JEE Main 2023 (Memory based)

1st February 2023 - Shift 2

Answer & Solutions

PHYSICS

1. Ratio of acceleration due to gravity on the surface of planet 1 and planet 2 is x while the ratio of radii of respective planets is y. The ratio of respective escape velocity on the surface of planet 1 and planet 2 is equal to

A. $\sqrt{\frac{x}{y}}$
B. $\frac{x}{y}$
C. $\sqrt{xy}$
D. xy

Answer (C)

Solution:

Escape velocity can be given as:

$$v_e = \sqrt{\frac{2GM}{R} \times \frac{R}{R}} = \sqrt{2gR}$$

$$\text{So,} \frac{v_1}{v_2} = \sqrt{\frac{g_1}{g_2} \times \frac{R_1}{R_2}} = \sqrt{xy}$$

2. In a hydrogen atom, an electron makes a transition from 3^{rd} excited state to ground state. Find the energy of the photon emitted.

A. $10.8\ eV$
B. $13.6\ eV$
C. $12.75\ eV$
D. $8.6\ eV$

Answer (C)

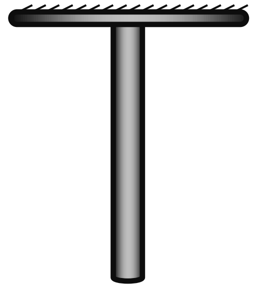
Solution:

$$\Delta E = 13.6(1)^2 \left[1 - \frac{1}{4^2}\right]\ eV$$
$$= 13.6 \times \frac{15}{16} = 12.75\ eV$$

3. A uniform rod of mass $10\ kg$ and length $6\ m$ is hanged from the ceiling as shown. Given area of cross-section of rod $3\ mm^2$ and Young's modulus is $2 \times 10^{11} \frac{N}{m^2}$. Find extension in the rod's length. [use $g = 10\ m/s^2$]

A. $1\ mm$
B. $0.5\ mm$

C. $0.25\ mm$
D. $1.2\ mm$



Answer (B)

Solution:

Young's modulus, $Y = 2 \times 10^{11} \frac{N}{m^2}$.
Area $= 3\ mm^2$
Mass of the rod $= 10\ kg$

We know that:

$$\Delta L = \left(\frac{mgL}{2AY}\right) = \frac{10 \times 10 \times 6}{2 \times 3 \times 10^{-6} \times 2 \times 10^{11}} = \frac{1}{2} \times 10^{-3}\ m = 0.5\ mm$$

4. For a heat engine based on Carnot cycle source is at temperature $600\ K$. Now if source temperature is $doubled$ then efficiency also gets doubled while keeping the sink temperature same at $x\ kelvin$. Value of x is equal to

A. $400\ K$
B. $600\ K$
C. $200\ K$
D. $300\ K$

Answer (A)

Solution:

Let the initial efficiency is x and sink temperature is T thus.

$$x = 1 - \frac{T}{600}$$

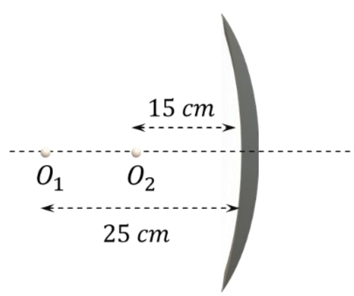
$$2x = 1 - \frac{T}{1200}$$

$$\frac{1}{2} = \frac{1 - \frac{T}{600}}{1 - \frac{T}{1200}} \Rightarrow \frac{T}{800} = \frac{1}{2}$$

$$T = 400\ K$$

5. Two point objects O_1 and O_2 are placed on principle axis of concave mirror of radius of curvature $40\ cm$. Find the distance between the two images.

A. $160\ cm$
B. $40\ cm$
C. $100\ cm$
D. $80\ cm$



Answer (A)

Solution:

For O_1:

$u = -25\ cm$
$f = -20\ cm$
$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v_1} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{20} + \frac{1}{25} = -\frac{1}{100} \Rightarrow v_1 = -100\ cm$$

For O_2:

$u = -15\ cm$
$f = -20\ cm$

$$\frac{1}{v_2} = -\frac{1}{20} + \frac{1}{15} = \frac{1}{60} \Rightarrow v_2 = +60$$
$$|v_1 - v_2| = [60 - (-100)] = 160\ cm$$

6. A train (moving with initial speed $= 20\ m/s$) applies brakes to stop at the incoming station which is $500\ m$ ahead. If brakes are applied after moving $250\ m$, then how much beyond the station train would stop?

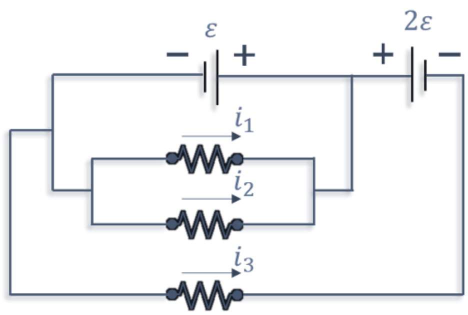
A. $125\ m$
B. $500\ m$
C. $250\ m$
D. $400\ m$

Answer (C)

Solution:
The train needs $500\ m$ to stop.
So, it will move beyond the station by $500\ m - 250\ m = 250\ m$

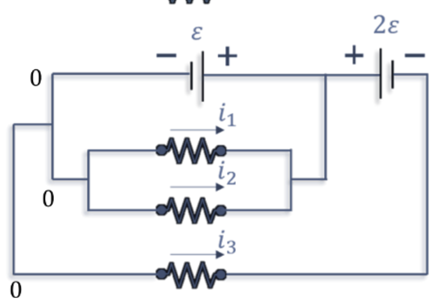
7. Consider the following circuit. All resistors have resistance $10\ \Omega$ each. Find $\left|\frac{i_1+i_2}{i_3}\right|$

A. 2
B. 1
C. 3
D. $1/3$



Answer (A)

Solution:



Magnitudes of current:
$I_1 = -\varepsilon/R$
$I_2 = -\varepsilon/R$
$I_3 = \varepsilon/R$

$$\left|\frac{I_1 + I_2}{I_3}\right| = \frac{2\varepsilon/R}{\varepsilon/R} = 2$$

8. Assertion (A): For making a voltmeter, we prefer a voltmeter of resistance of $4000\ \Omega$ over a voltmeter of resistance $1000\ \Omega$.
Reason (R): Voltmeter should be of higher resistance such that it draws less current from the circuit.

A. A and R both are true. R is correct explanation of A.
B. A and R both are true. R is not the correct explanation of A.
C. A is true but R is false.
D. A is false but R is true.

Answer (A)

Solution:

The reason is correctly explaining the statement as, if more current is drawn the net resistance of circuit will change and we cannot get correct value of potential. To avoid this, we choose higher resistance.

