 - 4x^5)^4(1 - x)^{-4} = (1 - 4x^5 + \cdots)(1 - x)^{-4}$
$= {}^{4+7-1}C_7 - 4 \cdot {}^{4+2-1}C_2 = {}^{10}C_7 - 4 \cdot {}^5C_2 = 80$
If $a + b + c + d = 11$
x^{11} coefficient in in $(1 - x^5)^4(1 - x)^{-4} = (1 - 4x^5 + 6x^{10})(1 - x)^{-4}$

$= {}^{4+11-1}C_{11} - 4 \cdot {}^{4+6-1}C_6 + 6 \cdot {}^{4+1-1}C_1$
$= {}^{14}C_{11} - 4 \cdot {}^9C_6 + 6 \cdot 4 = 364 - 336 + 24 = 52$
Total matrices $= 20 + 52 + 80 + 52 = 204$

18. If $\frac{{}^{2n+1}P_{n-1}}{{}^{2n-1}P_n} = \frac{11}{21}$, then $n^2 + n + 15$ equals ______.

Answer (45)

Solution:

$\frac{{}^{2n+1}P_{n-1}}{{}^{2n-1}P_n} = \frac{11}{21}$
$\Rightarrow \frac{(2n+1)!(n-1)!}{(n+2)!(2n-1)!} = \frac{11}{21}$
$\Rightarrow \frac{(2n+1)(2n)}{(n+2)(n+1)n} = \frac{11}{21}$
$\Rightarrow \frac{2(2n+1)}{(n+1)(n+2)} = \frac{11}{21}$
$\Rightarrow 84n + 42 = 11n^2 + 33n + 22$

$\Rightarrow 11n^2 - 51n - 20 = 0$
$\Rightarrow 11n^2 - 55n + 4n - 20 = 0$
$\Rightarrow (11n + 4)(n - 5) = 0$
$\Rightarrow n = 5$ or $n = -\frac{4}{11}$ (not possible)
$\Rightarrow n = 5$
$\Rightarrow n^2 + n + 15 = 25 + 5 + 15 = 45$

19. $\int_0^{\alpha} \frac{x}{\sqrt{x+\alpha}-\sqrt{x}} dx = \frac{16+20\sqrt{2}}{15}$ then α is equal to __________.

Answer (2)

Solution:

$$\int_0^{\alpha} \frac{x}{\sqrt{x+\alpha}-\sqrt{x}} dx = \int_0^{\alpha} \frac{1}{\alpha}\left[(x+\alpha)^{\frac{3}{2}} - \alpha(x+x)^{\frac{1}{2}} + x^{\frac{3}{2}}\right] dx$$
$$= \frac{1}{\alpha}\left[\frac{2}{5}(x+\alpha)^{\frac{5}{2}} - \alpha\frac{2}{3}(x+\alpha)^{\frac{3}{2}} + \frac{2}{5}x^{\frac{5}{2}}\right]_0^{\alpha}$$
$$= \frac{1}{\alpha}\left[\frac{2}{5}\cdot(2\alpha)^{\frac{5}{2}} - \frac{2\alpha}{3}(2\alpha)^{\frac{3}{2}} + \frac{2}{5}\alpha^{\frac{5}{2}} - \frac{2}{5}\alpha^{\frac{5}{2}} + \frac{2}{3}\alpha^{\frac{5}{2}}\right]$$
$$= \frac{1}{\alpha}\left(\frac{2^{\frac{7}{2}}\alpha^{\frac{5}{2}}}{5} - \frac{2^{\frac{5}{2}}\alpha^{\frac{5}{2}}}{3} + \frac{2}{3}\alpha^{\frac{5}{2}}\right)$$
$$= \alpha^{\frac{3}{2}}\left(\frac{2^{\frac{7}{2}}}{5} - \frac{2^{\frac{5}{2}}}{3} + \frac{2}{3}\right)$$
$$= \frac{\alpha^{\frac{3}{2}}}{15}\left(3\cdot 2^{\frac{7}{2}} - 5\cdot 2^{\frac{5}{2}} + 10\right)$$
$$= \frac{\alpha^{\frac{3}{2}}}{15}\left(24\sqrt{2} - 20\sqrt{2} + 10\right)$$
$$= \frac{\alpha^{\frac{3}{2}}}{15}\left(4\sqrt{2} + 10\right) = \frac{16+20\sqrt{2}}{15}$$
$$\Rightarrow \alpha = 2$$