9. According to the shown $P-T$ graph of three processes, temperature at point O is equal to

A. $0°C$
B. $-373°C$
C. $100°C$
D. $-273°C$

Answer (D)

Solution:

All the gases will cease to exist at $-273°\,C$, therefore the pressure will be zero so the temperature of point O is $-273°C$

10. A wire of length l, cross-sectional area A is pulled as shown. Y is the Young's modulus of wire. Find the elongation in wire if: $F=100\ N, A=10\ cm^2,\ l=1\ m,\ Y=5\times10^{10}\ N/m^2$

A. $10^{-6}\ m$
B. $10^{-5}\ m$
C. $2\times10^{-6}\ m$
D. $2\times10^{-5}\ m$

Answer (C)

Solution:

$$\Delta l=\frac{Fl}{AY}=2\times10^{-6}$$

11. In a YDSE Setup, if a mica sheet of thickness 't' and refractive index μ is inserted in front of one of the slits. Find the number of fringes by which the central fringe gets shifted.
[Given: λ, D and d are wavelength of light, distance between slits and screen and slit separation respectively]

A. $\frac{\mu t}{\lambda}$
B. $\frac{(\mu-1)t}{\lambda}$
C. $\frac{(\mu+1)t}{\lambda}$
D. $\frac{(2\mu-1)t}{\lambda}$

Answer (B)

Solution:

Path difference due to Mica sheet = $(\mu-1)t$

Number of fringes shift = $\frac{(\mu-1)t\times D/d}{(\lambda D/d)}=(\mu-1)t/\lambda$

12. For a photoelectric setup, threshold frequency is f_0. For incident frequency of $2f_0$, stopping potential is V_1 for incident frequency of $5f_0$, stopping potential is V_2. Find $\frac{V_1}{V_2}$.

A. $1/5$
B. $1/2$
C. $1/3$
D. ¼

Answer (D)

Solution:

$$eV_1 = h(2f_0) - hf_0 = hf_0$$

$$eV_2 = h(5f_0) - hf_0 = 4hf_0$$

$$\frac{V_1}{V_2} = \frac{f_0}{4f_0} = \frac{1}{4}$$

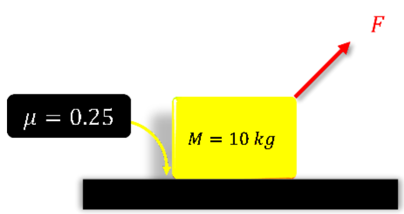
13. A block is acted upon by a force F as shown. If $M = 10\ kg$ and coefficient of friction is 0.25, find minimum F so that block slides.

A. $\frac{200}{4\sqrt{3}+1}\ N$

B. $\frac{200}{4\sqrt{3}-1}\ N$

C. $\frac{100}{4\sqrt{3}+1}\ N$

D. $50\ N$



Answer (A)

Solution:

$$F \sin 30° + \mathrm{N} = Mg$$
$$F \cos 30° = \mu \mathrm{N}$$
$$\Rightarrow F = \frac{200}{1 + 4\sqrt{3}}\ N$$

14. If universal gravitational constant(G), Plank's constant(h) and speed of light (c) are taken as fundamental quantities then dimension of mass is equal to

A. $\sqrt{\frac{Gh}{c}}$

B. $\sqrt{\frac{G}{hc}}$

C. $\sqrt{\frac{h}{Gc}}$

D. $\sqrt{\frac{hc}{G}}$

Answer (D)

Solution:

$[m] = [G]^x[h]^y[c]^z$

$[M] = [M^{-1}L^3T^{-2}]^x[ML^2T^{-1}]^y[LT^{-1}]^z$

So, $y - x = 1 \ldots\ldots\ldots..(1)$

$3x + 2y + z = 0 \ldots\ldots\ldots\ldots.(2)$

$-2x - y - z = 0 \ldots\ldots\ldots..(3)$

On solving (1), (2) and (3)

$$x = -\frac{1}{2}, y = \frac{1}{2}, z = \frac{1}{2}$$

$$m = \sqrt{\frac{hc}{G}}$$

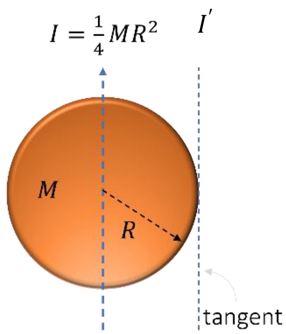
15. For uniform disc, moment of inertia about diameter is $\frac{MR^2}{4}$, where M is mass and R is radius of disc. Find the moment of inertia about tangent parallel to diameter.

A. $\frac{3}{4}MR^2$
B. $\frac{5}{4}MR^2$
C. $\frac{3}{2}MR^2$
D. $\frac{5}{2}MR^2$

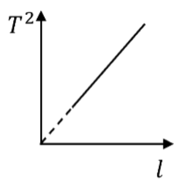
Answer (B)

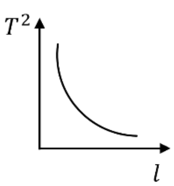
Solution:

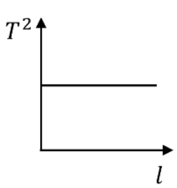
$$I' = \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2$$

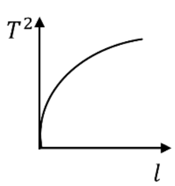


16. Which of the following graphs best represents the relation between square of time period and length of a simple pendulum?

A.


B.


C.


D.


Answer (A)

Solution:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T^2 = \frac{4\pi^2}{g} l$$

Thus, graph between T^2 and l is a straight line passing through origin.

17. A uniform wire of resistance R is folded into a regular polygon of n sides. Find the equivalent resistance of this system between any two adjacent points.

A. $\frac{n-1}{n} R$

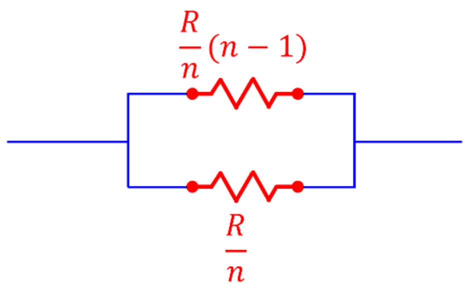
B. $\frac{n-1}{n^2} R$

C. $\frac{n-1}{n^3} R$

D. $\frac{n+1}{n^2} R$

Answer (B)

Solution:



$$R_{eq} = \frac{\frac{R}{n}(n-1) \times \frac{R}{n}}{\frac{R}{n}(n-1) + \frac{R}{n}}$$

$$= \frac{(n-1)\frac{R}{n}}{n} = \frac{n-1}{n^2} R$$

18. Which of the following is correct for $zener$ diode.

1) It acts as $voltage$ regulator.
2) It is used in $forward\ bias$.
3) It is used in $reverse\ bias$.
4) It is used as switch in series.

A. $(1)\ and\ (4)$
B. $(2)\ and\ (3)$
C. $(1)\ and\ (3)$
D. $(2)\ and\ (4)$

Answer (C)

Solution:

Zener diode acts as voltage regulator. It is used in reverse bias.

19. Choose the correct statement regarding a ground-to-ground projectile:

A. Kinetic energy is zero at highest point.
B. Potential energy is highest at highest point.
C. Horizontal component of velocity increases.
D. Vertical component of velocity remains constant.

Answer (B)

Solution:

Potential energy is highest at maximum height.

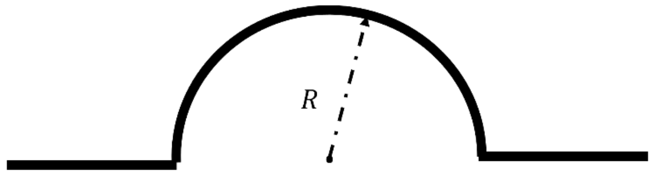
20. The electromagnetic wave, the ratio of energy caried by electric field to that by magnetic field is

Answer (1)

Solution:

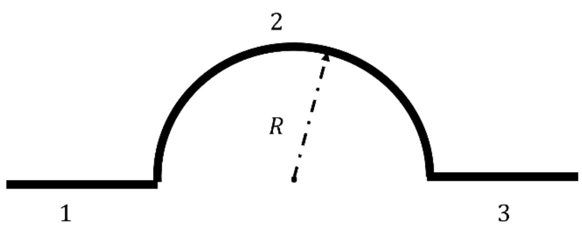
Both electric field and magnetic field carries same energy.

21. An infinite wire is bent in the shape as shown in the figure with portion AOB being semi-circular of radius R. If current i flows through the wire, then magnetic field at the centre O is equal to $\frac{\mu_0 i}{kR}$. Value of k is equal to



Answer (4)

Solution:



Magnetic field due to section 1 and 3 of the wire will be zero as centre is in the line of the wire, therefore field will be due to section 2 only.
Thus,

$$B = \frac{\mu_0 i}{4\pi R} \times \pi = \frac{\mu_0 i}{4R}$$

22. If a force F applied on an object moving along y-axis varies with y-coordinate as

$$F = 3 + 2y^2$$

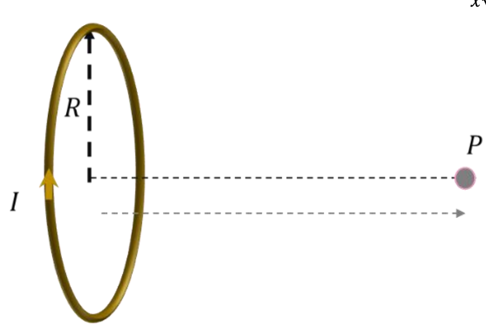
The work done in displacing the body from $y = 2\ m$ to $y = 5m$ is __________ J.

Answer (87)

Solution:

$$\text{work done} = \int_{y_1}^{y_2} F\, dy$$
$$= \int_2^5 (3 + 2y^2) dy$$
$$= \left[3y + \frac{2}{3}y^3\right]$$
$$= 15 + \frac{250}{3} - 6 - \frac{16}{3}$$
$$= 9 + \frac{234}{3}$$
$$= 87\, J$$

23. The magnetic field induced at point P on axis as shown in figure is $\frac{\mu_0 I}{x\sqrt{5}R}$. Find x



Answer (10)

Solution:

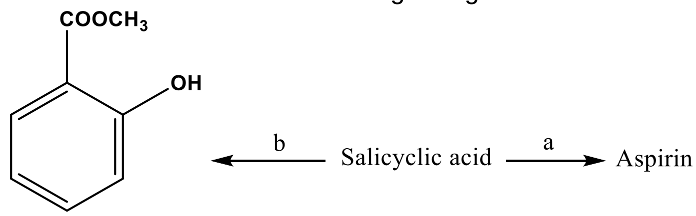
$$B_P = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{2\mu}{(R^2+r^2)^{\frac{3}{2}}}\right)$$
$$= \left(\frac{\mu_0}{2\pi}\right)\left(\frac{I \times \pi R^2}{(R^2+r^2)^{3/2}}\right)$$
$$= \frac{\mu_0 I R^2}{2(R^2+r^2)^{3/2}}$$

As $r = 2R$

$$B_P = \frac{\mu_0 I R^2}{2(R^2+4R^2)^{3/2}} = \frac{\mu_0 I}{10\sqrt{5}\,R}$$

CHEMISTRY

1. Find out the correct statement regarding a and b.



A. (a) Methanol/H^+
(b) Ethanoic anhydride

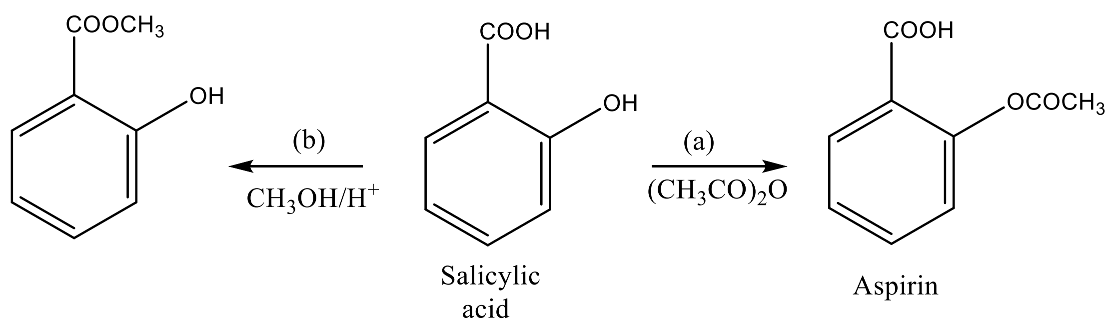
B. (a) Ethanol/H^+
(b) Ethanoic anhydride

C. (a) Ethanoic anhydride
(b) Methanol/H^+

D. (a) Ethanoic anhydride
(b) Ethanol/H^+

Answer (C)

Solution:



2. Assertion: Gypsum is used to slow down the setting of cement.
Reason: Gypsum is unstable at higher temperatures

A. Both (A) and (R) are correct
B. (A) is correct and (R) is incorrect
C. (A) is incorrect and (R) is correct
D. Both (A) and (R) are incorrect

Answer (A)

Solution:

Gypsum is added in small amount to slow down the setting of cement. So, assertion is correct.

Gypsum is thermally unstable at high temperature as it undergoes dehydration at $373\ K$ to form calcium sulphate hemihydrate and upon heating above 373 K it converts to dead burnt plaster ($CaSO_4$.

So, Reason is correct.

3. Compare the enthalpy of vaporization (ΔH_{vap}) for H_2O, D_2O, and T_2O.

A. $H_2O > D_2O > T_2O$
B. $H_2O > T_2O > D_2O$
C. $T_2O > D_2O > H_2O$
D. $T_2O > H_2O > D_2O$

Answer (C)

Solution:

Enthalpy of vaporization (ΔH_{vap}) α strength of intermolecular force of attraction.

And strength of intermolecular forces of attraction is α mass

Therefore, the correct order of enthalpy of vaporization (ΔH_{vap}) is $T_2O > D_2O > H_2O$.

4. Consider the following reaction
$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$
Select the incorrect statement about the above equilibrium reaction

A. On adding He gas at constant volume, equilibrium shift in forward reaction
B. On adding He gas at constant pressure, equilibrium shift in forward reaction
C. On adding He gas at constant pressure, equilibrium shift in backward reaction
D. On adding He gas at constant volume, equilibrium shift in backward reaction

Answer (B)

Solution:

On adding He gas at constant volume equilibrium remains unaffected.

On adding He gas at constant pressure equilibrium shift in that direction which number of gaseous molecules are greater.

Hence the correct answer is option (B).

5. Identify the correct order of bond dissociation energy of halogens.

A. $F_2 > Cl_2$
B. $Br_2 > F_2$
C. $I_2 > F_2$
D. $Br_2 > Cl_2$

Answer (B)

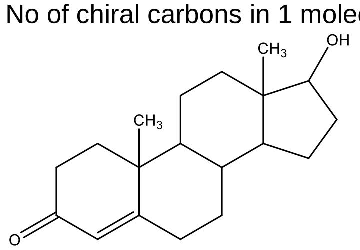
Solution:

The correct bond dissociation energy of halogens is

$Cl_2 > Br_2 > F_2 > I_2$

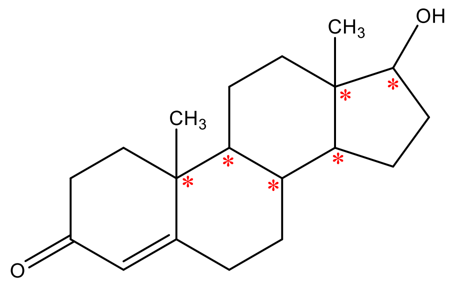
The bond dissociation energy of F_2 is less than Cl_2 *and* Br_2 because of lp – lp repulsions in case of F_2.

6. No of chiral carbons in 1 molecule of testosterone.



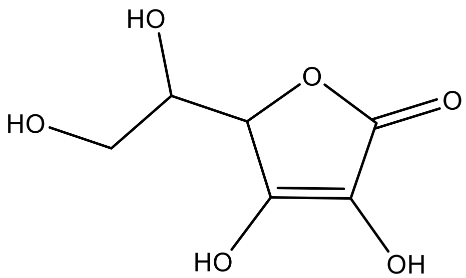
Answer (6)

Solution:



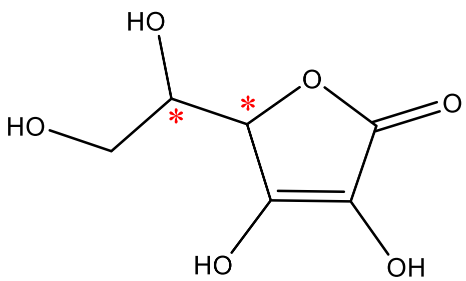
Testosterone

7. Find the number of asymmetric carbons in structure of Vitamin C.



Answer (2)

Solution:



2 – Chiral Carbons

8. For a first order reaction, half life ($t_{1/2}$) is 50 min, find $t_{3/4}$ (in minutes) of the reaction?

Answer (100)

Solution:

$t_{3/4}$ is the time taken for consumption of 3/4th of the reactant and it is equal to 2 times the , $t_{1/2}$.

$1 \xrightarrow{t_{1/2}} \frac{1}{2} \xrightarrow{t_{1/2}} \frac{1}{4}$

Therefore, $t_{3/4}$ will be 100 minutes.

9. Which of the following option is Nessler's reagent?

A. $K_2[HgI_4]$

B. $K_2Cr_2O_7$

C. $K_4[Fe(CN)_6]$
D. $K_3[Cu(CN)_4]$

Answer (A)

Solution:

Nessler's reagent is $K_2[HgI_4]$

10. Find out depression in freezing point (ΔT_f) for CH_3COOH (α = 20%) dissolved in aqueous solution having 10% (w/w) CH_3COOH in solution. Given K_f of water = $1.86 \frac{K.kg}{mole}$

A. 4.13 K
B. 2.13 K
C. 1.13 K
D. 0.13 K

Answer (A)

Solution:

$(\Delta T_f) = (i)(K_f)(m)$
$(\Delta T_f) = (i)(1.86)(m)$
Let's calculate molality
$$m = \frac{\%w/w \times 10 \times W_{solution}}{MM_{solute} \times W_{solvent}}$$

$$molality = \frac{10 \times 10 \times 100}{(60)(90)} = \frac{100}{54}$$

Let's calculate vant Hoff's factor (i)

$CH_3COOH \rightleftharpoons CH_3COO^- + H^+$

1	0	0
(1 - $\propto$)	$\propto$	$\propto$

$$i = \frac{total\ final\ moles}{total\ initial\ moles} = \frac{1+\propto}{1} == 1.2$$

$$= (1.2) \times (1.86) \times \frac{100}{54}$$

$$= 4.133\ K$$

11. The spin only magnetic moment of Mn^{2+} in $[Mn(H_2O)_6]^{2+}$ is:

A. 2.87 B.M.
B. 3.87 B.M.
C. 5.91 B.M.
D. 1.73 B.M.

Answer (C)

Solution:

Mn^{2+} in $[Mn(H_2O)_6]^{2+}$ has $(t_{2g})^3(e_g)^2$ configuration. Thus, total unpaired electrons are 5.

Hence, spin only magnetic moment = $\sqrt{5(5+2)} = 5.91\ B.M.$

12. Consider the H_2O_2 and O_2F_2 molecules where X and Y are O-O bond length in H_2O_2 and O_2F_2 respectively. Compare X and Y.

A. X > Y

B. X < Y

C. X = Y

D. X and Y cannot be compared

Answer (A)

Solution:

Both H_2O_2 and O_2F_2 have open book like structure. According to Bent's rule, the more electronegative atom in a molecule extracts higher p-character. In H_2O_2, O atom is more electronegative than H-atom and hence extracts higher p-character. In O_2F_2, F atom is more electronegative than O atom and hence extracts higher p-character. Therefore, O-atom in O_2F_2 will have highest s-character. Hence, O-O bond length in H_2O_2 (X) will be more than O-O bond length in O_2F_2 (Y).

13. Which of the following act as a tranquilizer.

A. Aminoglycoside

B. Chloramphenicol

C. Aspirin

D. Valium

Answer (D)

Solution:

Aminoglycoside - Antibiotic

Chloramphenicol – Antibiotic

Aspirin - Analgesic

Valium - Tranquilizer

14. Which of the following order is correct regarding magnitude of first electron gain enthalpy.

A. Cl < F

B. O < S

C. Te < O

D. S < Se

Answer (B)

Solution:

ΔH_{ege} decreases down the group due to decrease in Z_{eff}

ΔH_{ege} also decreases due to interelectronic repulsions.

Therefore, the expected order in case of Group-16 elements is O > S > Se > Te but due to interelectronic repulsions in O the actual order becomes S > Se > Te > O.

The expected order in case of Group-17 elements is F > Cl > Br > I but due to interelectronic repulsions in F the actual order becomes Cl > F > Br > I.

15. Which of the following given complexes has 2 isomers.

A. $[Co(NH_3)_5NO_2]^{2+}$

B. $[Co(NH_3)_5Cl]^{2+}$

C. $[Co(NH_3)_5I]^{2+}$
D. $[Co(NH_3)_5Br]^{2+}$

Answer (A)

Solution:

$[Co(NH_3)_5NO_2]^{2+}$ can show linkage isomerism. So, the correct answer is option(A).

16. Which of the following industry contributes maximum to global warming?

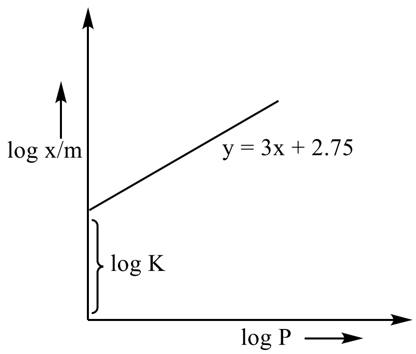
A. Oil industry
B. Fertilizer industry
C. Paper industry
D. Ice factory

Answer (A)

Solution:

Oil industry contributes maximum to the global warming.

17. Consider the given graph. Find the value of $\frac{1}{n} + \log K$



A. 2.75
B. 3.75
C. 6.75
D. 5.75

Answer (D)

Solution:

$log\frac{x}{m} = \log K + \frac{1}{n}\log P$

On comparison with y = 3x +2.75

We have, log K = 2.75

$$\frac{1}{n} = 3$$

$$\frac{1}{n} + \log K = 3 + 2.75 = 5.75$$

18. Which of the following reactions will not result in the formation of H_2O_2

A. $BaO_2.8H_2O(s) + H_2SO_4(aq)$

B. $2 - ethylanthraquinol \xrightarrow{O_2}$

C. $KO_2 + H_2O \rightarrow$

D. $Na_2O + H_2O \rightarrow$

Answer (D)

Solution:

$BaO_2.8H_2O + H_2SO_4\ (aq) \rightarrow BaSO_4(s) + H_2O_2 + 8H_2O$

$2 - ethylanthraquinol \xrightarrow{O_2} H_2O_2 + 2 - ethylanthraquinone$

$2KO_2 + 2H_2O \rightarrow 2KOH + O_2 + H_2O_2$

$Na_2O + H_2O \rightarrow 2NaOH$

Hence the correct answer is option (D)

19. A electron in Be^{3+} goes from n=4 to n=2 . Find out energy released in eV (Ground state energy of H- atom = 13.6 eV)

E. 40.8 eV
F. 122.4 eV
G. 217. 6 eV
H. 21.17 eV

Answer (A)

Solution:

Energy released,

$= 13.6 \times Z^2 \left(\frac{1}{2^2} - \frac{1}{4^2}\right)$

$= 13.6 \times 16 \times \left(\frac{1}{4} - \frac{1}{16}\right)$

$= 13.6 \times 16\left(\frac{3}{16}\right) = 40.8\ eV$

20. The correct order of bond strength of C- C, Si -Si, Ge – Ge, Sn – Sn is

A. C-C > Si-Si > Ge-Ge > Sn-Sn

B. C-C > Si- Si > Ge- Ge ≈ Sn - Sn

C. C- C > Si -Si < Ge – Ge < Sn – Sn

D. C- C > Si – Si > Sn – Sn > Ge - Ge

Answer (A)

Solution:

Bond strength decreases on moving down for carbon family

21. Which of the following option contains all the isoelectronic species?

A. N^{3-}, O^{2-}, F, Na

B. $S^{-2}, Cl^{-}, K^{+}, Ca^{2+}$

C. NH_3, CH_4, PF_5, Na^+
D. Ne, Na^+, F, N^{3-}

Answer (B)

Solution:

$S^{-2}, Cl^-, K^+, Ca^{2+}$ all the species contain 18 electrons

22. An atom forms two lattices FCC and BCC. The edge length of FCC lattice is 2.5Å and edge length of BCC lattice is 2Å. Then find the ratio of density of FCC to density of BCC.

Answer (1)

Solution:

For FCC,

$$d_{fcc} = \frac{4 \times M}{a^3} \text{ ---------(1)}$$

For BCC,

$$d = \frac{2 \times M}{a^3} \text{ ---------(2)}$$

$$\frac{d_{fcc}}{d_{bcc}} = \frac{4 \times M}{(2.5)^3} \times \frac{(2)^3}{2 \times M} = 1.024 \approx 1$$

MATHEMATICS

1. If the term independent of x in the expansion of $\left(x^{\frac{2}{3}} + \frac{\alpha}{x^3}\right)^{22}$ is 7315, then $|\alpha|$ is:

A. 1
B. 2
C. 0
D. 3

Answer (A)

Solution:

$T_{r+1} = {}^{22}C_r \left(x^{\frac{2}{3}}\right)^{22-r} \left(\frac{\alpha}{x^3}\right)^r$

$\Rightarrow \frac{2(22-r)}{3} - 3r = 0$

$\Rightarrow 44 - 2r - 9r = 0$

$\Rightarrow r = 4$

$\therefore T_5 = {}^{22}C_4 \alpha^4 = 7315$

$\Rightarrow \alpha^4 = \frac{7315}{7315} = 1$

$\Rightarrow |\alpha| = 1$

2. The value of $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x+\frac{\pi}{4}}{2-\cos 2x} dx$ is:

A. $\frac{3\pi^2}{\sqrt{6}}$

B. $\sqrt{3}\pi^2$

C. $\frac{\pi^2}{6\sqrt{3}}$

D. $\frac{6\pi^2}{\sqrt{3}}$

Answer (C)

Solution:

$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x dx}{2-\cos 2x} + \frac{\pi}{4} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{2-\cos 2x}$

$= 0 + \frac{\pi}{4} \cdot 2 \int_0^{\frac{\pi}{4}} \frac{dx}{2-\cos 2x}$

$= \frac{\pi}{2} \int_0^{\frac{\pi}{4}} \frac{\sec^2 x dx}{1+3\tan^2 x}$

Now, $\tan x = t$

$= \frac{\pi}{2} \int_0^1 \frac{dt}{1+3t^2}$

$= \frac{\pi}{2} \left[\frac{\tan^{-1}(\sqrt{3}t)}{\sqrt{3}}\right]_0^1$

$= \frac{\pi}{2\sqrt{3}} \left(\frac{\pi}{3}\right) = \frac{\pi^2}{6\sqrt{3}}$

3. The area determined by $xy < 8$, $y < x^2$ and $y > 1$ is:

A. $4\ln 2 - \frac{14}{3}$
B. $4\ln 2 + \frac{20}{3}$
C. $8\ln 4 - \frac{14}{3}$
D. $8\ln 4 - \frac{20}{3}$

Answer (C)

Solution:

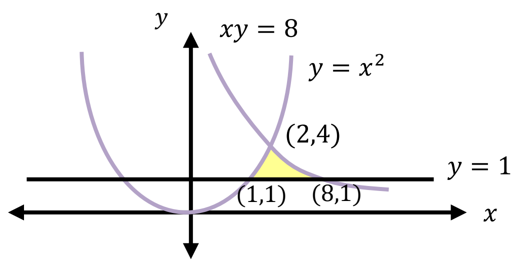
Area $= \int_1^2 (x^2 - 1)\,dx + \int_2^8 \left(\frac{8}{x} - 1\right) dx$

$= \frac{x^3}{3} - x\Big|_1^2 + (8\ln x - x)|_2^8$

$= \left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - 1\right) + (8\ln 8 - 8) - (8\ln 2 - 2)$

$= \frac{4}{3} + 8\ln 4 - 6$

$= 8\ln 4 - \frac{14}{3}$



4. If $f(x) + f\left(\frac{1}{1-x}\right) = 1 - x$, then $f(2)$ equals:

A. $\frac{1}{4}$
B. $-\frac{5}{4}$
C. $\frac{3}{4}$
D. $-\frac{3}{4}$

Answer (B)

Solution:

$f(x) + f\left(\frac{1}{1-x}\right) = 1 - x \cdots (i)$
Put $x = 2$ in (i)
$f(2) + f(-1) = -1 \cdots (ii)$
Put $x = -1$ in (i)
$f(-1) + f\left(\frac{1}{2}\right) = 2 \cdots (iii)$
Put $x = \frac{1}{2}$ in (i)
$f\left(\frac{1}{2}\right) + f(2) = \frac{1}{2} \cdots (iv)$
$(ii) + (iv) - (iii)$ gives,
$f(2) + f(-1) + f\left(\frac{1}{2}\right) + f(2) - f(-1) - f\left(\frac{1}{2}\right) = -1 + \frac{1}{2} - 2$
$\Rightarrow 2f(2) = -1 + \frac{1}{2} - 2$
$\Rightarrow 2f(2) = -\frac{5}{2}$
$\Rightarrow f(2) = -\frac{5}{4}$

5. If $f(x) = x^x, x > 0$ then $f''(2) + f'(2)$ is:

A. $10 + 12\ln 2 + 4(\ln 2)^2$
B. $10 + 4(\ln 2)^2$

C. $10 + 12\ln 2$

D. $2^{\ln 2} + (\ln 2)^2$

Answer (A)

Solution:

$$f(x) = x^x$$
$$f'(x) = x^x(1 + \ln x)$$
$$\therefore f'(2) = 4(1 + \ln 2)$$
$$f''(x) = \frac{x^x}{x} + x^x(1 + \ln x)^2$$
$$\Rightarrow f''(2) = 2 + 4(1 + \ln 2)^2$$
$$\Rightarrow f''(2) + f'(2) = 4 + 4\ln 2 + 6 + 8\ln 2 + 4(\ln 2)^2$$
$$\Rightarrow f''(2) + f'(2) = 10 + 12\ln 2 + 4(\ln 2)^2$$

6. Which of the following is a tautology?

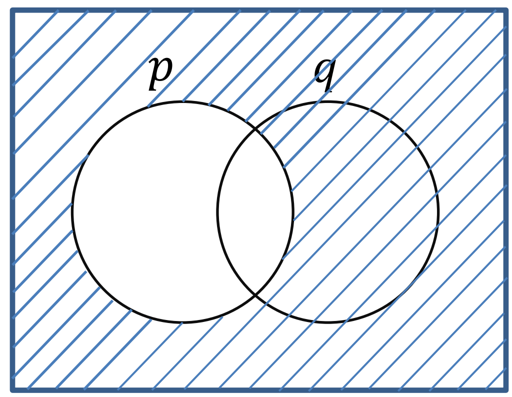
A. $p \to (\sim p \wedge q)$
B. $p \to (p \vee q)$
C. $p \to (\sim p \vee q)$
D. $p \to (\sim p \wedge \sim q)$

Answer (B)

Solution:

$a.$ $p \to (\sim p \wedge q)$

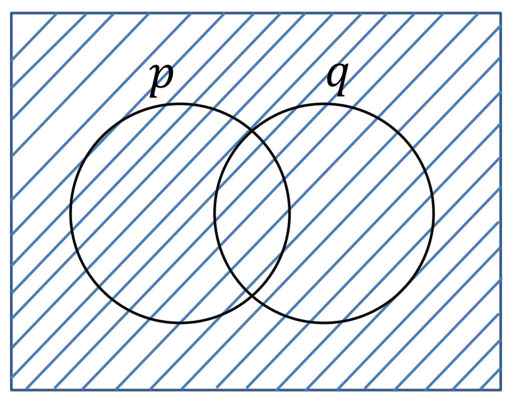
$\cong (\sim p) \vee (\sim p \wedge q)$



$b.$ $p \to (p \vee q)$

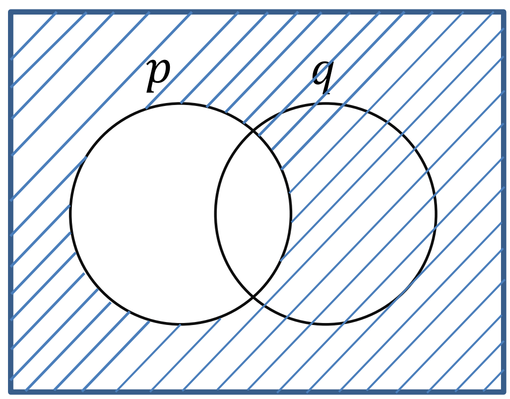
$\cong (\sim p) \vee (p \vee q)$

It can be inferred from the diagram it represents tautology.



$c.$ $p \to (\sim p \vee q)$

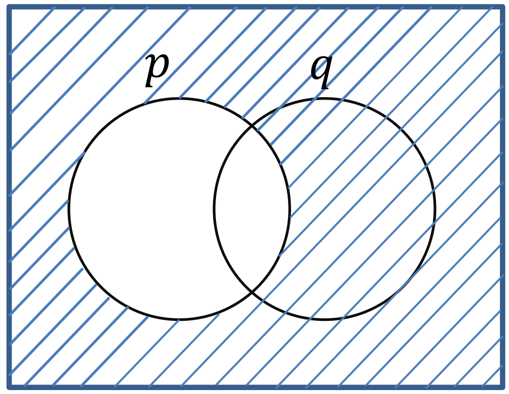
$\cong (\sim p) \vee (\sim p \vee q)$



$d.\quad p \to (\sim p \wedge \sim q)$

$\cong (\sim p) \vee (\sim p \wedge \sim q)$

$\cong (\sim p) \vee (p \vee q)'$



7. If the system of equations

$\alpha x + y + z = 1,$

$x + \alpha y + z = 1,$

$x + y + \alpha z = \beta$ has infinitely many solutions, then:

A. $\alpha = 1,\ \beta = 1$

B. $\alpha = 1,\ \beta = -1$

C. $\alpha = -1,\ \beta = -1$

D. $\alpha = -1,\ \beta = 1$

Answer (A)

Solution:

For infinite solutions

$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$

$\Rightarrow \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & 1 \\ \beta & 1 & \alpha \end{vmatrix} = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & 1 & 1 \\ 1 & \beta & \alpha \end{vmatrix} = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \beta \end{vmatrix} = 0$

Clearly $\alpha = 1 = \beta$ makes all the equations identical i.e., three coincidence planes.

$\therefore \alpha = 1 = \beta$

8. If $A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$, then which of the following is true?

A. $A^{30} = A^{25}$

B. $A^{30} + A^{25} + A = I$

C. $A^{30} - A^{25} + A = I$

D. $A^{30} = A^{25} + A$

Answer (C)

Solution:

$A = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

$A^2 = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

$$\Rightarrow A^2 = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

$$A^3 = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I$$

$(a)\ A^{30} = (A^3)^{10} = (-I)^{10} = I$

$A^{25} = (A^3)^8 \cdot A = (-I) \cdot A = A$

$\Rightarrow A^{30} \neq A^{25}$

$(b)\ \ A^{30} + A^{25} + A = I + A + A = I + 2A \neq I$

$(c)\ \ A^{30} - A^{25} + A = I - A + A = I$

$(d)\ \ A^{30} - A^{25} - A = I - A - A = I - 2A \neq 0$

9. 2 unbiased die are thrown independently. A is the event such that the number on the first die is less than second die. B is the event, such that number on the first die is even and number on the second die is odd. C is the event such that first die shows odd number and second die shows even number. Then:

A. $n\big((A \cup B) \cap C\big) = 6$

B. A and B are mutually exclusive events

C. A and B are independent events

D. $n(A) = 18,\ n(B) = 6,\ n(C) = 6$

Answer (A)

Solution:

$A = \{(1,2),\ (1,3),\ (1,4),\ (1,5),\ (1,6), (2,3),\ (2,4),\ (2,5),\ (2,6)(3,4),\ (3,5),\ (3,6)(4,5),\ (4,6), (5,6)\}$

$n(A) = 15$

$B = \{(2,1),\ (2,3),\ (2,5),\ (4,1),\ (4,3),\ (4,5),\ (6,1),\ (6,3),\ (6\ 5)\}$

$n(B) = 9$

$C = \{(1,2), (1,4),\ (1,6),\ (3,2),\ (3,4),\ (3,6),\ (5,2),\ (5,4),\ (5,6)\}$

$n(C) = 9$

$\big((A \cup B) \cap C\big) = \{(1,2), (1,4), (1,6), (3,4), (3,6), (5,6)\}$

$\Rightarrow n\big((A \cup B) \cap C\big) = 6$

$A \cap B = \{(2,3),\ (2,5),\ (4,5)\}$

$P(A \cap B) = \frac{3}{36} = \frac{1}{12}$

$P(A) = \frac{15}{36},\ P(B) = \frac{9}{36},\ P(C) = \frac{9}{36}$

$P(A) \cdot P(B) = \frac{15}{36} \cdot \frac{9}{36} = \frac{5}{48}$

$\Rightarrow A$ and B are not independent events

10. If $\frac{dy}{dx} = \frac{x^2+3y^2}{3x^2+y^2},\ y(1) = 0$, then:

A. $\frac{2x^2}{(x-y)^2} = \ln|x - y| + \frac{2x}{x-y}$

B. $\frac{2x}{(x-y)^2} = \ln|x-y| + 1$

C. $\frac{2x^2}{(x-y)^2} = \ln|x-y| + \frac{y}{x-y}$

D. $\frac{2x}{(x-y)^2} = \ln|x-y| + \frac{y}{x-y}$

Answer (A)

Solution:

$\frac{dy}{dx} = \frac{x^2+3y^2}{3x^2+y^2}$

Let $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{1+3v^2}{3+v^2}$

$\Rightarrow x\frac{dv}{dx} = \frac{1+3v^2}{3+v^2} - v = \frac{-v^3+3v^2-3v+1}{v^2+3}$

$\Rightarrow \frac{(v^2+3)}{-v^3+3v^2-3v+1}dv = \frac{1}{x}dx$

$\Rightarrow \int \frac{(v^2+3)}{(1-v)^3}dv = \frac{1}{x}dx$

$\Rightarrow \int \frac{1}{(1-v)}dv - \int \frac{2}{(1-v)^2}dv + \int \frac{4}{(1-v)^3}dv = \int \frac{1}{x}dx$

$\Rightarrow -\ln|1-v| - \frac{2}{(1-v)} + \frac{2}{(1-v)^2} = \ln|x| + C$

$\because y(1) = 0 \Rightarrow v(1) = 0 \Rightarrow C = 0$

$\therefore \frac{2}{\left(1-\frac{y}{x}\right)^2} = \ln\left|1-\frac{y}{x}\right| + \frac{2}{1-\frac{y}{x}} + \ln x$

$\Rightarrow \frac{2x^2}{(x-y)^2} = \ln|x-y| + \frac{2x}{x-y}$

11. $\vec{a} = \hat{\imath} - \hat{\jmath} + \hat{k}, \vec{b} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k}, \vec{c} = 4\hat{\imath} + 5\hat{\jmath} - \hat{k}$. If $\vec{r}.\vec{b} = 0$ and $\vec{r} \times \vec{a} = \vec{b} \times \vec{c}$, then $\vec{r}$ is equal to:

A. $-12\hat{\imath} - 8\hat{\jmath} + \hat{k}$

B. $-12\hat{\imath} - \frac{23}{3}\hat{\jmath} + \hat{k}$

C. $12\hat{\imath} + \frac{23}{3}\hat{\jmath} + \hat{k}$

D. $12\hat{\imath} + 8\hat{\jmath} + \hat{k}$

Answer (B)

Solution:

$\vec{r} \times \vec{a} = \vec{b} \times \vec{c}$

$\vec{b} \times (\vec{r} \times \vec{a}) = \vec{b} \times (\vec{b} \times \vec{c})$

$\Rightarrow (\vec{b}.\vec{a})\vec{r} - (\vec{b}.\vec{r})\vec{a} = (\vec{b}.\vec{c})\vec{b} - (\vec{b}.\vec{b})\vec{c}$

$\Rightarrow 6\vec{r} = -8(2\hat{\imath} - 3\hat{\jmath} + \hat{k}) - 14(4\hat{\imath} + 5\hat{\jmath} - \hat{k})$

$\Rightarrow 6\vec{r} = -72\hat{\imath} - 46\hat{\jmath} + 6\hat{k}$

$\Rightarrow \vec{r} = -12\hat{\imath} - \frac{23}{3}\hat{\jmath} + \hat{k}$

12. If $2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $x \in (0,\ 1)$ has:

A. 2 solutions for $x < \frac{1}{2}$

B. 2 solutions for $x > \frac{1}{2}$

C. 1 solutions for $x < \frac{1}{2}$

D. 1 solutions for $x > \frac{1}{2}$

Answer (C)

Solution:

$2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Put $x = \tan\theta$ we get,

$2\tan^{-1}\left(\tan\left(\frac{\pi}{4} - \theta\right)\right) = \cos^{-1}\cos 2\theta$

$\Rightarrow 2\left(\frac{\pi}{4} - \theta\right) = 2\theta$

$\Rightarrow \theta = \frac{\pi}{8}$

$\Rightarrow \tan^{-1} x = \frac{\pi}{8}$

$\Rightarrow x = \tan\frac{\pi}{8} = \sqrt{2} - 1$

$\therefore x = \sqrt{2} - 1$ only 1 solution for $x < \frac{1}{2}$

13. Number of non-negative integral solutions of $x + y + z = 21$ if $x \geq 1,\ y \geq 3,\ z \geq 6$ are ___________.

Answer (78)

Solution:

$\because x + y + z = 21\ \ [x \geq 1,\ y \geq 3,\ z \geq 6]$

$\Rightarrow (x-1) + (y-3) + (z-6) = 11$

$\Rightarrow x_1 + y_1 + z_1 = 11$

Where, $x_1 \geq 0,\ y_1 \geq 0,\ z_1 \geq 0$

Total ${}^{11+3-1}C_{3-1}$ solutions

${}^{13}C_2 = \frac{13!}{2!11!} = 6 \times 13 = 78$

14. Total 6 digit numbers using the digits 4, 5, 9 which are divisible by 6 are ___________.

Answer (81)

Solution:

We have,

For this, 4 will be fixed as unit place digit

		Total number
Case I:	$4's \rightarrow 6$ times	1
Case II:	$4's \rightarrow 4$ times $5's \rightarrow 1$ times $9's \rightarrow 1$ times	$\frac{5!}{3!} = 20$
Case III:	$4's \rightarrow 3$ times	

$5's \rightarrow 3$ times $\quad \frac{5!}{2!3!} = 10$

Case IV: $4's \rightarrow 3$ times
$9's \rightarrow 3$ times $\quad \frac{5!}{2!3!} = 10$

Case V: $4's \rightarrow 2$ times
$5's \rightarrow 2$ times $\quad \frac{5!}{2!2!} = 30$
$9's \rightarrow 2$ times

Case VI: $4's \rightarrow 1$ times
$5's \rightarrow 1$ times $\quad \frac{5!}{4!} = 5$
$9's \rightarrow 4$ times

Case VII: $4's \rightarrow 1$ times
$5's \rightarrow 4$ times $\quad \frac{5!}{4!} = 5$
$9's \rightarrow 1$ times

Total numbers $= 81$

15. Let $3\ A.P.'s$ be

$S_1 = 2, 5, 8, 11, \cdots 394$

$S_2 = 1, 3, 5, 7, \cdots$

And $S_3 = 2,\ 7,\ 12,\ \cdots 397$
Then sum of common terms of these three $A.P.'\ s$ is __________.

Answer (2561)

Solution:

Common terms in $S_1,\ S_2,\ S_3$ are
$= 2,\ 17,\ 32,\ 47,\ \cdots$
S_2 has all odd numbers up to 397
Common terms in $S_1,\ S_2,\ S_3$ are
$= 17,\ 47,\ 77,\ \cdots, 377$
Sum of terms $= \frac{13}{2}(17 + 377)$
$= 2561$

16. Let $f(x) = |(x-3)(x-2)| - 3x + 2$ for $x \in [1,3]$. If M and m are absolute maximum & absolute minimum value of $f(x)$, then $|m| + |M|$ equals ______.

Answer (8)

Solution:

$$|(x-3)(x-2)| = \begin{cases} (x-2)(x-3), & x \in [1,2) \\ -(x-2)(x-3), & x \in [2,3] \end{cases}$$
$$f(x) = \begin{cases} x^2 - 5x + 6 - 3x + 2, & x \in [1,2) \\ -x^2 + 5x - 6 - 3x + 2, & x \in [2,3] \end{cases}$$
$$f(x) = \begin{cases} x^2 - 8x + 8, & x \in [1,2) \\ -x^2 + 2x - 4, & x \in [2,3] \end{cases}$$
$$f'(x) = \begin{cases} 2x - 8, & x \in [1,2) \\ -2x + 2, & x \in [2,3] \end{cases}$$
$$\Rightarrow f'(x) = \begin{cases} < 0, & x \in [1,2) \\ < 0, & x \in [2,3] \end{cases}$$
$\Rightarrow f'(x)$ is strictly decreasing in $[1,3]$
$f(1) = f(x)_{max} = M = 2 - 3 + 2 = 1$

$f(3) = f(x)_{min} = m = 0 - 9 + 2 = -7$
$\therefore |m| + |M| = |-7| + |1| = 8$

17. Let $X_1,\ X_2,\ X_3,\ \cdots,\ X_7$ is an A.P such that $X_1 < X_2 < X_3 \cdots < X_7,\ X_1 = 9,\ \sigma = 4$. The value of $\bar{X} + X_6$ is equal to __________.

Answer (34)

Solution:

Let the series be $a - 3d,\ a - 2d,\ a - d,\ a,\ a + d,\ a + 2d,\ a + 3d$
$a - 3d = 9$
Now if we shift the origin, the variance remains same
$\therefore$ for $-3d,\ -2d,\ -d,\ 0,\ d,\ 2d,\ 3d$
$\Rightarrow 16 = \frac{2}{7}(9d^2 + 4d^2 + d^2) - (\bar{X})^2$
$\Rightarrow 16 = \frac{2}{7}(14)d^2 - (0)^2$
$\Rightarrow d = 2$
$a - 3d = 9$
$\Rightarrow a = 15$
$\bar{X} = 15$
$X_6 = a + 2d = 19$
$\bar{X} + X_6 = 15 + 19 = 34$